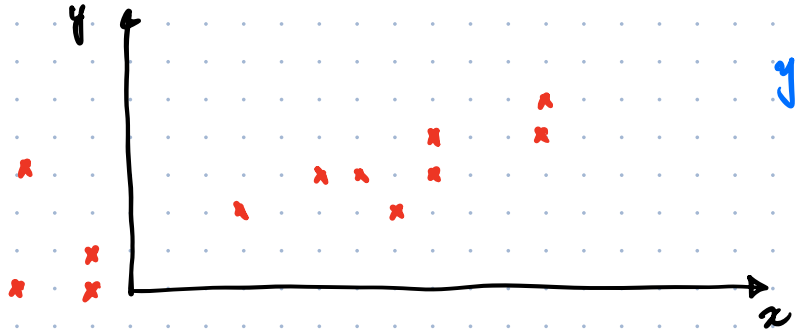
Model Fitting



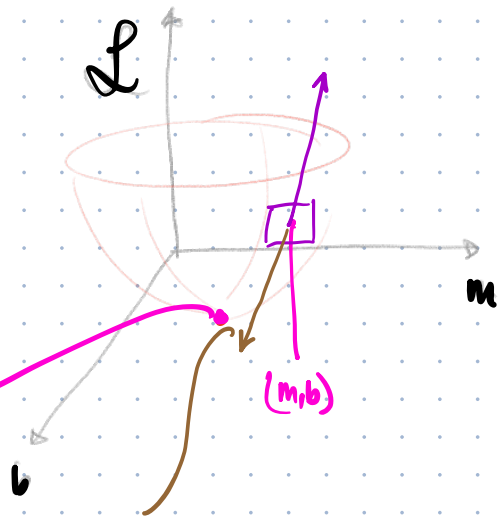
$y = mx + b$

$$\text{Loss fn. } (m,b) = \mathcal{L}(m,b) = \frac{1}{N}\sum_{i=1}^{N}(mx_i + b - y_i)^2$$

- $\frac{1}{N}$: # data
- $mx_i + b$: prediction
- y_i: ground truth

Compute gradient.

$$\begin{bmatrix} \frac{\partial \mathcal{L}}{\partial m} \\ \frac{\partial \mathcal{L}}{\partial b} \end{bmatrix}$$

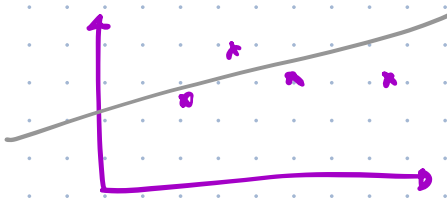


$(m^*, b^*) \rightarrow \nabla \mathcal{L} = 0$

$-\nabla_\theta \mathcal{L}$

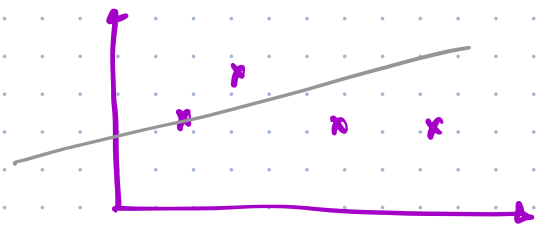
$$\theta_{new} = \theta_{current} - \alpha \nabla_\theta \mathcal{L} \qquad \theta_{current} = \begin{bmatrix} m \\ b \end{bmatrix}$$

α: Learning Rate.



$$\theta = (A^T A)^{-1} A^T b$$

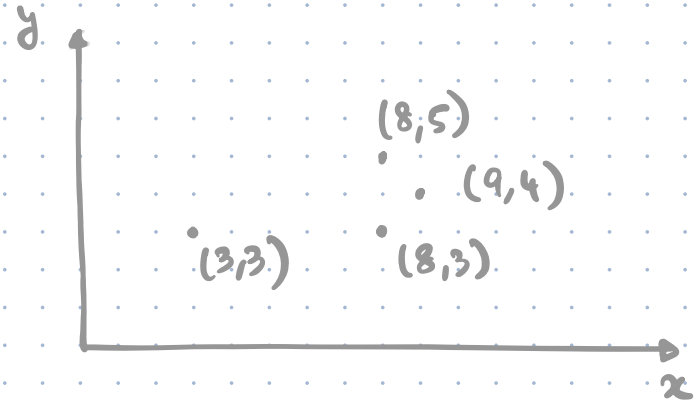
$$\theta = \begin{bmatrix} m \\ b \end{bmatrix}$$



$ax + by + c = 0$ $\quad (a,b)^T(a,b) = 1$

$$A \begin{bmatrix} a \\ b \end{bmatrix} = \emptyset$$

SVD A^TA and pick eigenvector corresponding to the smallest eigenvalue.



Fitting $y = mx + b$

$$A = \begin{bmatrix} 3 & 1 \\ 8 & 1 \\ 9 & 1 \\ 8 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 3 \\ 4 \\ 5 \end{bmatrix} \quad x = \begin{bmatrix} m \\ b \end{bmatrix}$$

System of linear Eq.

$$Ax = b$$

Soln:

$$x = (A^TA)^{-1}A^Tb$$

Data points: $(x_1, y_1, z_1) \cdots (x_n, y_n, z_n)$

Plane: $ax + by + cz + d = 0$ $\quad (a,b,c)^T(a,b,c) = 1$

$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}(ax_i + by_i + cz_i + d)^2$$

$$\frac{\partial \mathcal{L}}{\partial d} = \frac{2}{N}\sum_{i=1}^{N}(ax_i + by_i + cz_i + d)$$

Set $\frac{\partial \mathcal{L}}{\partial d} = 0$, we get

$$\frac{1}{N}\sum_{i=1}^{N}(ax_i + by_i + cz_i + d) = 0$$

$$\Rightarrow a\langle x \rangle + b\langle y \rangle + c\langle z \rangle + \frac{Nd}{N} = 0$$

$\frac{1}{N}\sum_{i=1}^{N} ax_i + \frac{1}{N}\sum$

$\Rightarrow d = -a\langle x\rangle - b\langle y\rangle - c\langle z\rangle$

$$ax + by + cz \underbrace{- a\langle x\rangle - b\langle y\rangle - c\langle z\rangle}_{d} = \emptyset$$

$$\Rightarrow a(x - \langle x\rangle) + b(y - \langle y\rangle) + c(z - \langle z\rangle) = \emptyset$$

$$\Rightarrow \begin{bmatrix} x - \langle x\rangle & y - \langle y\rangle & z - \langle z\rangle \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \emptyset$$

$$\begin{bmatrix} x_1 - \langle x\rangle & y_1 - \langle y\rangle & z_1 - \langle z\rangle \\ & & \\ x_N - \langle x\rangle & y_N - \langle y\rangle & z_N - \langle z\rangle \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \emptyset$$

$A \qquad x = \emptyset$

SVD. A^TA Smallest eigenvalue / eigenvector.

B

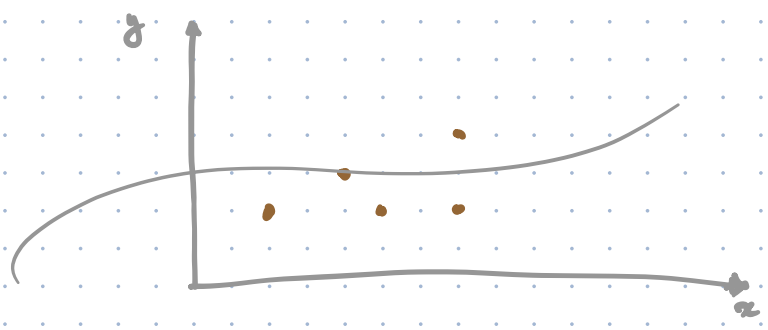
x	y	z
3	3	1
8	5	1
8	3	3
9	4	1

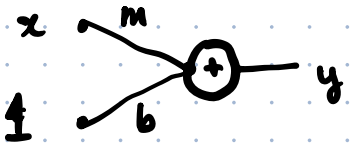
$\langle x\rangle = 7$

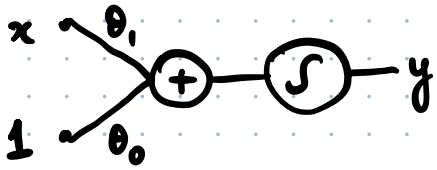
$\langle y\rangle = 15/4$

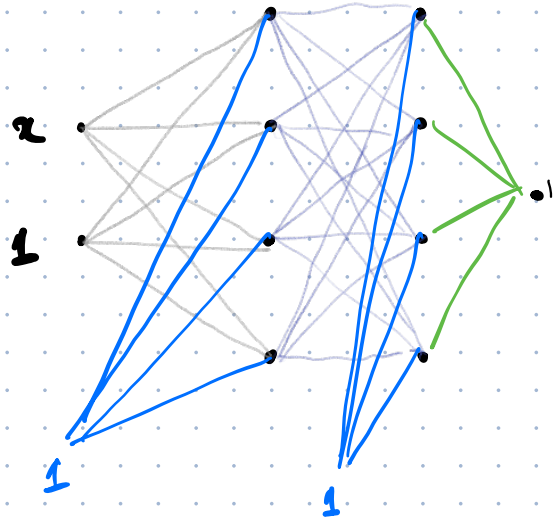
$\langle z\rangle = 3/2$

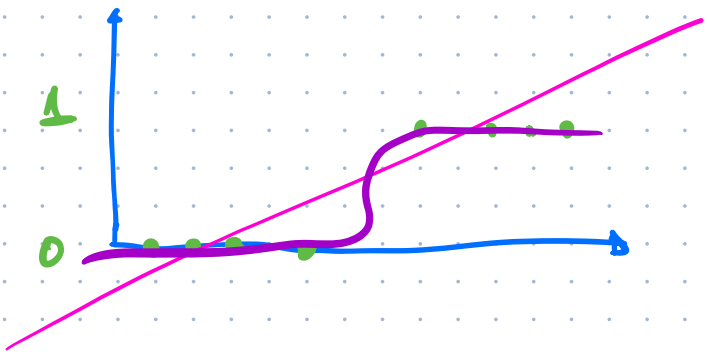
$$A = \begin{bmatrix} -4 & -3/4 & -1/2 \\ 1 & 5/4 & -1/2 \\ 1 & -3/4 & 3/2 \\ 2 & 1/4 & -1/2 \end{bmatrix} \quad X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$




x
m
+
y
1
b


x
θ1
+
S
y
1
θ0


x
1
1
1
1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