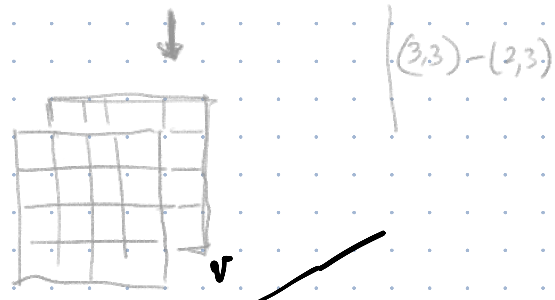
Frame t Frame $t+1$ u v

loc (red pixel) at $t = (1,1)$

loc (red pixel) at $t+1 = (0,2)$

shift $= (0,2) - (1,1) = \underbrace{(-1,1)}$

Optical Flow (u, v)



Optical Flow

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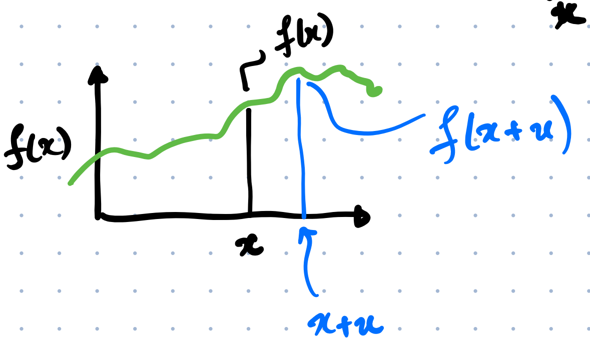
① $I(x,y,t) = \underbrace{I(x+u, y+v, t+1)}_{*}$ Brightness Constancy Constraint.

$f(x+u) = f(x) + u f_x(x) + \frac{u^2}{2!} f_{xx}(x) + \cdots$ (TAYLOR'S SERIES)



First derivative of $f(x) = \frac{\partial f}{\partial x}$

Second derivative of $f(x) = \partial^2 f / \partial x^2$.

$* \; I(x+u, y+v, t+1) = I(x,y,t) + u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} + \frac{\partial I}{\partial t}$

1st term   2nd term.

$$u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} + \frac{\partial I}{\partial t} = I(x+u, y+v, t+1) - I(x,y,t)$$

$$= \varnothing \quad (1)$$

$$\Rightarrow \quad u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} = -\frac{\partial I}{\partial t}$$

$$\Rightarrow \quad \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)\cdot(u,v) = -\frac{\partial I}{\partial t}$$

$$\Rightarrow \quad \begin{bmatrix} I_x & I_y \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

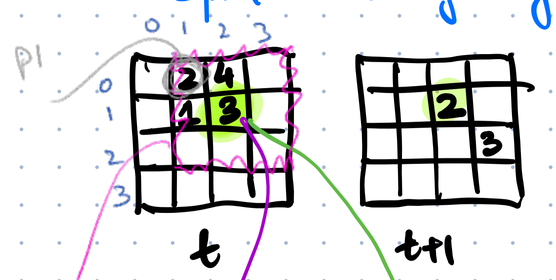
For a particular pixel.

?

$I_{t+1}[x,y] - I_t[x,y]$

Spatial image gradients. (Sobel Filters)

$I_x(1,2)$
$= I(1,2) - I(1,1)$
$= 3 - 1$
$= 2$



Bot

3

$I_x = 2$

$I_y = -1$

$I_t = -1$

$$\begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -(-1)$$

(2) Spatial Coherence

③ Nine Equations

$$\begin{bmatrix} I_x(P_i) & I_y(P_i) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t(P_i) \quad \text{where} \quad i \in [1, 9]$$

$P_1 = (0, 1)$

$$\begin{bmatrix} I_x(P_1) & I_y(P_1) \\ I_x(P_2) & I_y(P_2) \\ \vdots & \\ I_x(P_9) & I_y(P_9) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(P_1) \\ I_t(P_2) \\ \vdots \\ I_t(P_9) \end{bmatrix}$$

Spatial coherence.

$$\underset{\in \mathbb{R}^{9\times 2}}{A} \quad \underset{\in \mathbb{R}^{2\times 1}}{\vec{x}} = \underset{\in \mathbb{R}^{9\times 1}}{\vec{b}}$$

$$\vec{x} = (A^T A)^{-1} A^T \vec{b}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \dots$$

$$\nabla I \begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

$[I_x \quad I_y]$

$$\Rightarrow \nabla I \begin{bmatrix} u_e + u_p \\ v_e + v_p \end{bmatrix} = -I_t$$

$$\Rightarrow \overbrace{\nabla I \begin{bmatrix} u_e \\ v_e \end{bmatrix}}^{\text{dot-product}\ \to\ \emptyset} + \overbrace{\nabla I \begin{bmatrix} u_p \\ v_p \end{bmatrix}} = -I_t$$

along edge

$\perp$ edge

$\perp$ edge