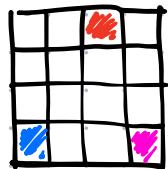
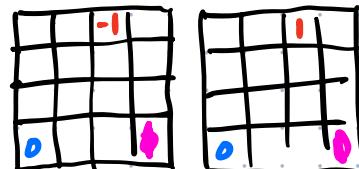


Frame t



Frame $t+1$



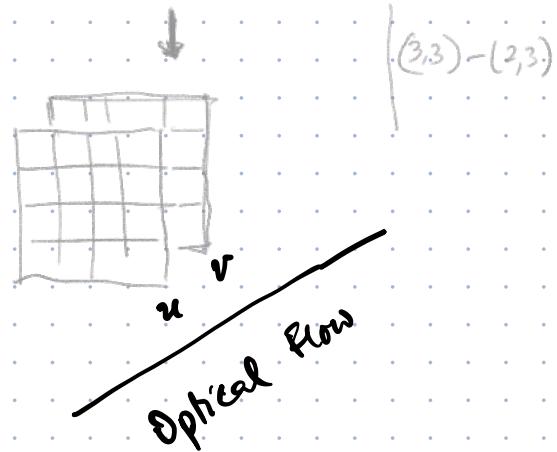
u v

loc(red pixel) at $t = (1,1)$

loc(red pixel) at $t+1 = (0,2)$

$$\text{Shift} = (0,2) - (1,1) = (-1, 1)$$

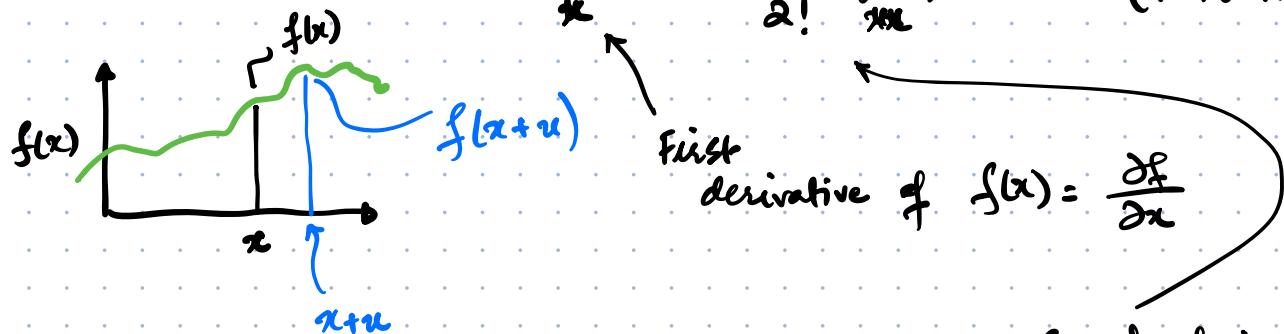
Optical Flow
(u, v)



$$\textcircled{1} \quad I(x, y, t) = \underbrace{I(x+u, y+v, t+1)}_{*}$$

Brightness Constancy
Constraint.

$$f(x+u) = f(x) + u f'(x) + \frac{u^2}{2!} f''(x) + \dots \quad (\text{TAYLOR'S SERIES})$$



Second derivative of
 $f(x) = \frac{\partial^2 f}{\partial x^2}$

$$I(x+u, y+v, t+1) = I(x, y, t) + u \frac{\partial I}{\partial x} + v \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t}$$

↑ ↑ ↑

1st term 2nd term .

$$u \frac{\partial I}{\partial x} + v \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t} = I(x+u, y+v, t+1) - I(x, y, t)$$

$\Rightarrow ①$

$$\Rightarrow u \frac{\partial I}{\partial x} + v \frac{\partial I}{\partial y} = - \frac{\partial I}{\partial t}$$

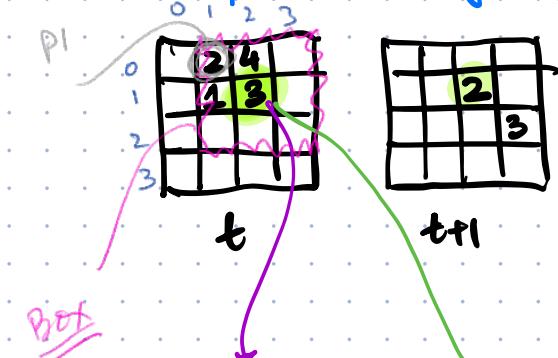
$$\Rightarrow \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \cdot (u, v) = - \frac{\partial I}{\partial t}$$

$$\Rightarrow \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

For a particular pixel.

$I_{t+1}[x,y] - I_t[x,y]$

Spatial image gradients. (Sobel Filters)



③

$$\begin{aligned} I_x &= 2 \\ I_y &= -1 \\ I_t &= -1 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -(-1)$$

$$\begin{aligned} I_x(1,2) &= I(1,2) - I(1,1) \\ &= 3 - 1 \\ &= 2 \end{aligned}$$

② Spatial Coherence

(3)

Nine Equations

$$\begin{bmatrix} I_x(p_i) & I_y(p_i) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t(p_i) \quad \text{where } i \in [1, 9]$$

$$P_1 = (0, 1)$$



$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_9) & I_y(p_9) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_9) \end{bmatrix}$$

↑
Spatial coherence.

$$\begin{array}{l} \mathbf{A} \\ \in \mathbb{R}^{9 \times 2} \end{array} \quad \begin{array}{l} \vec{x} \\ \in \mathbb{R}^{2 \times 1} \end{array} = \begin{array}{l} \vec{b} \\ \in \mathbb{R}^{9 \times 1} \end{array}$$

$$\vec{x} = (\vec{A}^T \vec{A})^{-1} \vec{A}^T \vec{b}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \dots$$

$$\nabla_I \begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

$$\Rightarrow \nabla I \begin{bmatrix} u_e + u_p \\ v_e + v_p \end{bmatrix} = -I_L$$

dot-product

$$\Rightarrow \nabla I \begin{bmatrix} u_e \\ v_e \end{bmatrix} + \nabla I \begin{bmatrix} u_p \\ v_p \end{bmatrix} = -I_L$$

The diagram illustrates the decomposition of a vector into components along and perpendicular to a given edge. A green vector labeled \vec{D} is shown originating from a point on a horizontal red line. A vertical red arrow labeled "along edge" points upwards from the line. A horizontal red arrow labeled " \perp edge" points to the right, perpendicular to the line. The vector \vec{D} is decomposed into two components: one along the edge and one perpendicular to the edge.