Linear Algebra Basics

Scalars: age = 4

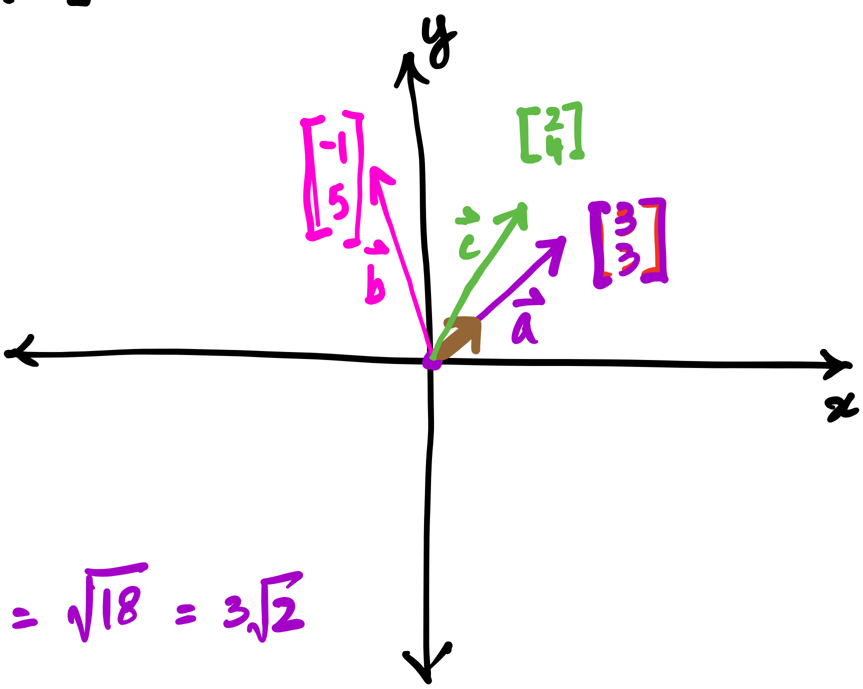
Vectors:

$$\begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \leftarrow \text{column vector} \qquad \begin{bmatrix} 2 & 3 & 4 & 6 & 7 \end{bmatrix}$$

row vector

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \qquad \vec{x}^T = \begin{bmatrix} 2 & 3 & 4 & 1 \end{bmatrix}$$

2D Examples:



$$\vec{a} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$|\vec{a}| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Q. How close (or similar) are two vectors?

$$\vec{a} - \vec{c} = \begin{bmatrix} 3-2 \\ 3-4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad |\vec{a} - \vec{c}| = \ell$$

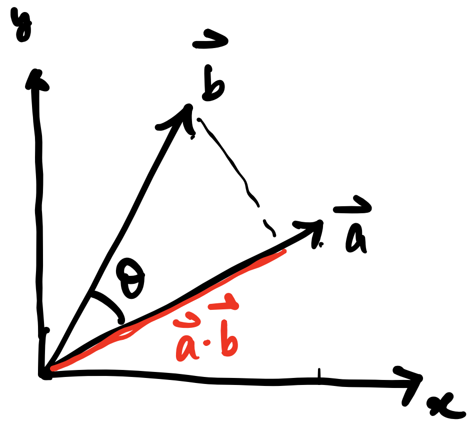
Case 1

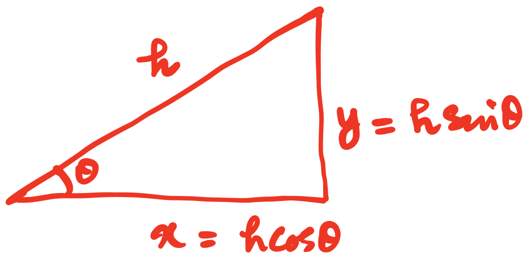
$$\vec{a} - \vec{b} = \begin{bmatrix} 3+1 \\ 3-5 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \qquad |\vec{a} - \vec{b}| = \ell$$

Angle between the two vectors:

Case 2

$$\underline{\vec{a}\cdot\vec{b}} = |a|\,|b|\cos\theta$$





Also: $$\vec{a}\cdot\vec{b} = \sum_{i=1}^{N} a_i b_i$$

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix}$$

inner product

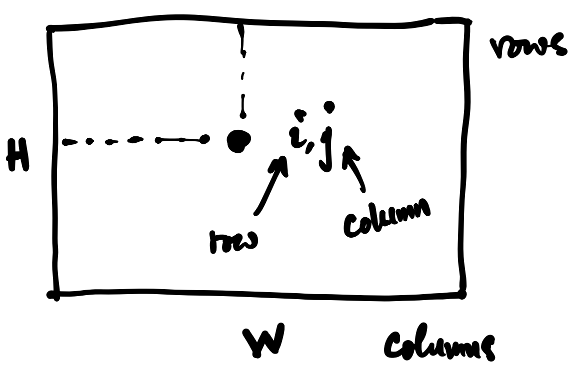
$$= \vec{a}^T \vec{b}$$

$$= \underset{1\times N}{\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_N \end{bmatrix}} \underset{N\times 1}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_N \end{bmatrix}}$$

Outer-products:

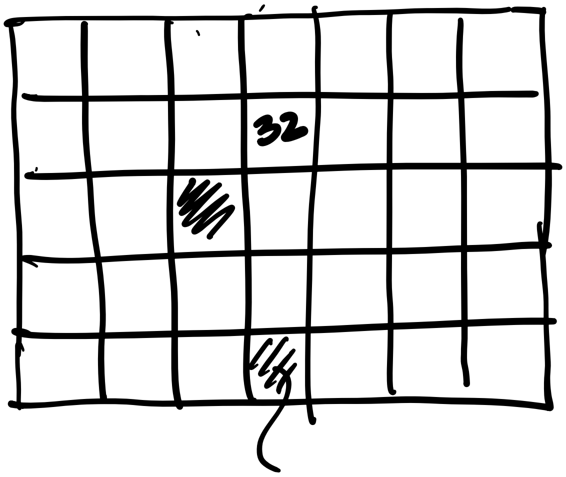
$$\underset{(N\times 1)}{\vec{a}}\ \underset{(1\times N)}{\vec{b}^T} = \underset{(N\times N)}{A}$$

Matrices



Aside: $\vec{y} = A\vec{x} \rightsquigarrow |A\vec{x}|$

Grayscale



8-bits : 256 levels   **char**

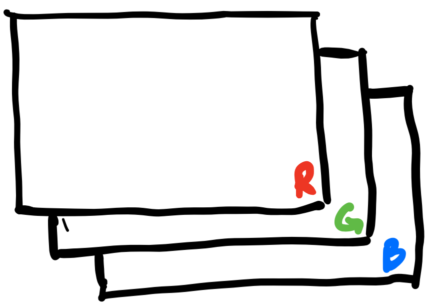
[0, 255]

↑ Black   ↑ White

$\downarrow$ convert it into floats

[0.0, 1.0]

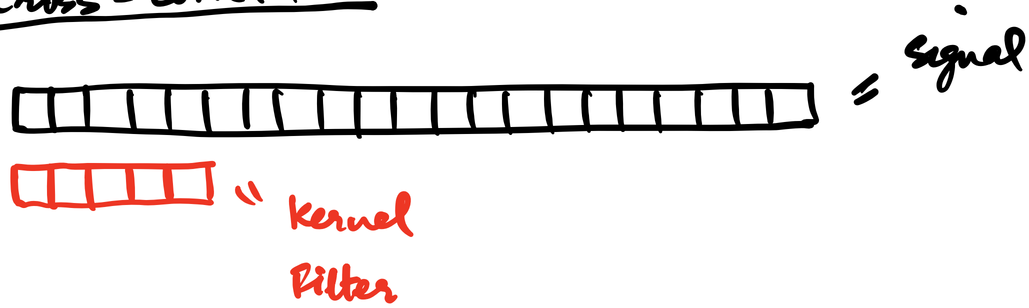
Color images



8-bit, RGB

4096 x 1024

Cross-correlation



Example:

$I = $

1	-1	3	0	5	1	4
0	1	2	3	4	5	6

$f = $

1	1	2

$cc(I, f) = $

		2				
0	1	2	3	4	5	6

$(1)(-1) + (3)(1) + (0)(2)$

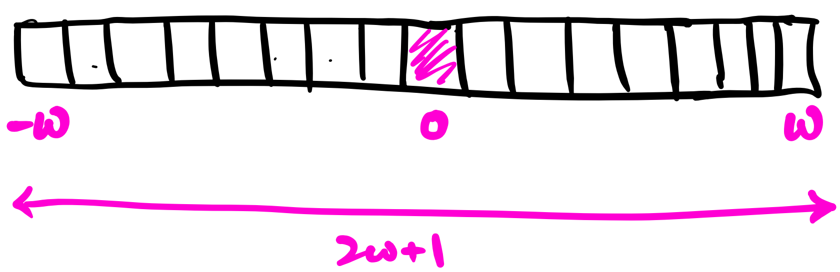
$= 2$

for $i = 1$ to 5

$O[i] = I[i-1] \times f[0] + I[i] \times f[1] + I[i+1] \times f[2]$

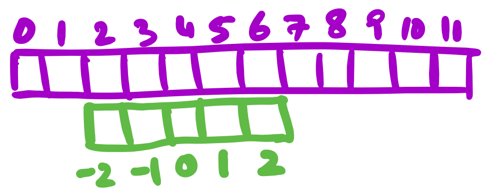
output

Filters / kernel:

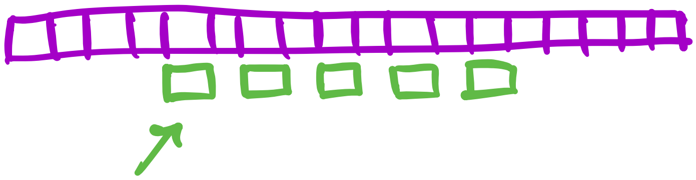


Half-width $= w$

$$cc(I, f)_i = \sum_{j=-w}^{+w} I[i-j]\, f[j]$$



1. Dilation



half-width = 2