Sample Midterm Exam

Computer Vision (CSCI 4220U)

Practice Exam (120 min.)

Family name: ______________________________

Given names: ______________________________

Student number: ______________________________

Question	Marks		
1	____________	/	20
2	____________	/	15
3	____________	/	15
4	____________	/	15
5	____________	/	15
6	____________	/	20
Total	____________	/	100

Instructions

- This is a closed book exam.
- Write in pen.
- Non-programmable calculators are permitted.
- Write "**I do not know**" to receive 10% marks in lieu of an answer.
- Total pages: 15.

Question 1 [20 Marks] - Camera Geometry and Projection

Consider a camera with the following parameters:

- Intrinsic matrix: $\mathbf{K} = \begin{pmatrix} 400 & 0 & 320 \\ 0 & 400 & 240 \\ 0 & 0 & 1 \end{pmatrix}$
- Rotation matrix: $\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$
- Camera center in world coordinates: $\mathbf{C} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}$

Part A [20%]

What is the focal length of this camera (in pixels)? What is the principal point?

Part B [30%]

Compute the translation vector $\mathbf{t} = -\mathbf{RC}$.

Part C [25%]

Write down the full camera projection matrix $\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$.

Part D [25%]

Project the world point $\mathbf{X}_{\text{world}} = (2, 0, 10)^T$ to the image plane. Show all steps.

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Question 2 [15 Marks] - Linear Filtering

Part A [40%]

Consider the following two filters:

Filter $\mathbf{h}_1$:

$$\mathbf{h}_1 = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix}$$

Filter $\mathbf{h}_2$:

$$\mathbf{h}_2 = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

What type of filters are these? What do they detect?

Part B [30%]

Show that $\mathbf{h}_1$ is separable by expressing it as an outer product of two 1D vectors.

Part C [30%]

A 512×512 image is convolved with a 31×31 filter.

1. How many multiplications are needed per output pixel for direct 2D convolution?
2. If the filter is separable, how many multiplications are needed per output pixel?
3. What is the speedup factor?

Question 3 [15 Marks] - Harris Corner Detection

The Harris corner detector uses the structure tensor (second moment matrix):

$$\mathbf{M} = \sum_{(x,y)\in W} \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

Part A [30%]

Explain how the eigenvalues λ_1 and λ_2 of $\mathbf{M}$ are used to classify a point as:

1. Flat region
2. Edge
3. Corner

Part B [15%]

Given the following 2x2 matrix:

$$\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix}$$

Compute the determinant $\det(\mathbf{A})$.

Part C [15%]

In corner detection, suppose the structure tensor at a pixel is a diagonal matrix:

$$\mathbf{M} = \begin{pmatrix} 900 & 0 \\ 0 & 850 \end{pmatrix}$$

Compute the eigenvalues λ_1 and λ_2 of this matrix. Based on these eigenvalues, classify this point as a flat region, edge, or corner.

Part D [20%]

The Harris corner response is defined as $R = \det(\mathbf{M}) - k \cdot \mathrm{trace}(\mathbf{M})^2$.

Given $\mathbf{M} = \begin{pmatrix} 100 & 20 \\ 20 & 80 \end{pmatrix}$ and $k = 0.04$, compute R.

Part E [20%]

Why are corners considered good features to track in optical flow and stereo matching? Relate your answer to the structure tensor.

Question 4 [15 Marks] - RANSAC and Line Fitting

You are given 5 data points that should lie on a line:

Point	x	y
P_1	0	1.0
P_2	1	3.1
P_3	2	4.9
P_4	3	7.0
P_5	4	100

Part A [25%]

Using points P_1 and P_3, compute the line equation $y = mx + b$.

Part B [25%]

Using a distance threshold of $\epsilon = 0.5$, determine which of the remaining points (P_2, P_4, P_5) are inliers for the line found in Part A.

Part C [25%]

If the outlier ratio is $e = 0.2$ (20% outliers) and we want probability $p = 0.99$ of selecting at least one good sample, how many RANSAC iterations k are needed?

Use the formula: $k = \frac{\log(1-p)}{\log(1-(1-e)^n)}$ where $n = 2$ points are needed to fit a line.

Part D [25%]

Explain why RANSAC is preferred over least squares when outliers are present in the data.

Question 5 [15 Marks] - Homography and Optical Flow

Part A [35%] - Homography

A homography $\mathbf{H}$ maps points between two views of a planar surface:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim \mathbf{H} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

1. How many degrees of freedom does a general homography have?
2. Each point correspondence provides how many constraints?
3. What is the minimum number of point correspondences needed to compute $\mathbf{H}$?

Part B [35%] - Optical Flow

The optical flow constraint equation is:

$$I_x u + I_y v + I_t = 0$$

1. What does each term (I_x, I_y, I_t, u, v) represent?
2. Why can't this single equation determine both u and v? What is this problem called?
3. How does the Lucas-Kanade method overcome this problem?

Part C [30%]

In the Lucas-Kanade method, we solve the overdetermined system $\mathbf{A}\mathbf{v} = \mathbf{b}$ where:

$$\mathbf{A} = \begin{pmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \vdots \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} u \\ v \end{pmatrix}, \quad \mathbf{b} = -\begin{pmatrix} I_{t1} \\ I_{t2} \\ \vdots \end{pmatrix}$$

Write down the normal equations used to solve for $\mathbf{v}$.

Question 6 [20 Marks] - Stereo Vision

Two cameras are set up in a stereo configuration with:

- Baseline: $B = 0.1$ meters (10 cm)
- Focal length: $f = 500$ pixels
- The cameras have parallel optical axes (rectified stereo)

Part A [25%]

For a point at depth $Z = 2$ meters, compute the expected disparity d using the formula $d = \frac{fB}{Z}$.

Part B [25%]

If a different point has a measured disparity of $d = 25$ pixels, what is its depth?

Part C [25%]

What is an epipolar line? Why does the epipolar constraint simplify stereo matching?

Part D [25%]

In a rectified stereo pair:

1. What is special about the epipolar lines?
2. What does this mean for the stereo correspondence search?

End of exam.