Image Gradients

Computational Photography (CSCI 3240U) & Computer Vision (CSCI 4220U)

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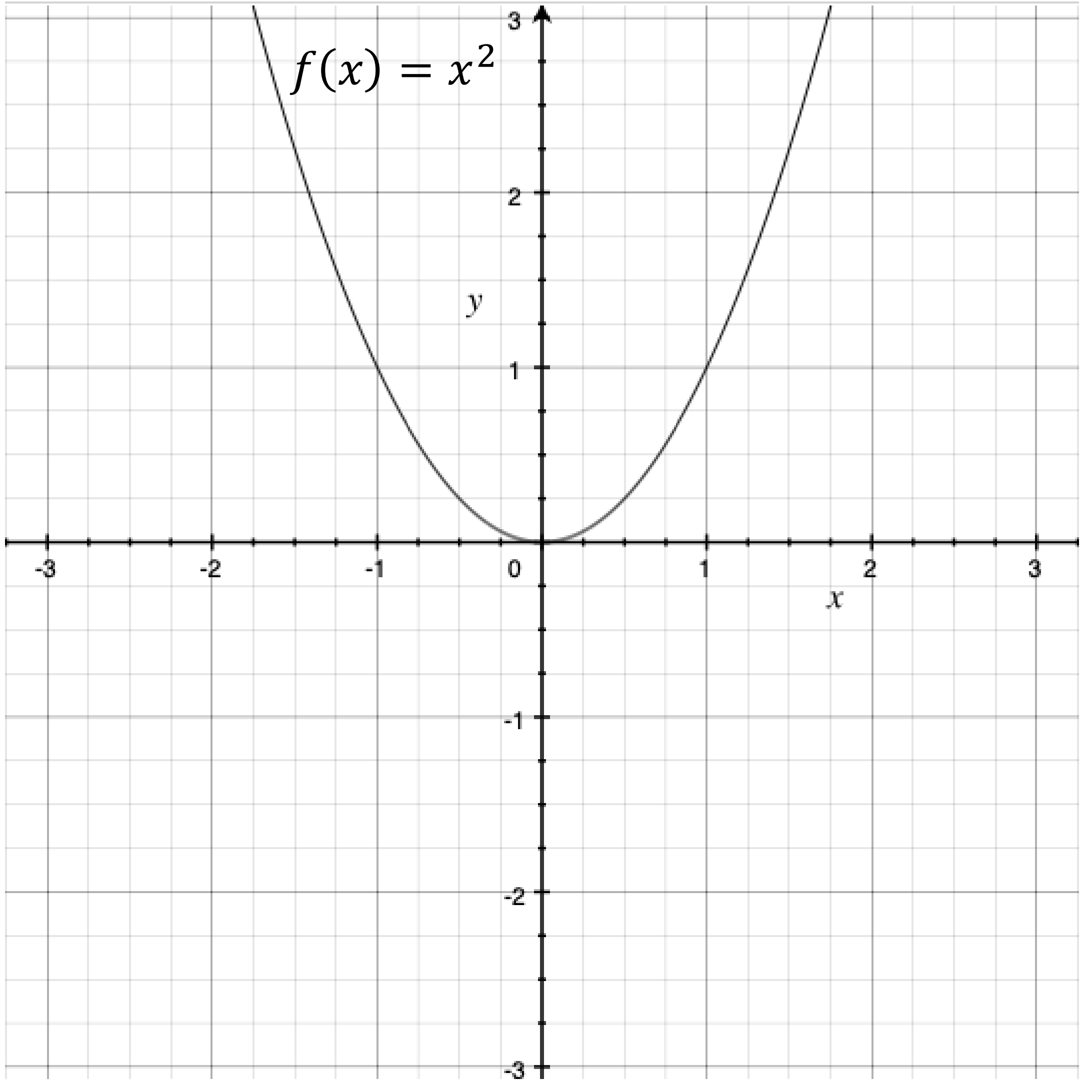
Today’s lecture

- Why do we care about image gradients?
- Computing image gradients
- Sobel filters
- Gradient magnitude and directions
- Visualizing image gradients

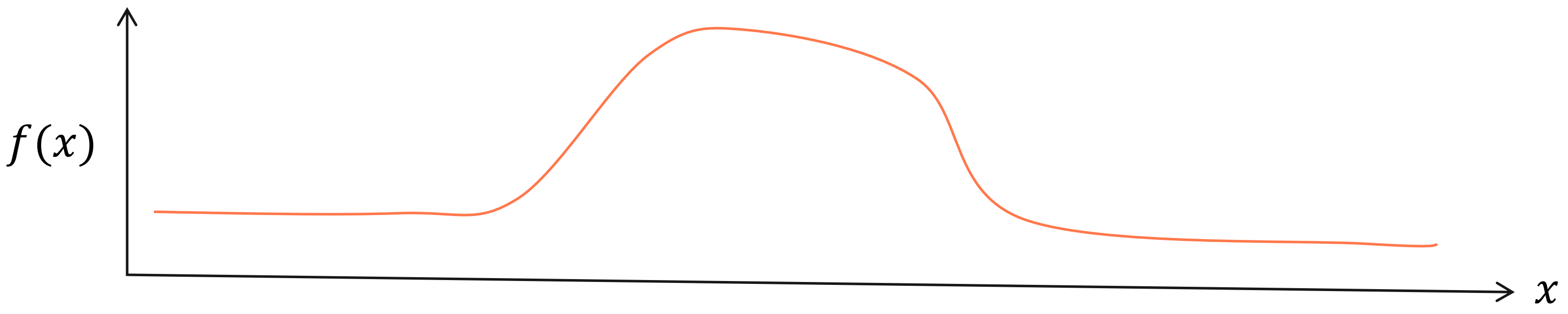
Derivative

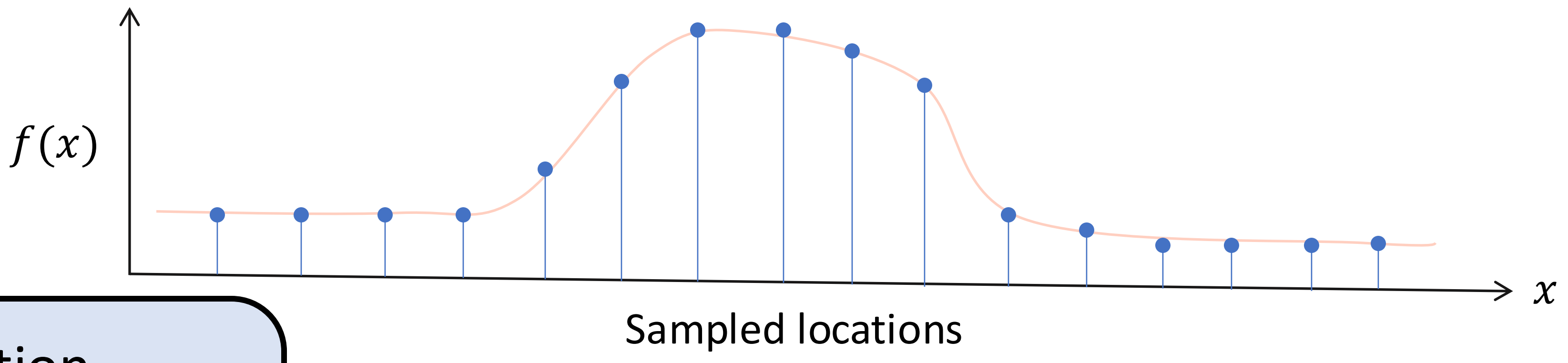
$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$



Derivative

Given a function $f(x)$, we are interested in computing $\frac{df}{dx}$





Definition

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

Finite Difference Approximation

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

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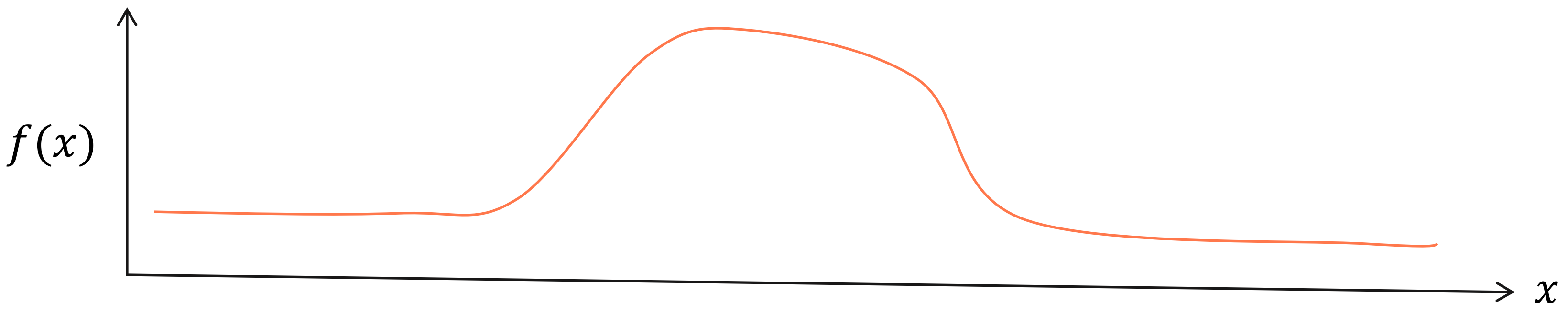

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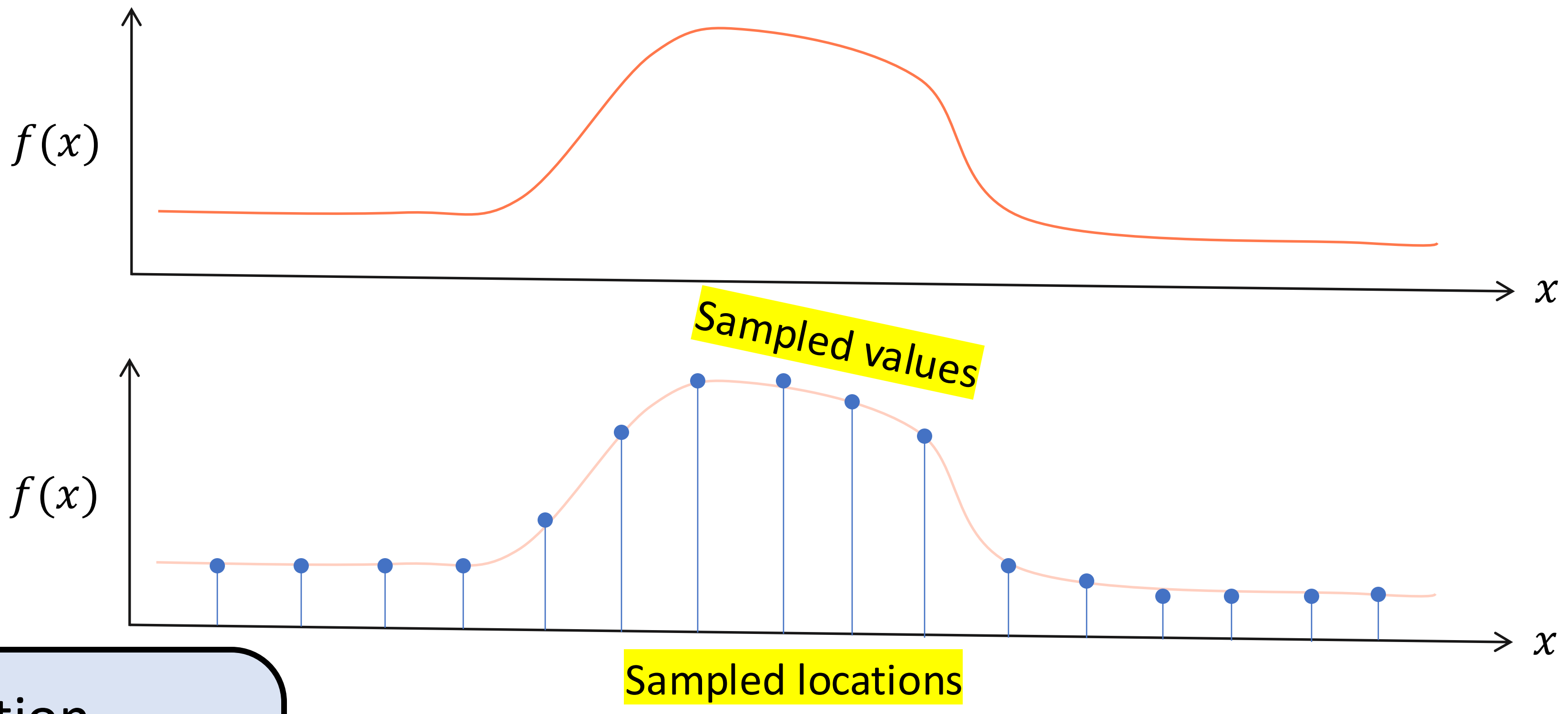

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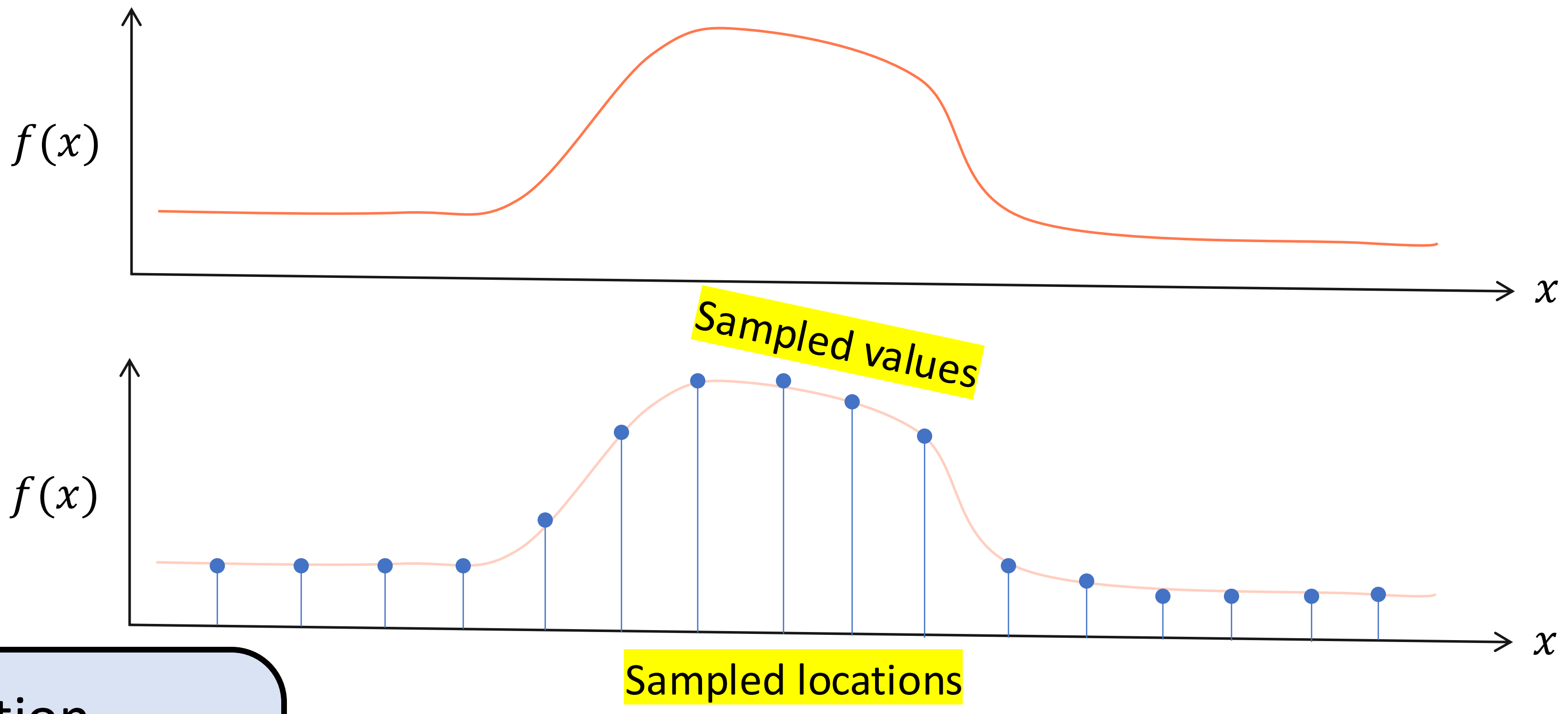

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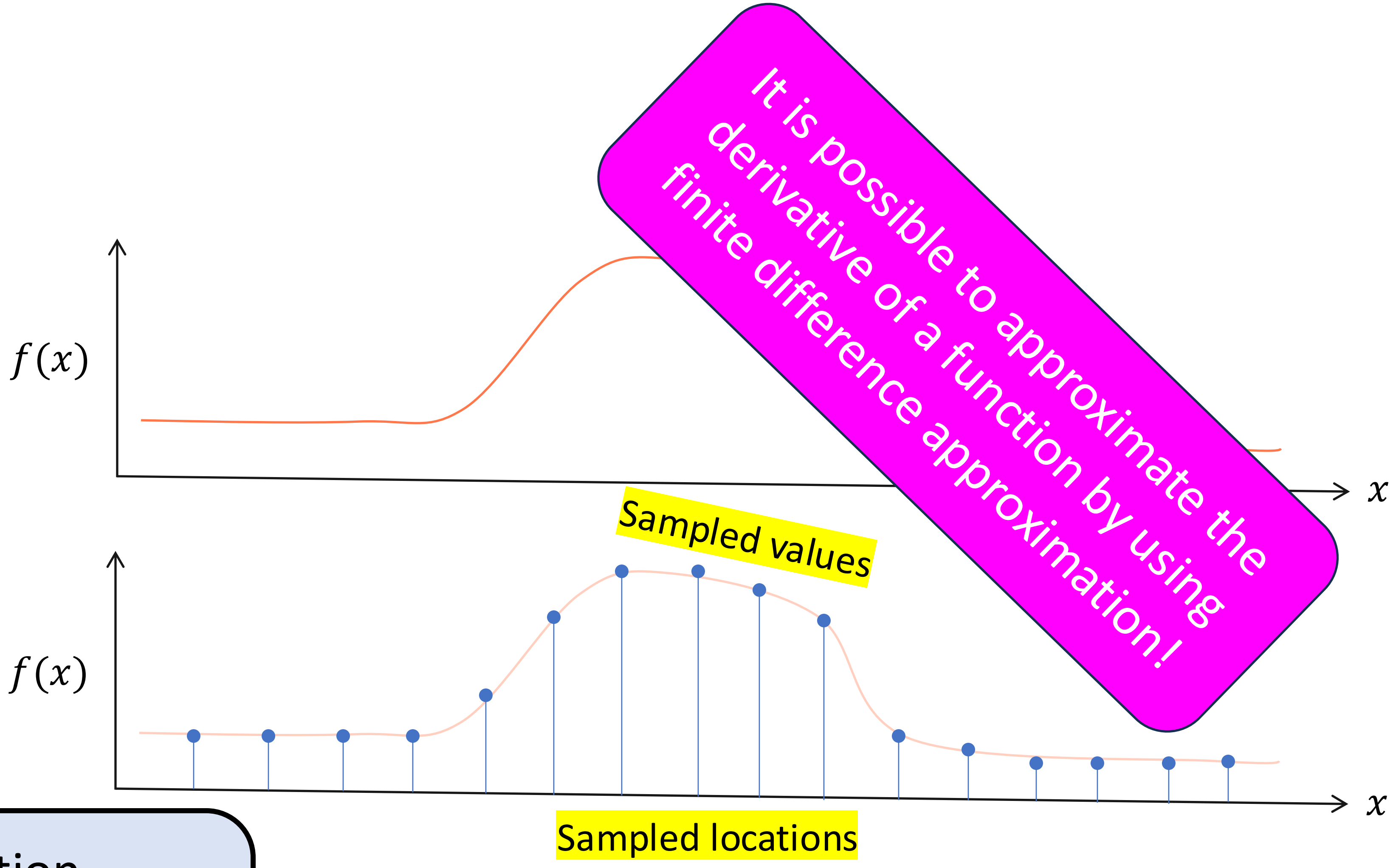

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Use finite difference approximation to compute image derivatives

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

$I =$

0	1	2	3	4	5	6
1	**1**	**9**	**8**	**6**	**0**	**0**

$I' =$

$I'' =$

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$I' =$

0	**8**	**-1**	**-2**	**-6**	**0**	**?**

$I'' =$

8	**-9**	**-1**	**-4**	**6**	**?**	**?**

Use convolution to implement finite difference approximation

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

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0	1	2	3	4	5	6

$I' =$

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$I * [1, -1] =$

Use convolution to implement finite difference approximation

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$I =$

1	**1**					**0**
0	1				5	6

Key observation: we are able to approximate the derivative of a function by convolving it with an appropriate filter. In this example, convolution with filter $[1, -1]$ results in finite difference approximation. Recall that it is a convolution operation, so the filter is flipped when applying to the signal.

$I' =$

0	**8**	**-1**	**-2**	**-6**	**0**	**?**

$I * [1, -1] =$

0	**8**	**-1**	**-2**	**-6**	**0**	**?**

Partial derivatives

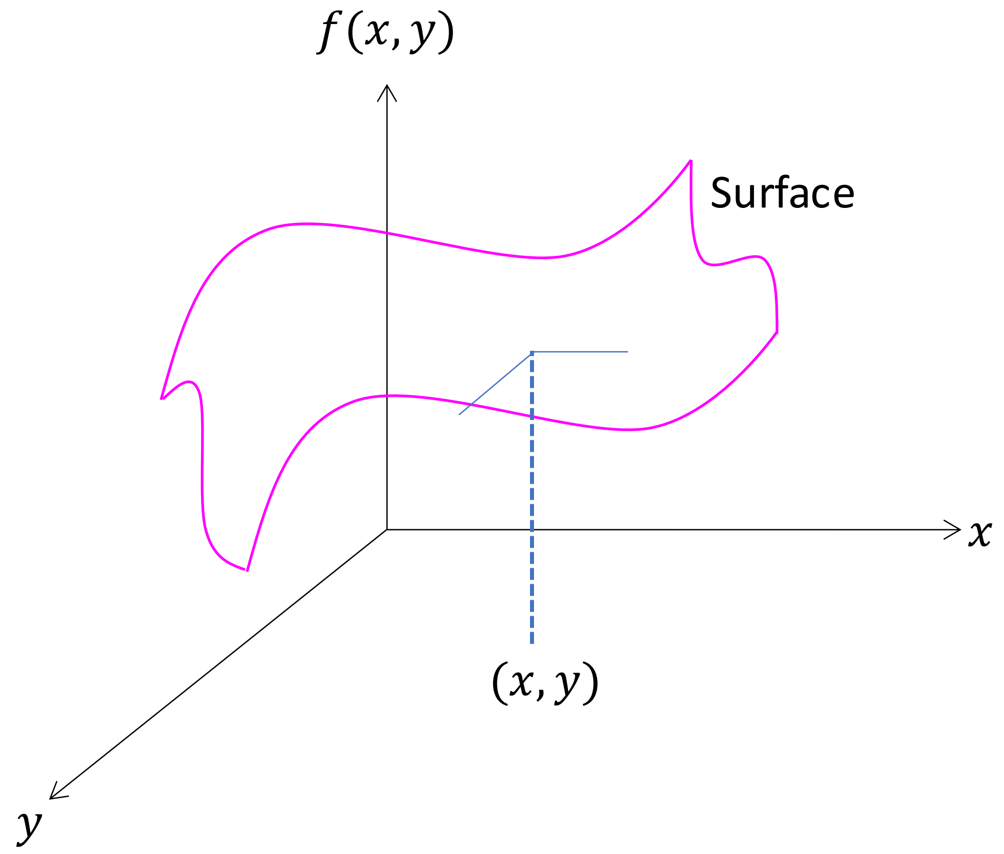


Image derivatives in x and y directions

$I =$

1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

$I_x = I * [1, -1] =$

$I_y = I * [1, -1]^T =$

Image gradient ∇I

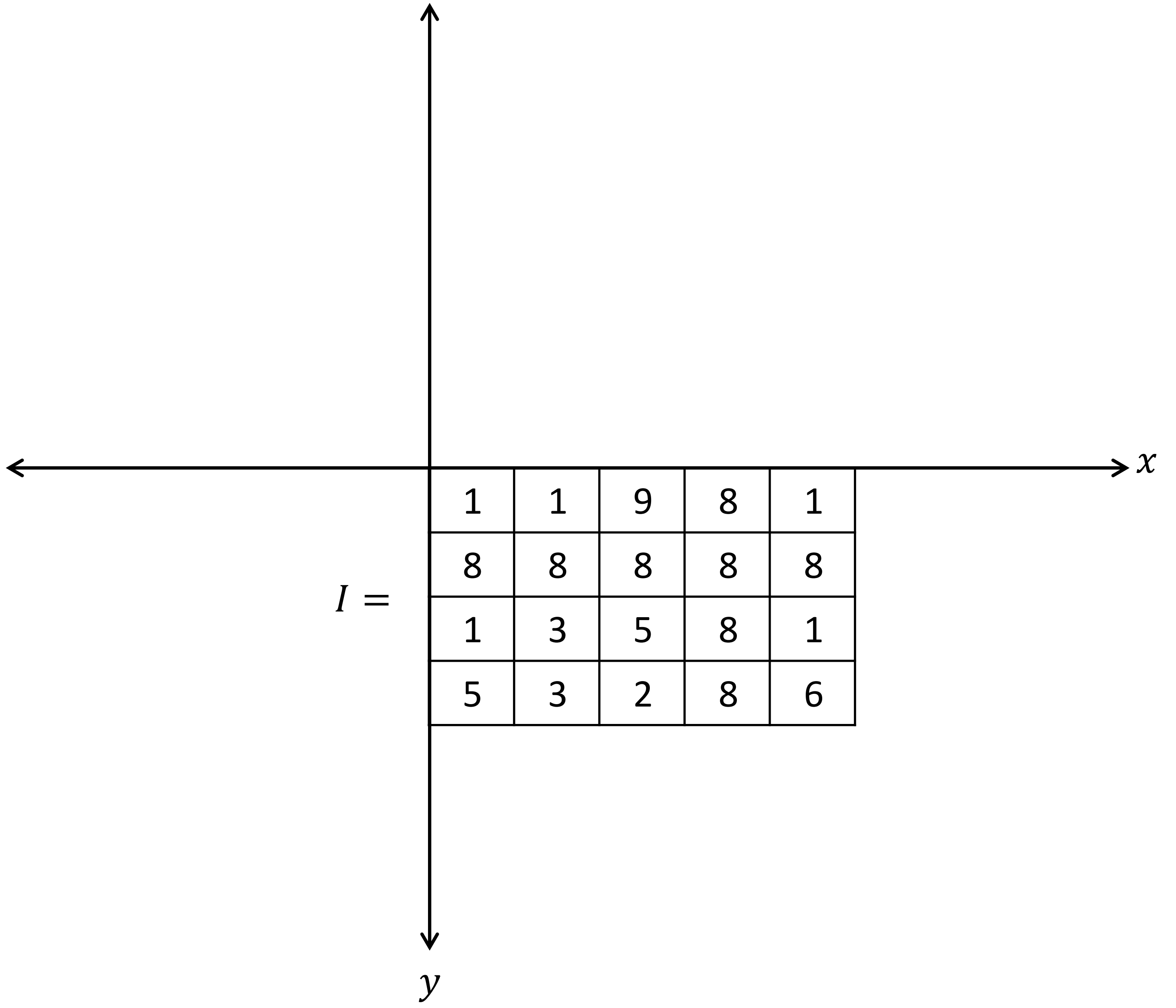
$$\nabla I = \left[\frac{\partial I(x,y)}{\partial x}, \frac{\partial I(x,y)}{\partial y}\right]$$

$I_x =$

0	8	-1	-7	
0	0	0	0	
2	2	3	-7	
-2	-1	6	-2	

$I_y =$

7	7	-1	0	7
-7	-5	-3	0	7
4	0	-3	0	5



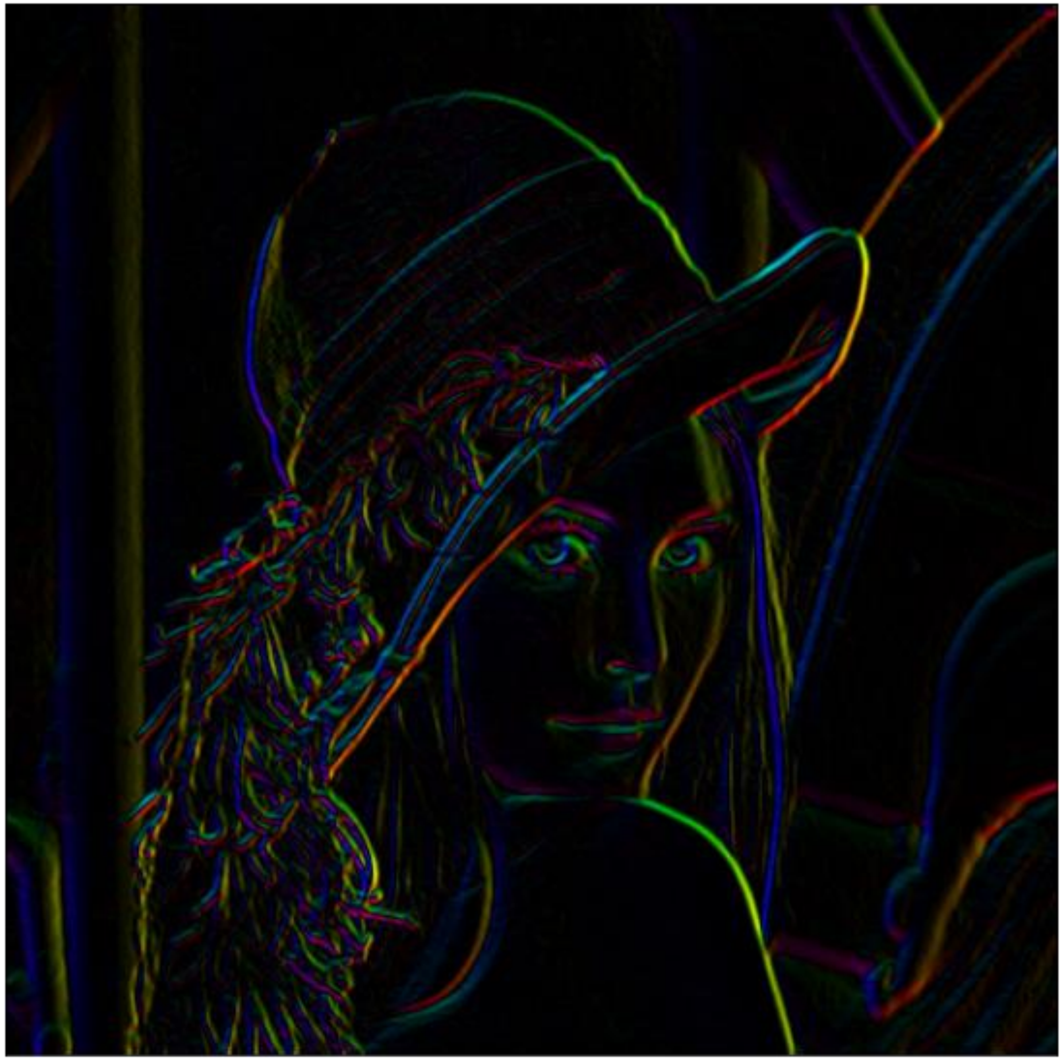
$I =$

1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

Gradient direction and magnitude

$$\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

$$\theta = \tan^{-1}\left(\frac{\partial I}{\partial y} \Big/ \frac{\partial I}{\partial x}\right)$$



Filters for computing image derivatives

Sobel

$H_x =$

1	0	-1
2	0	-2
1	0	-1

$H_y =$

1	2	1
0	0	0
-1	-2	-1

Prewire

$H_x =$

1	0	-1
1	0	-1
1	0	-1

$H_y =$

1	1	1
0	0	0
-1	-1	-1

Roberts

$H_x =$

0	-1
1	0

$H_y =$

1	0
0	-1

Image gradients

- Image derivatives and gradients highlight edge pixels

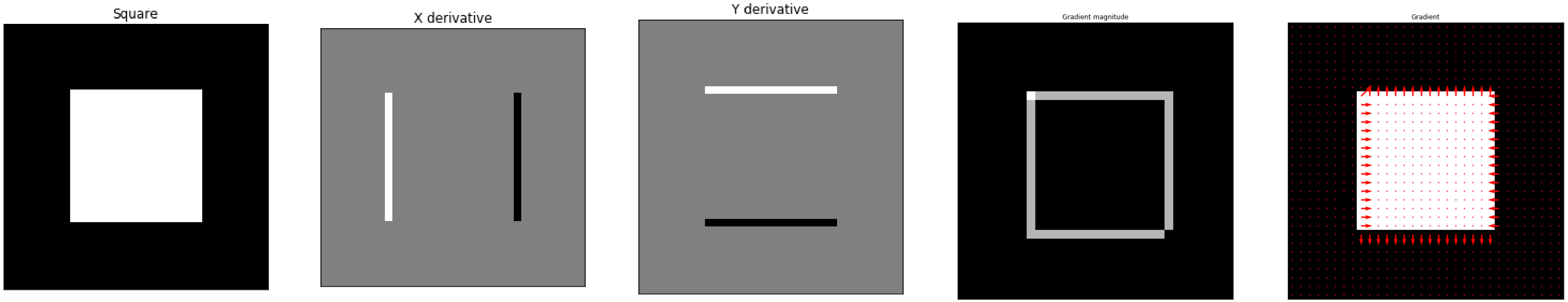
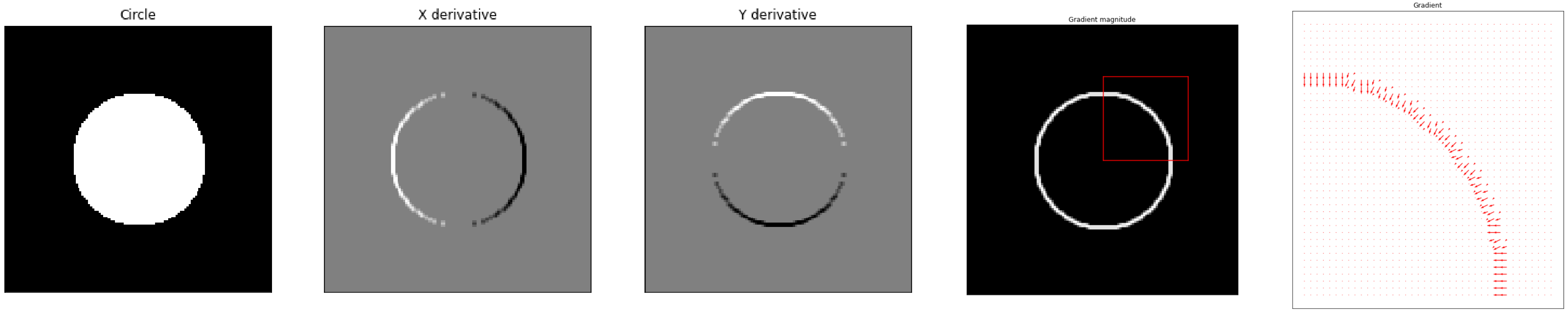


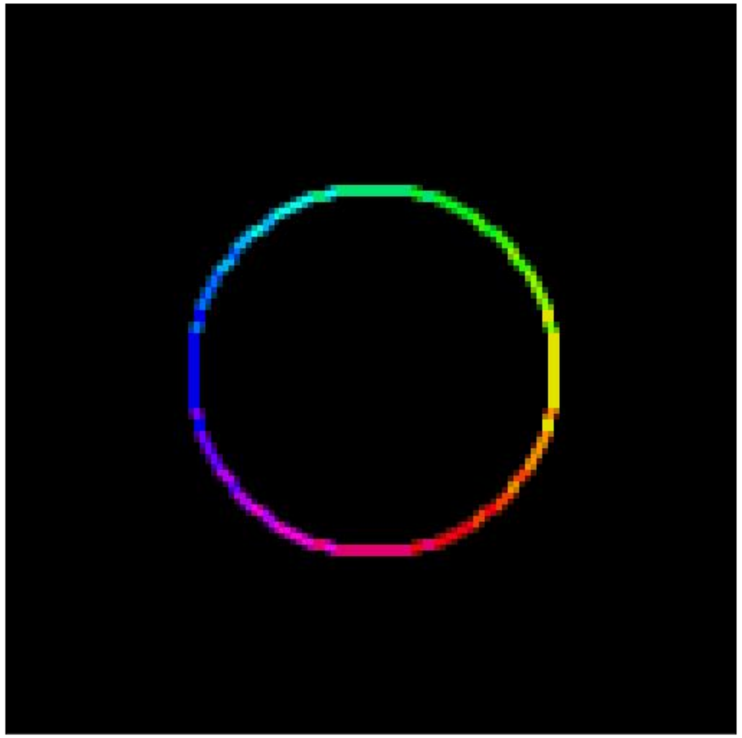
Image gradients

- Image derivatives and gradients highlight edge pixels



Visualizing image gradients

- Use color to visualize gradients (or any 2D field)
  http://csundergrad.science.uoit.ca/courses/cv-notes/notebooks/07-image-derivatives.html



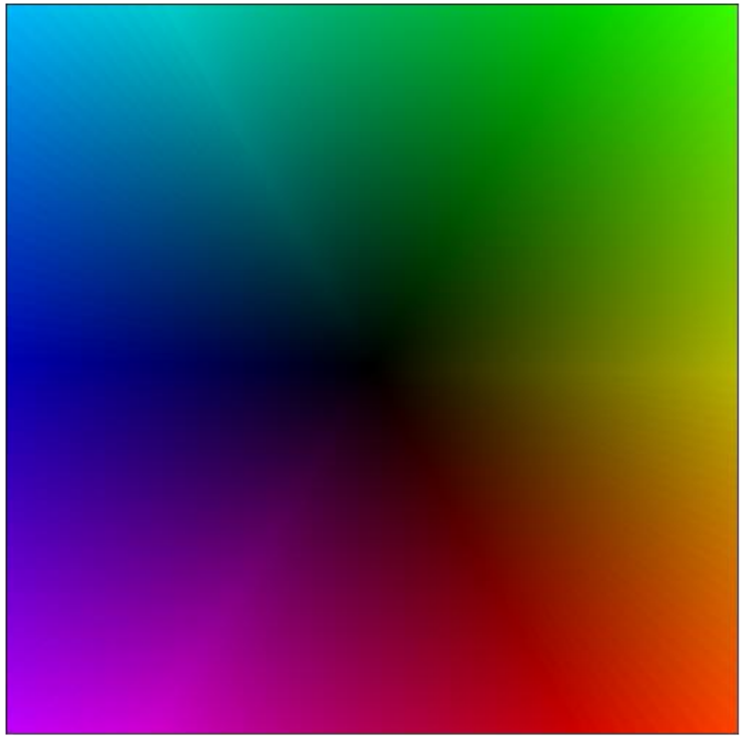
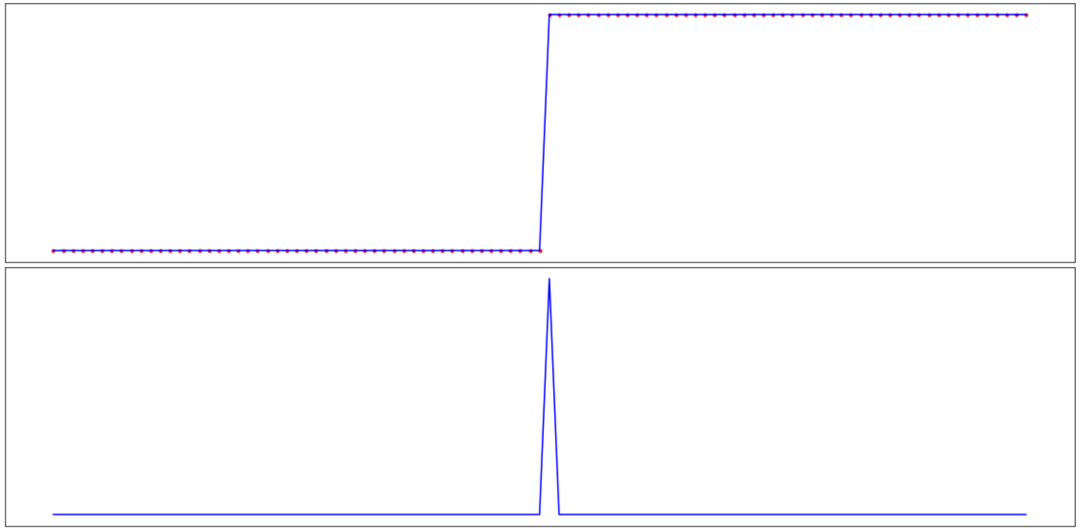
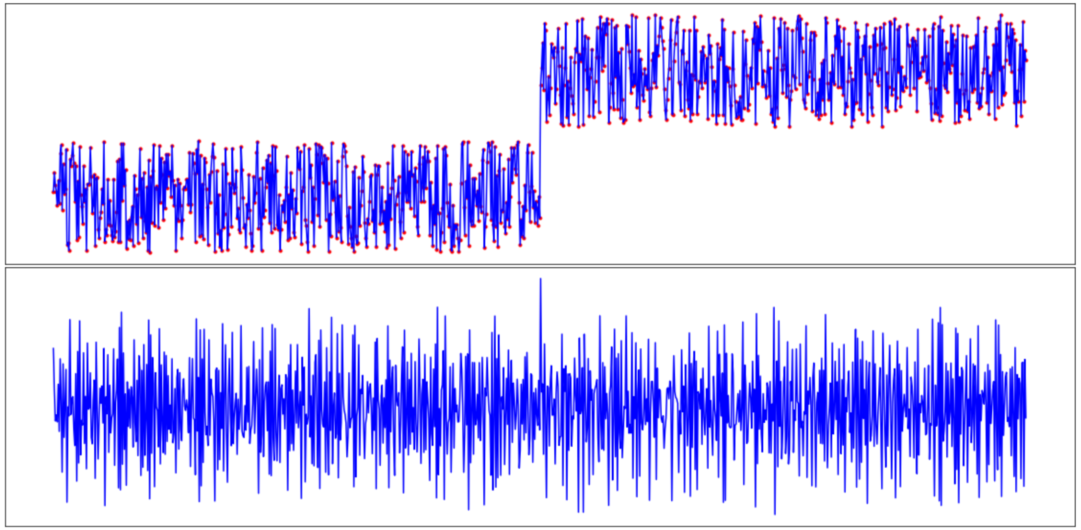


Image noise and gradients



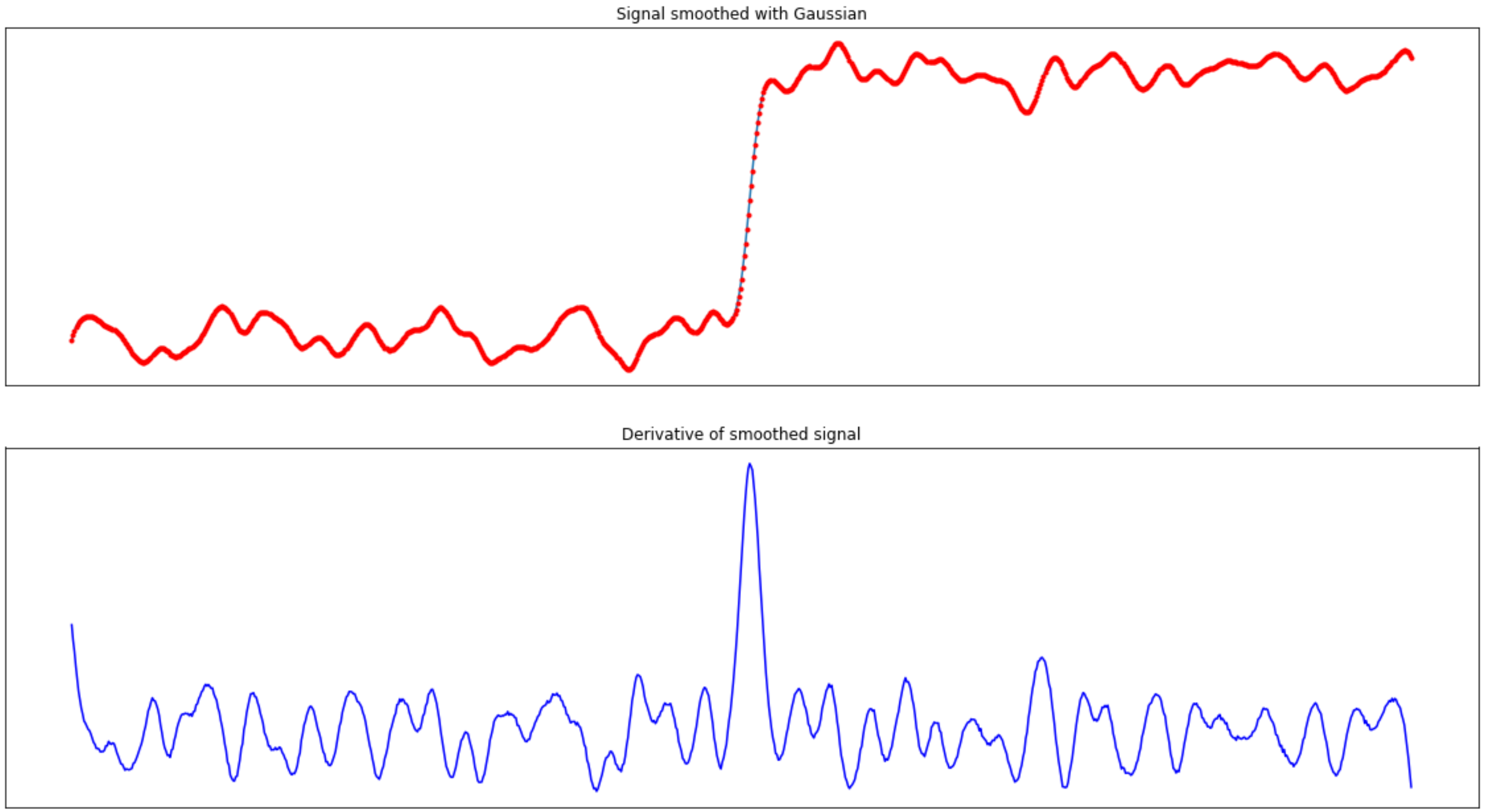
Noisy signal

Using gradient to find edges in the noisy signal



Computing gradients in practice

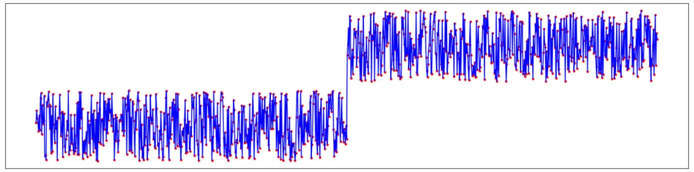
- Gaussian blur the signal before computing gradients



Computing gradients in practice

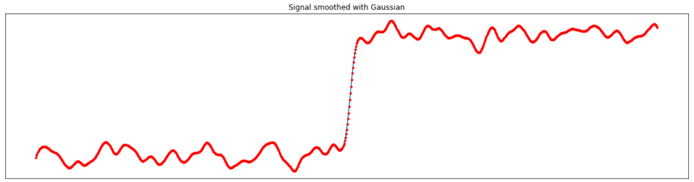
- Option 1
  - Filter the signal with a Gaussian kernel
  - Filter the signal with an appropriate kernel to compute gradient

$*$



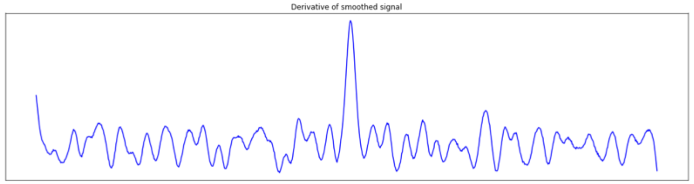
Noisy signal

$=$



After filtering with a Gaussian kernel

1st derivative of the smoothed signal



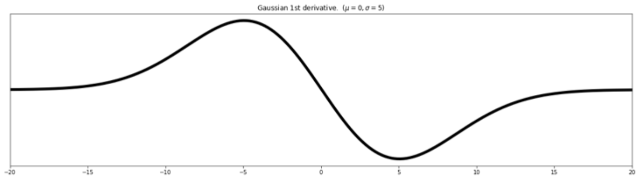
Computing gradients in practice

First derivative of the signal

- Option 2: use superposition principle
  - Compute derivative of the Gaussian filter and store the result
  - Filter the (noisy) signal with derivative of the Gaussian

Gaussian

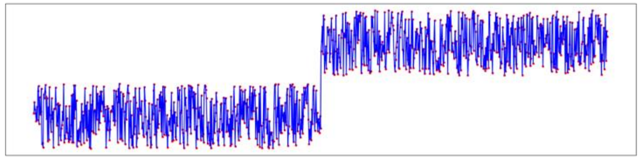
1st derivative of the Gaussian



Stored

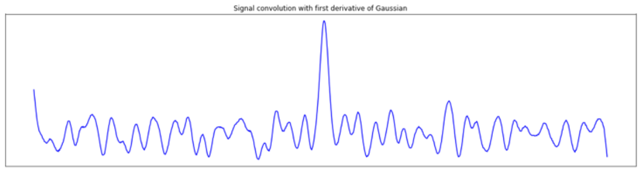
Saves one convolution at runtime

$*$



Noisy signal

$=$



1st derivative of the signal

Summary

- Image gradients
- Finite-difference approximation filters
- Gradient magnitude and direction
- Image noise and gradients