Jan 31, 2024

Today's lecture:

- Model fitting in the presence of outliers
- RANSAC

Robust line Fitting

1. Missing data
2. Multiple data sources

Lets fit a linear model $y = mx + b$ to this data.

Pick two pts at random.

Fit a line through these pts

Q. How well does this line represent the data?

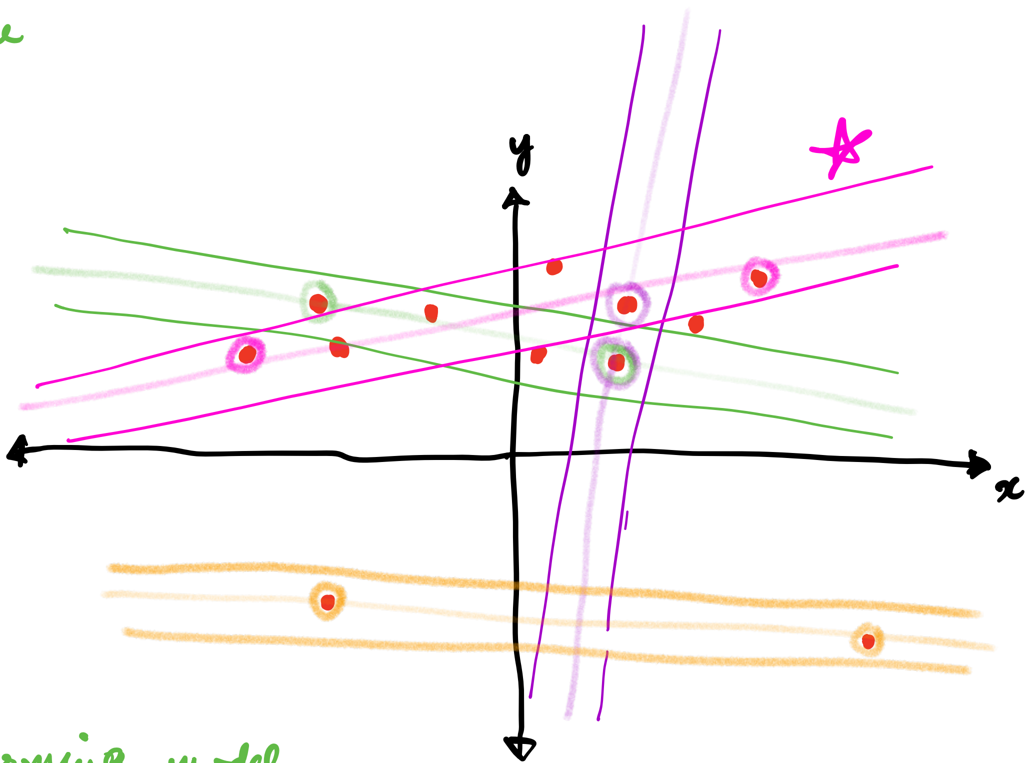
→ Count data points within some distance of the line.

Keep track of the best performing model

Repeat.

↓

Pick the best performing model



Fitting a 2D line to points (x_1, y_1) and (x_2, y_2)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \rightsquigarrow y = mx + c$$

Detours: Fit the following quadratic model $ax^2 + bx + c = y$ to data points (x_i, y_i), where $i \in [1, n]$.

① Is this a linear model?

② Can I express it as $A\vec{x} = \vec{b}$ or $A\vec{x} = \vec{0}$?

③ Can I use RANSAC to fit this model?
"Any model" can be fitted using RANSAC.

RANSAC

- Determine
  - s : the smallest number of points required for fitting
  - N : the number of iterations
  - d : threshold to decide whether or not a data points fits the model
  - T : the number of nearby points required to assert that the model fits well
- Until N iterations have occured
  - Draw a sample of s points <u>uniformly</u> and <u>independently</u>
  - <u>Fit model</u> to the set of s points

    least squares

    Total least squares
  - For each data point outside the set s
    - Test the distance and if it is less than d then this is a good point
  - If there are T or more good points
    - <u>Refit the model</u> using these T points
- Use the best fit using <u>fit error</u> as the criteria.

Q. How many iterations N are needed for success probability equal to p

Probability of sampling an outlier : e

Samples required for fitting our model : s

Probability of selecting an inlier : 1 − e

Probability of selecting s inliers in a row: $(1-e)^s$

Probability of a bad sample
(i.e., one of the s points are outliers): $1-(1-e)^s$

Probability of getting N bad samples:
$[1-(1-e)^s]^N$

$P(A \cap B) = P(A|B)\,P(B)$
$P(A \cap B) = P(B|A)\,P(A)$
$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$

Probability of getting at least one good sample in N tries: $1-[1-(1-e)^s]^N$

Let $p = 1-[1-(1-e)^s]^N$

p: prob. of success

e: Outlier probability

N: # iterations

$$p = 1-[1-(1-e)^s]^N$$

$$\Rightarrow [1-(1-e)^s]^N = 1-p$$

$$\Rightarrow \log[1-(1-e)^s]^N = \log(1-p)$$

$$\Rightarrow N \log[1-(1-e)^s] = \log(1-p)$$

$$\Rightarrow N = \frac{\log(1-p)}{\log[1-(1-e)^s]}$$

Pros:

- Robust to outliers
- Fit models with large number of parameters

Cons:

- Slow

Applications

- Image stitching

- 3D scene analysis
    - Calibration
    - Fundamental matrix