

IDEA: FIT A LINE.

$$y = mx + c$$

↑      ↑  
?      ?

At least two points.

Model fitting : find the model parameters  $(m, c)$   
that leads to smallest distance.

x	y	Model
$x_1$	$y_1$ ← ground truth.	$\hat{y}_1 = mx_1 + c$
$x_i$	$y_i$	$\hat{y}_i = mx_i + c$
$x_n$	$y_n$	$\hat{y}_n = mx_n + c$

input  
↑  
Training data.

Use the ground truth & model predictions to compute the error (model fit).

$$\epsilon_i = y_i - \hat{y}_i$$

$$= y_i - mx_i - c$$

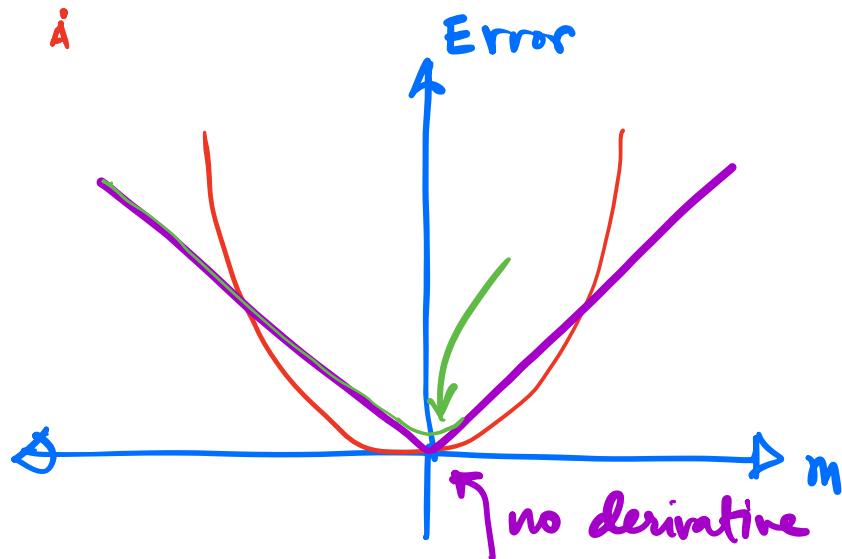
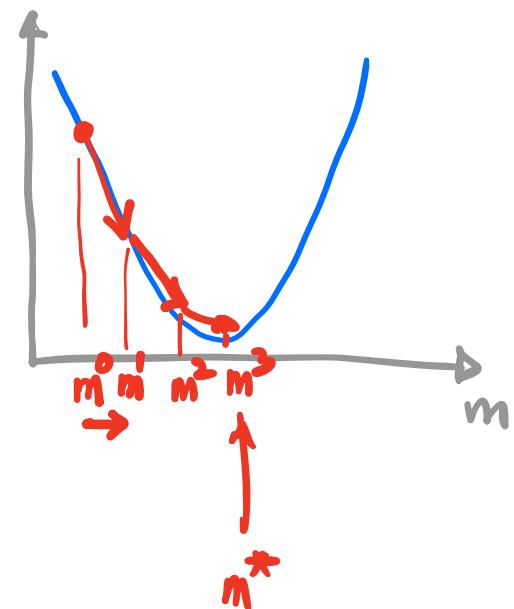
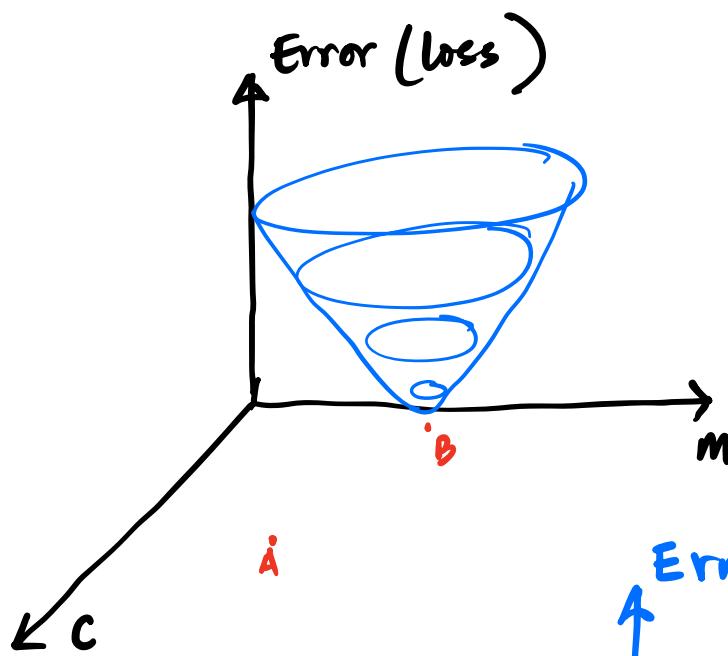
$$\text{Total error (MSE)} = \frac{1}{n} \sum_{i=1}^n \epsilon_i^2$$

Mean Squared Error

Contender:  
Sum of absolute differences

$$= \sum_{i=1}^n |\epsilon_i|$$

Loss : How to minimize loss?



$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - mx_i - c)^2 = \epsilon$$

We need to compute derivatives w.r.t. both  $m$  &  $c$ .

$$\begin{aligned}\frac{\partial \epsilon}{\partial m} &= \frac{1}{n} \sum_{i=1}^n 2(y_i - mx_i - c)(-x_i) \\ &= -\frac{1}{n} \sum_{i=1}^n 2x_i(y_i - mx_i - c) \\ \frac{\partial \epsilon}{\partial c} &= -\frac{1}{n} \sum_{i=1}^n 2(y_i - mx_i - c)\end{aligned}$$

To solve analytically set to 0 and solve for  $m$  &  $c$ .

Gradient of the error surface:

$$\begin{bmatrix} \frac{\partial \epsilon}{\partial m} \\ \frac{\partial \epsilon}{\partial c} \end{bmatrix}$$

Good news: For this problem, we are dealing with a quadratic surface.

So, we have a global minima (unique solution).