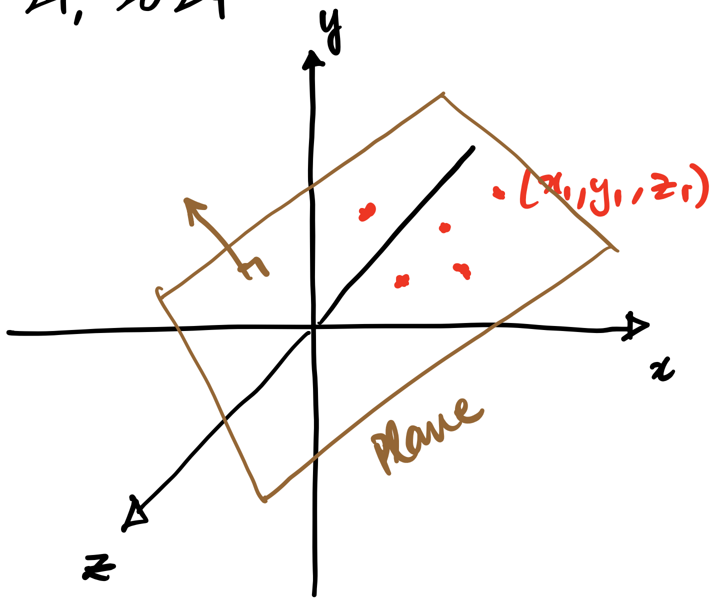
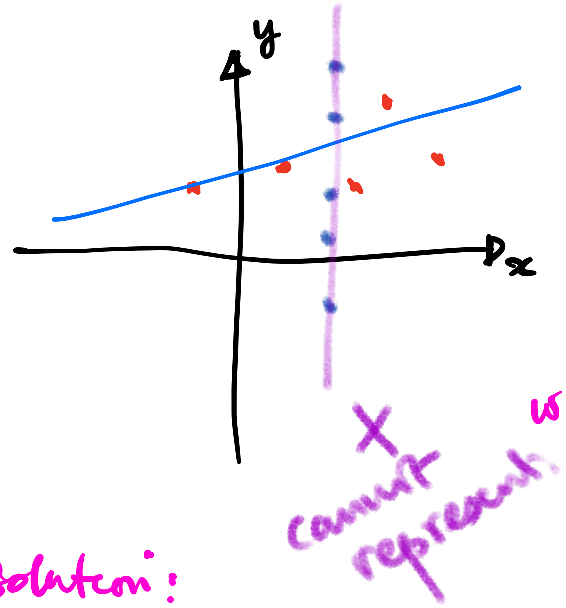
Jan 29, 2024



$$y = f(x, z; \theta)$$

Plane:

$$ax + by + cz + d = 0$$



line: $y = mx + c$

$$mx - y + c = 0$$

$$\downarrow$$

$$A\vec{x} = \vec{b}$$

where

$$\vec{x} = \begin{bmatrix} m \\ c \end{bmatrix}$$

$$A = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \quad \& \quad \vec{b} = \begin{bmatrix} y_1 \\ \vdots \\ y_2 \end{bmatrix}$$

Solution:

$$A\vec{x} = b$$

$$\Rightarrow A^T A \vec{x} = A^T b$$

$$\Rightarrow \vec{x} = (A^T A)^{-1} A^T b \leftarrow \text{Normal Equation}$$

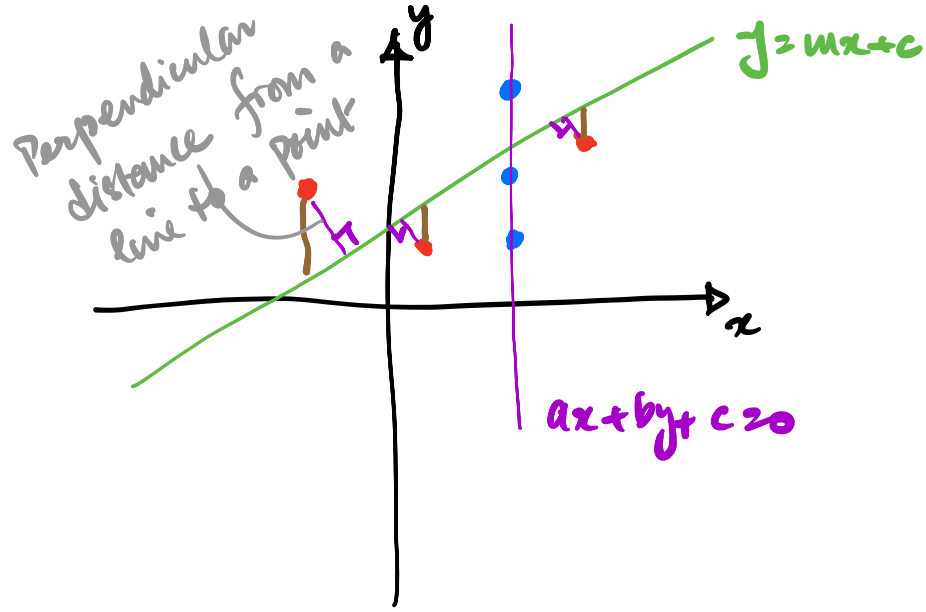
$$\uparrow$$

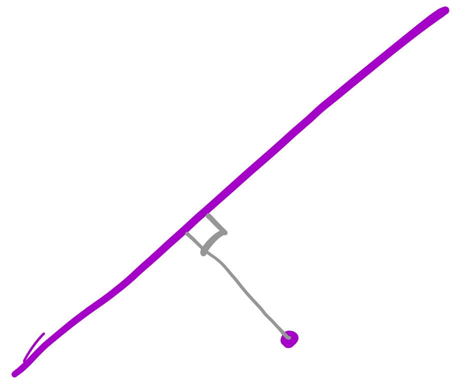
pseudo-inverse

line:

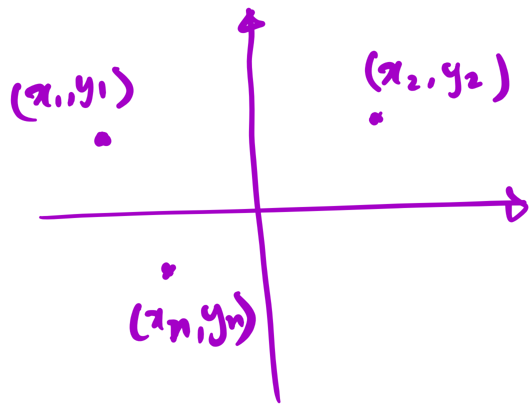
$$ax + by + c = 0 \rightarrow \text{To deal w/ vertical lines.}$$

To solve this we need to find the values of a, b, c.





$$a,b,c = \underset{a,b,c}{\operatorname{argmin}} \sum_{i=1}^{n} \underbrace{(ax_i + by_i + c)^2}_{\text{error}} \quad \text{s.t.} \quad (a,b)^T (a,b) = 1$$



$$A \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{0}$$

$$A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ & \vdots & \\ x_n & y_n & 1 \end{bmatrix}$$

use this

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ & \vdots & \\ x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \rightarrow A\vec{x} = \vec{0}$$

$$\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

There exists a trivial solution. $a=0$, $b=0$, $c=0$.

1. Compute eigenvalues and eigenvectors of A^TA.
2. Find the smallest eigenvalue.
3. The eigenvector corresponding to the smallest eigenvalue is the solution.

(SVD)

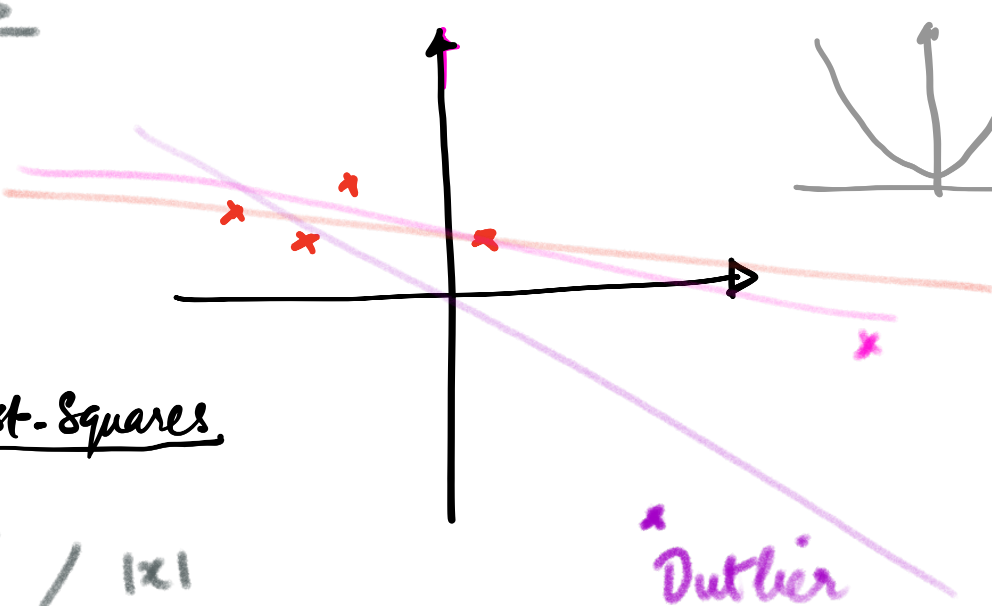
Problem:

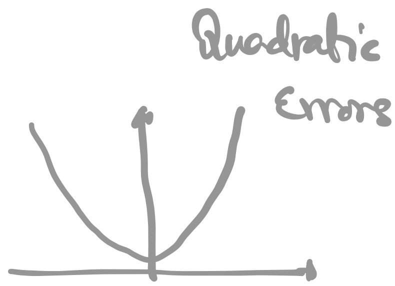
$\downarrow$

Model (linear) $\rightarrow$ $A\vec{x} = \vec{b}$

Model (linear) $\rightarrow$ $A\vec{x} = \vec{0}$

Outliers





Robust Least-Squares

absolute error