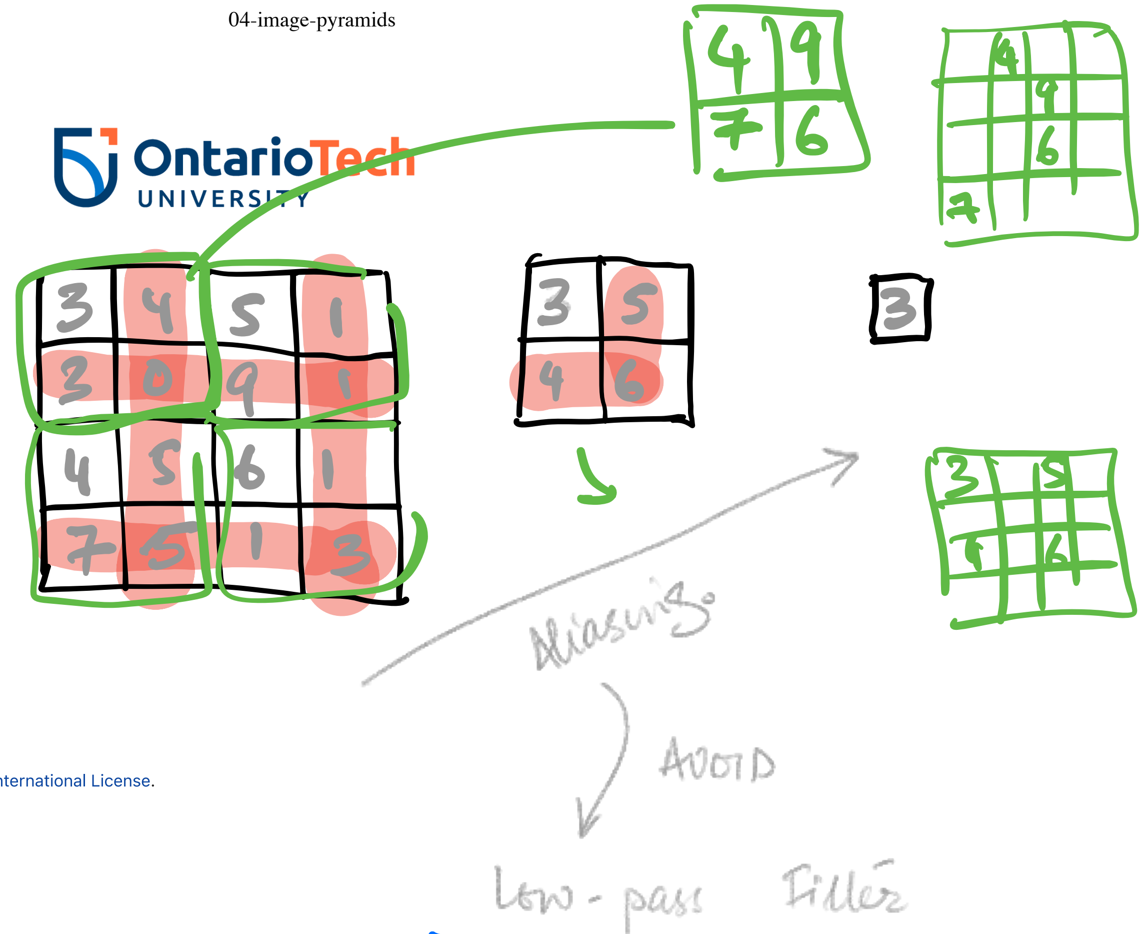

Image Pyramids

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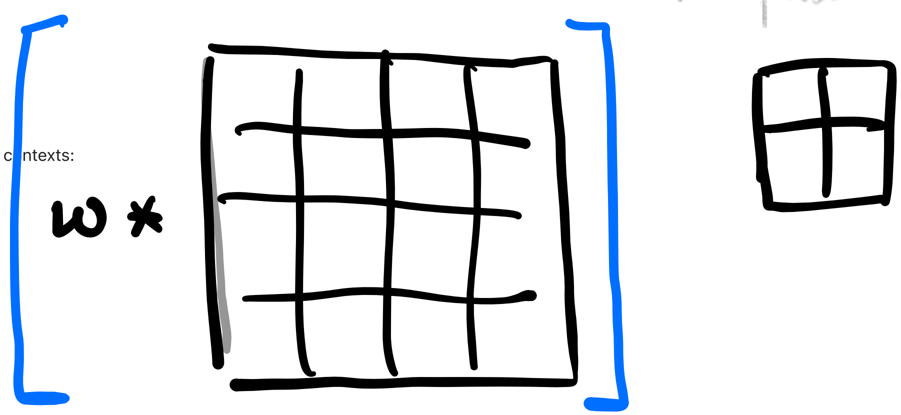
Lesson Plan

- Gaussian image pyramids
- Laplacian image pyramids
- Laplacian blending

Uses

Often used to achieve scale-invariant processing in the following contexts:

- template matching;
- image registration;
- image enhancement;
- interest point detection; and
- object detection.

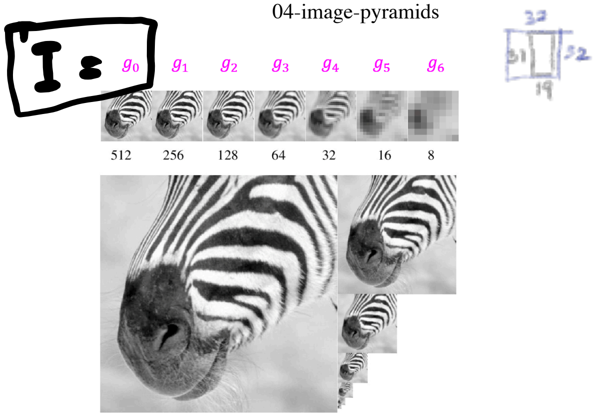


Gaussian Image Pyramid

The basic idea for constructing Gaussian image pyramid is as follows:

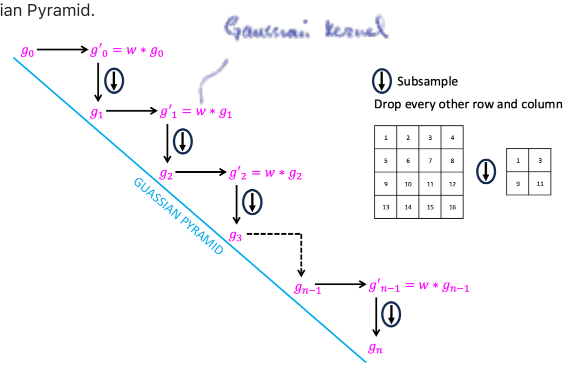
Let $g_0 = I$ and $i = 0$

1. Blur g_i with a Gaussian kernel w to create g_i'
2. Reduce g_i' dimensions by half by discarding every other row and and every other column to construct g_{i+1}
3. Set $i = i + 1$ and repeat this process until desired numbers levels are achieved or the image is reduced to size 1×1



Courtesy D. Forsyth

Starting with $g_0 = I$ the following figure depicts the construction of a Gaussian Pyramid.




```
In [3]: import numpy as np
        import math
        import cv2 as cv
        import matplotlib.pyplot as plt
```

Exercise 04-01

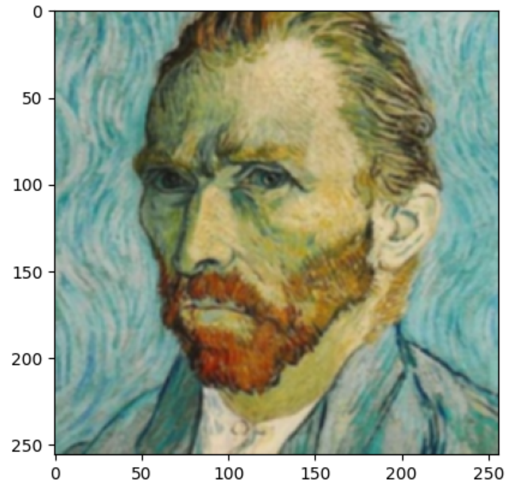
- Load image 'data/apple.jpg'
- Blur each channel with a 5-by-5 Gaussian kernel
- Construct the next level of Gaussian pyramid by discarding every other row or column

Solution:

```
In [4]: # %load solutions/image-pyramids/solution_01.py
        #I = cv.imread('data/apple.jpg')
        I = cv.imread('data/van-gogh.jpg')
        I = cv.cvtColor(I, cv.COLOR_BGR2RGB)
        I = cv.resize(I, (512, 512))

        print('Shape of I = {}'.format(I.shape))

        I[:,:,0] = cv.GaussianBlur(I[:,:,0], (5,5), 2, 2)
        I[:,:,1] = cv.GaussianBlur(I[:,:,1], (5,5), 2, 2)
        I[:,:,2] = cv.GaussianBlur(I[:,:,2], (5,5), 2, 2)

        I2 = I[::2,::2,:]
```

```
print('Shape of I2 = {}'.format(I2.shape))

plt.imshow(I2);
```

```
Shape of I = (512, 512, 3)
Shape of I2 = (256, 256, 3)
```



Implementation (Gaussian Image Pyramid)

In [5]:
```
# %load solutions/image-pyramids/gen_gaussian_pyramid.py
def gen_gaussian_pyramid(I, levels=6):
    G = I.copy()
    gpI = [G]
    for i in range(levels):
        G = cv.pyrDown(G)
        gpI.append(G)
    return gpI
```

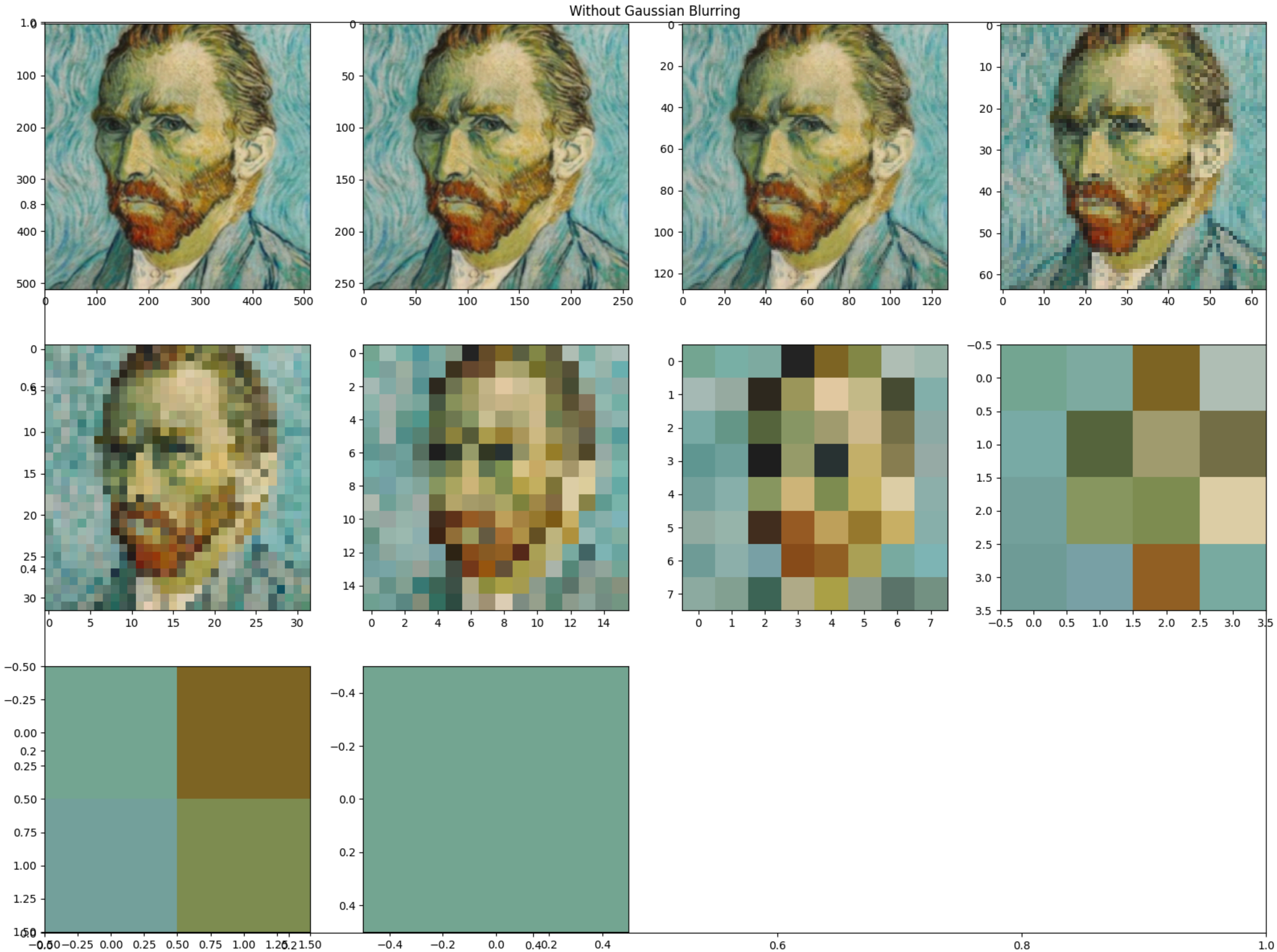
In [6]:
```
# %load solutions/image-pyramids/gen_gaussian_pyramid.py
def gen_pyramid(I, levels=6):
    G = I.copy()
    pI = [G]
    for i in range(levels):
        G = G[::2,::2,:]
        pI.append(G)
    return pI
```

In [7]:
```
foo = gen_gaussian_pyramid(I, levels=9)
boo = gen_pyramid(I, levels=9)

for i in foo:
    print(i.shape)

for i in boo:
    print(i.shape)
```

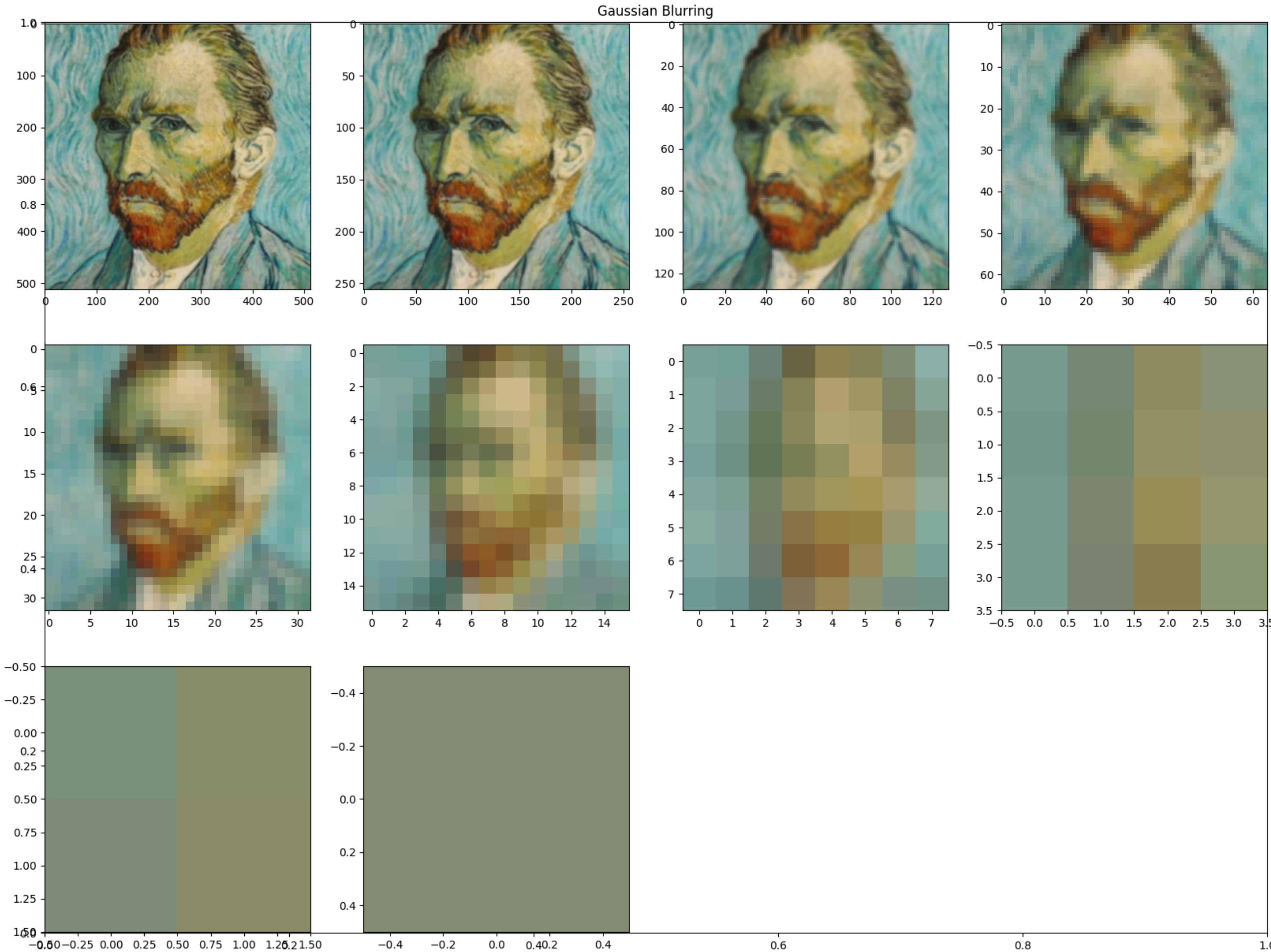
```
(512, 512, 3)
(256, 256, 3)
(128, 128, 3)
(64, 64, 3)
(32, 32, 3)
(16, 16, 3)
(8, 8, 3)
(4, 4, 3)
(2, 2, 3)
(1, 1, 3)
(512, 512, 3)
(256, 256, 3)
(128, 128, 3)
(64, 64, 3)
(32, 32, 3)
(16, 16, 3)
(8, 8, 3)
(4, 4, 3)
(2, 2, 3)
(1, 1, 3)
```

In [13]:
```
def show_pyramid(p):
    n = len(p)
    for i in range(n):
        c = 4
        r = math.ceil(n / c)
        ax = plt.subplot(r,c,i+1)
        ax.imshow(p[i])
```

In [14]:
```
plt.figure(figsize=(20,15))
plt.title('Without Gaussian Blurring')
show_pyramid(boo)
```



In [8]:
```
plt.figure(figsize=(20,15))
plt.title('Gaussian Blurring')
show_pyramid(foo)
```



Kernel for Gaussian blurring

OpenCV `pyrDown` method performs steps 1 and 2 above. It uses the following kernel to blur the image (in step 1).

$$\frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

In [9]:
```
A = cv.imread('data/apple.jpg')
B = cv.imread('data/orange.jpg')
A = cv.cvtColor(A, cv.COLOR_BGR2RGB)
B = cv.cvtColor(B, cv.COLOR_BGR2RGB)
```

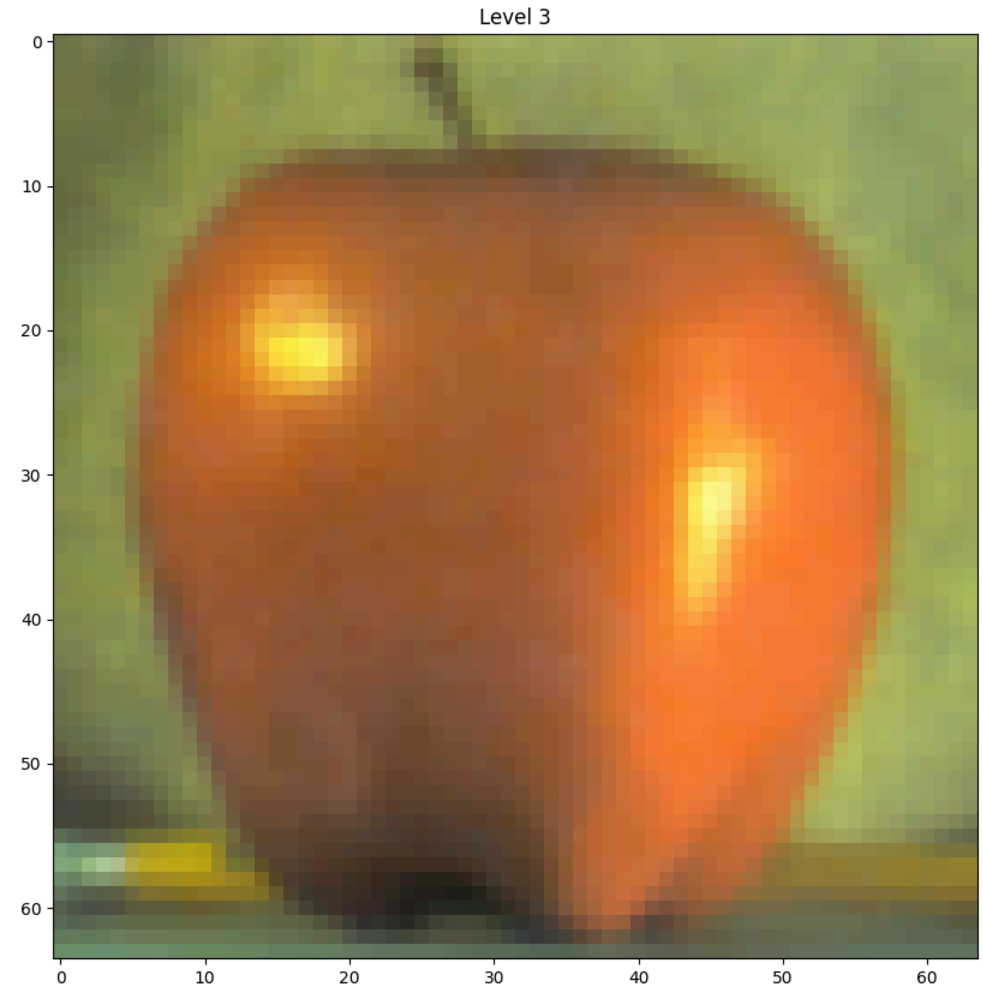
In [10]:
```
gpA = gen_gaussian_pyramid(A)
gpB = gen_gaussian_pyramid(B)
```

In [11]:
```
gp = gpB
num_levels = len(gp)
for i in range(num_levels):
    rows = gp[i].shape[0]
    cols = gp[i].shape[1]
    print('level={}: size={}x{}'.format(i, rows, cols))
```

```
level=0: size=512x512
level=1: size=256x256
level=2: size=128x128
level=3: size=64x64
level=4: size=32x32
level=5: size=16x16
level=6: size=8x8
```

In [12]:
```
gp = gpA
level = 3
plt.figure(figsize=(10,10))
plt.title('Level {}'.format(level))
plt.imshow(gp[level])
```

Out[12]: `<matplotlib.image.AxesImage at 0x14b22c4d0>`



Laplacian operator

We define Laplacian operator as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

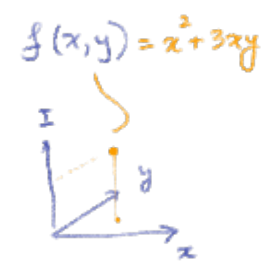
$\frac{\partial f}{\partial x} = 2x + 3y$

$\frac{\partial^2 f}{\partial x^2} = 2$

$\frac{\partial f}{\partial y} = 3x$

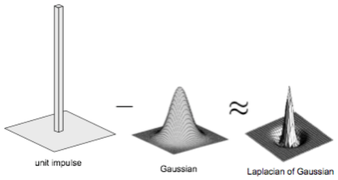
$\frac{\partial^2 f}{\partial y^2} = 0$

$\nabla^2 f = 2$



Approximation

In addition we can approximate the Laplacian of a Gaussian as follows:



Source: Lazebnik

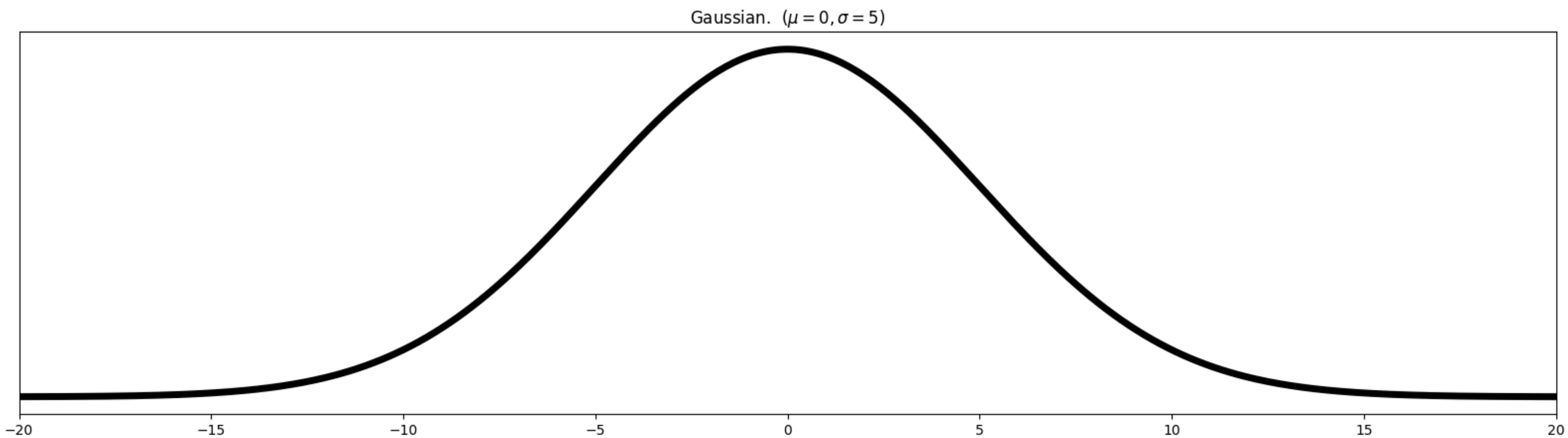
We use this property when constructing Laplacian image pyramids above.

Laplacian of a Gaussian

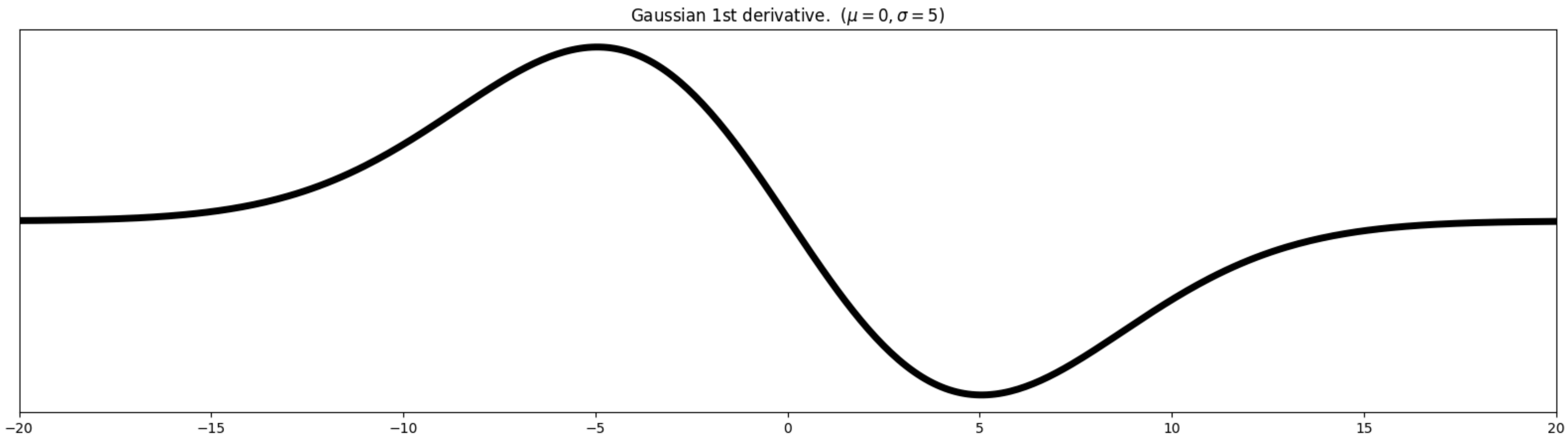
Approach 1: via computing derivatives

In [13]:
```
# 1D kernel for computing derivatives
Hx = np.array([1,-1], dtype='float32')
```

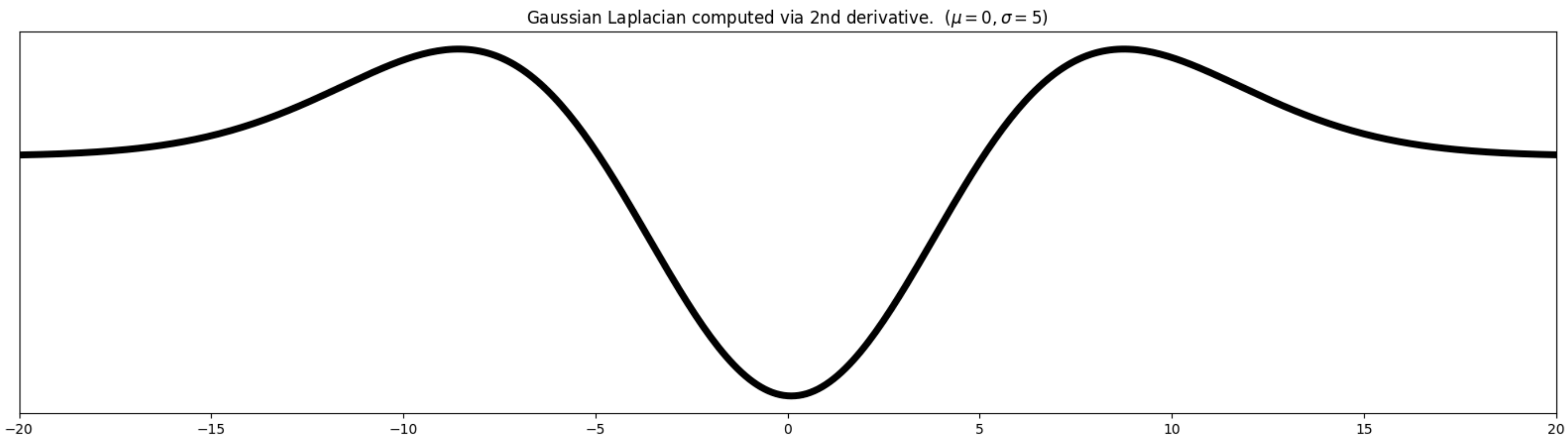
In [14]:
```
s = 4
ticks = np.linspace(-s,s,2*s+1)

gxx = np.linspace(-40,40,801)
mu = 0
sigma = 5
g = np.exp(- (gxx-mu)**2 / (2*(sigma**2)) ) / ( 2 * np.pi * sigma)

plt.figure(figsize=(20,5))
plt.plot(gxx, g, 'k', linewidth=5)
plt.xlim([-s*sigma, s*sigma])
plt.yticks([])
plt.title(r'Gaussian.  ($\mu=0, \sigma=5$)');
```



In [15]:
```
dg = np.convolve(g, Hx, 'same')

plt.figure(figsize=(20,5))
plt.plot(gxx, dg, 'k', linewidth=5)
plt.xlim([-s*sigma, s*sigma])
plt.yticks([])
plt.title(r'Gaussian 1st derivative.  ($\mu=0, \sigma=5$)');
```



In [16]:
```
ddg = np.convolve(dg, Hx, 'same')

plt.figure(figsize=(20,5))
plt.plot(gxx, ddg, 'k', linewidth=5)
plt.xlim([-s*sigma, s*sigma])
plt.yticks([])
plt.title(r'Gaussian Laplacian computed via 2nd derivative.  ($\mu=0, \sigma=5$)');
```

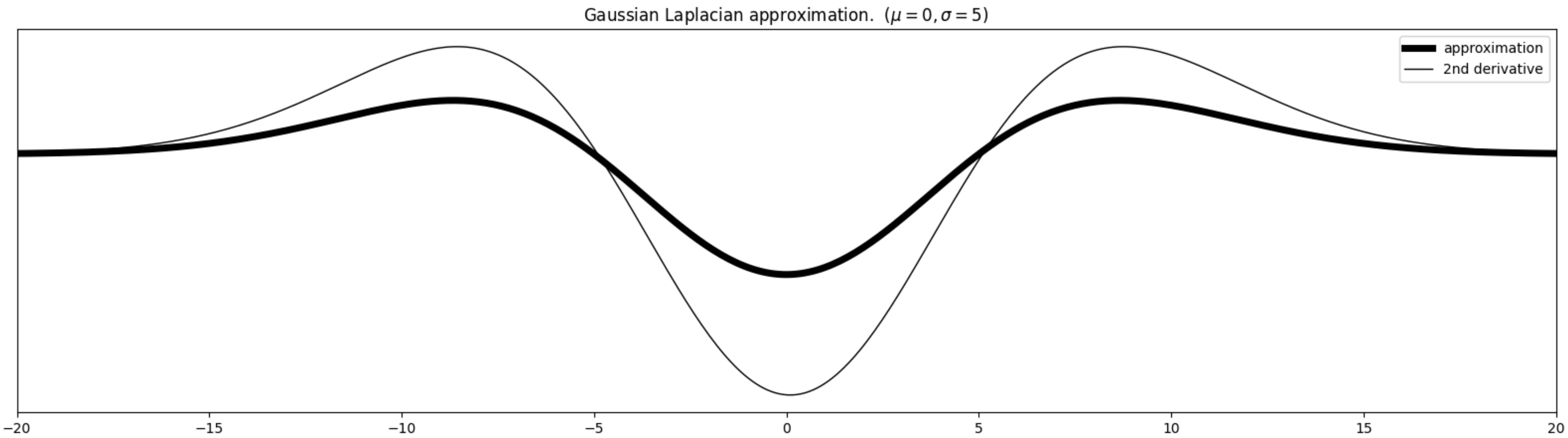


Approach 2: approximating Laplacian of a Gaussian

In [17]:
```
G_kernel = np.array([1., 4., 6., 4., 1.]) / 16.
g_smoothed = np.convolve(G_kernel, g, 'same')
```

In [18]:
```
s = 4
ticks = np.linspace(-s,s,2*s+1)

gxx = np.linspace(-40,40,801)
mu = 0
```

```
sigma = 5
g = np.exp(- (gxx-mu)**2 / (2*(sigma**2)) ) / ( 2 * np.pi * sigma)

G_kernel = np.array([1., 4., 6., 4., 1.]) / 16.
g_smoothed = np.convolve(G_kernel, g, 'same')

plt.figure(figsize=(20,5))
plt.plot(gxx, g_smoothed-g, 'k', linewidth=5, label='approximation')
plt.plot(gxx, ddg, 'k', linewidth=1, label='2nd derivative')
plt.xlim([-s*sigma, s*sigma])
plt.yticks([])
plt.legend()
plt.title(r'Gaussian Laplacian approximation.  ($\mu=0, \sigma=5$)');
```



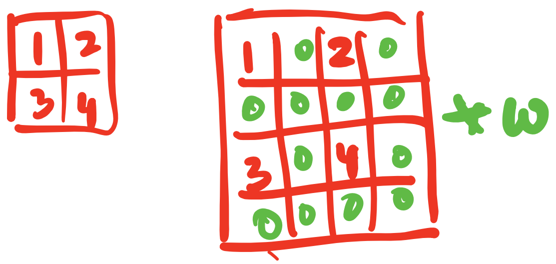
Laplacian Image Pyramid

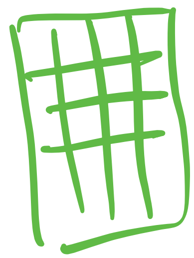
Proposed by Burt and Adelson is a bandpass image decomposition derived from the Gaussian pyramid. Each level encodes information at a particular spatial frequency. The basic steps for constructing Laplacian pyramids are:

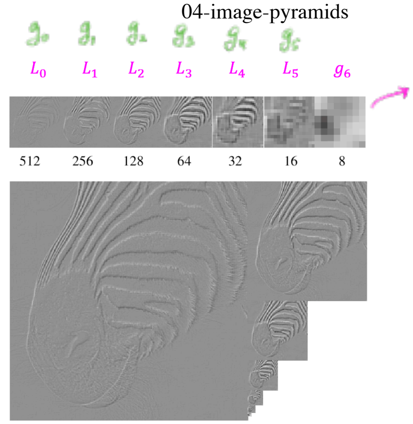
Construction (for image I)

Let $g_0 = I$ and set $i = 0$

1. Convolve g_i with Gaussan kernel (low-pass filter w) and down-sample by half to construct g_{i+1}. Downsample by discarding every other row and column.
2. Upsample g_{i+1} by inserting 0s between each row and column and interpolating the missing values by convolving it with w and create g'_i
3. Compute $L_i = g_i - g'_i$
4. Set $i = i + 1$ and repeat till the desired levels are reached.

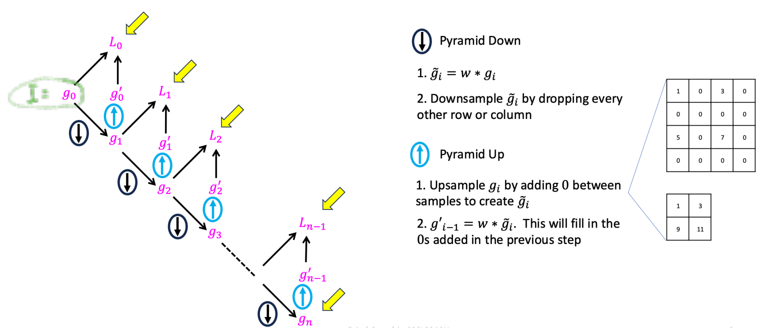






Courtesy D. Forsyth

Starting with image I, the following figure illustrates the construction of Laplacian pyramid.



Reconstructing the original image

It is possible to reconstruct the original image I from its Laplacian image pyramid consisting of n levels: $\{L_0, L_1, \cdots, L_{n-1}, g_n\}$.

Set $i = n$

1. Upsample g_i by inserting zeros between sample values and interpolate the missing values by convolving it with a filter w to get g'_{i-1}.
2. Set $g_{i-1} = L_{i-1} + g'_{i-1}$.
3. Set $i = i - 1$ and repeat till $i = 0$
4. Set $I = g_0$

Uses

Laplacian image pyramids are often used for compression. Instead of encoding g_0, we encode L_i, which are decorrelated and can be represented using far fewer bits.

Implementation (Laplacian Image Pyramid)

In [19]:
```
# %load solutions/image-pyramids/gen_laplacian_pyramid.py
def gen_laplacian_pyramid(gpI):
    """gpI is a Gaussian pyramid generated using gen_gaussian_pyramid method found in py file of the same name."""
    num_levels = len(gpI)-1
    lpI = [gpI[num_levels]]
    for i in range(num_levels,0,-1):
        GE = cv.pyrUp(gpA[i])
        L = cv.subtract(gpA[i-1],GE)
```

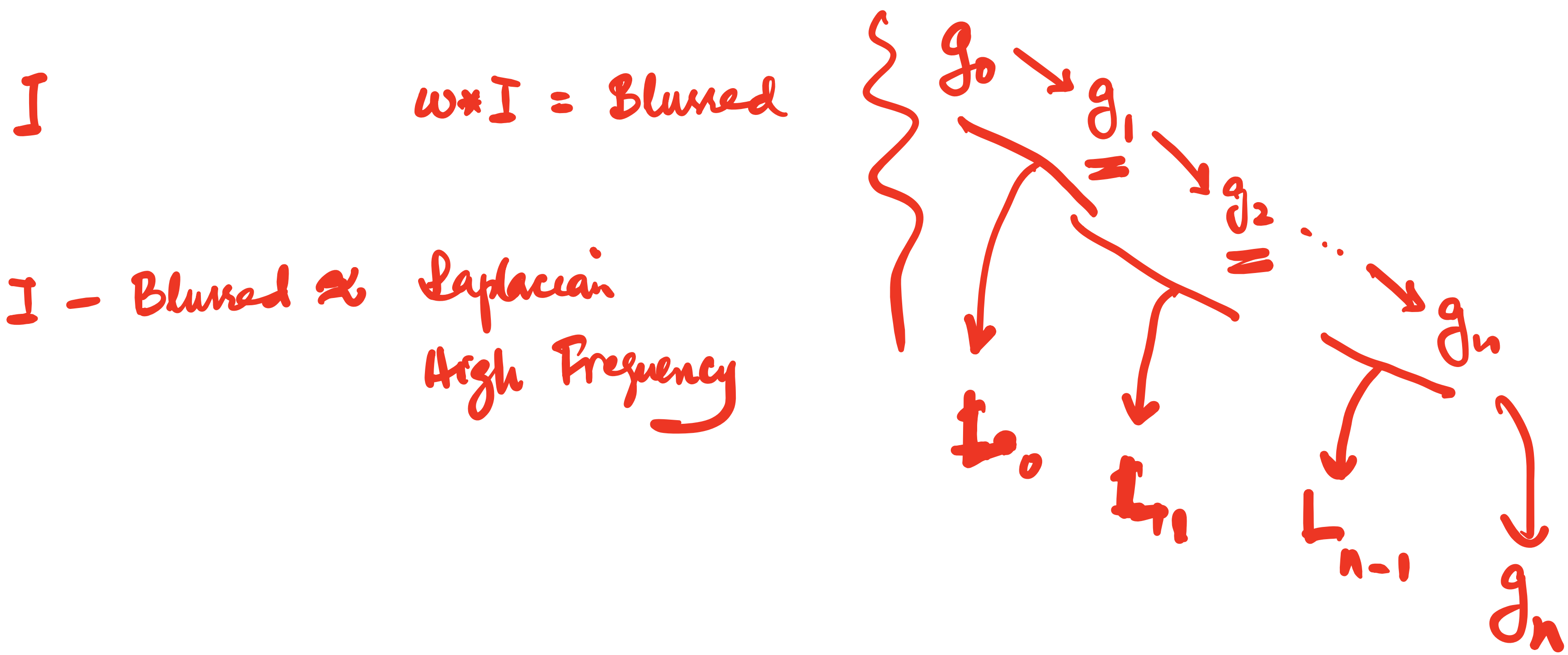
```
        lpI.append(L)
    return lpI
```

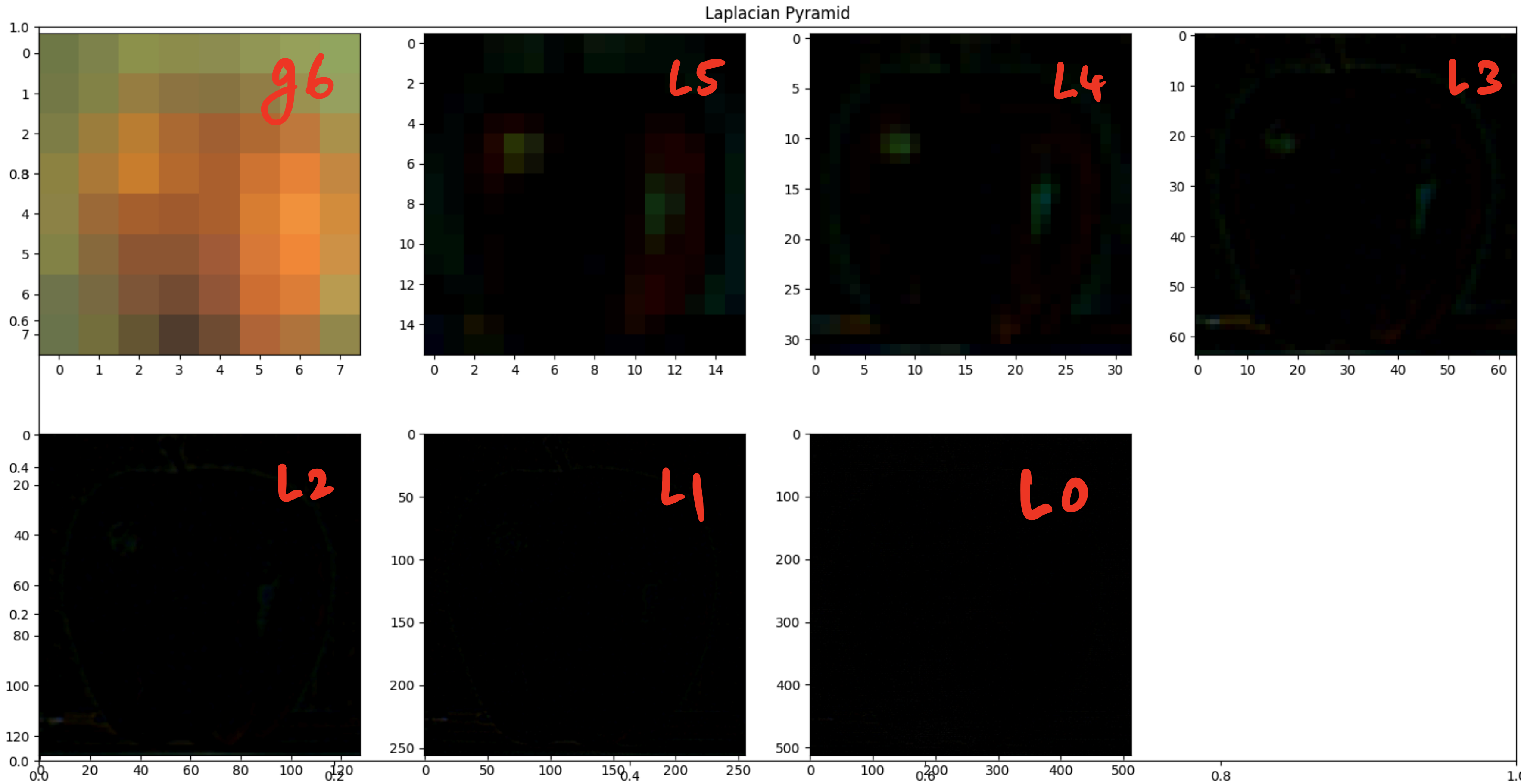
In [20]:
```
lpA = gen_laplacian_pyramid(gpA)
lpB = gen_laplacian_pyramid(gpB)
```

In [21]:
```
lp = lpA
num_levels = len(lp)
for i in range(num_levels):
    rows = lp[i].shape[0]
    cols = lp[i].shape[1]
    print('level={}: size={}x{}'.format(i, cols, rows))
```

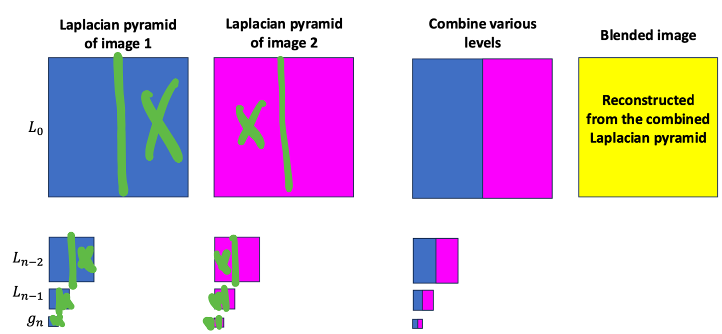
```
level=0: size=8x8
level=1: size=16x16
level=2: size=32x32
level=3: size=64x64
level=4: size=128x128
level=5: size=256x256
level=6: size=512x512
```

In [22]:
```
plt.figure(figsize=(20,10))
plt.title('Laplacian Pyramid')
show_pyramid(lpA)
```



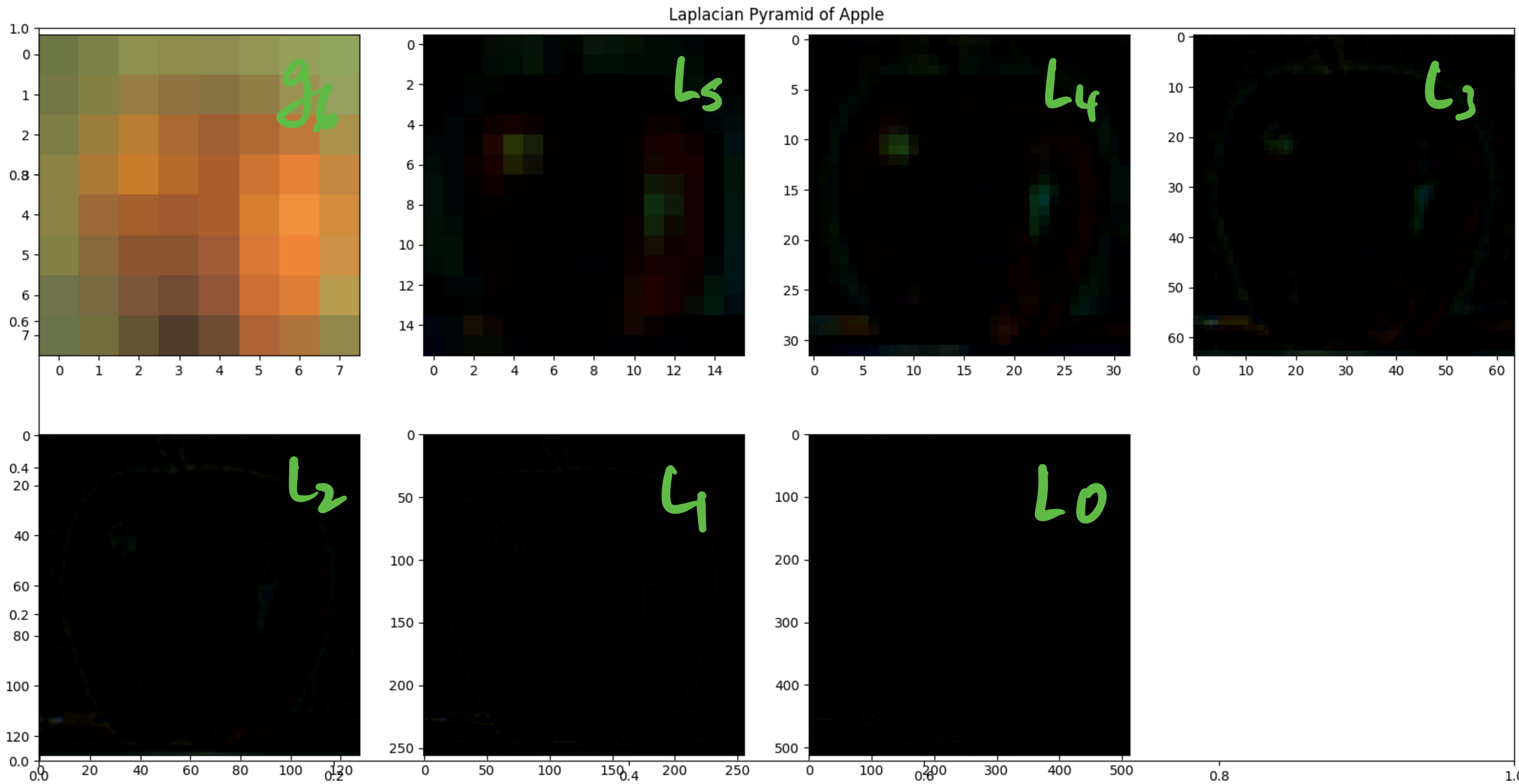


Laplacian Blending



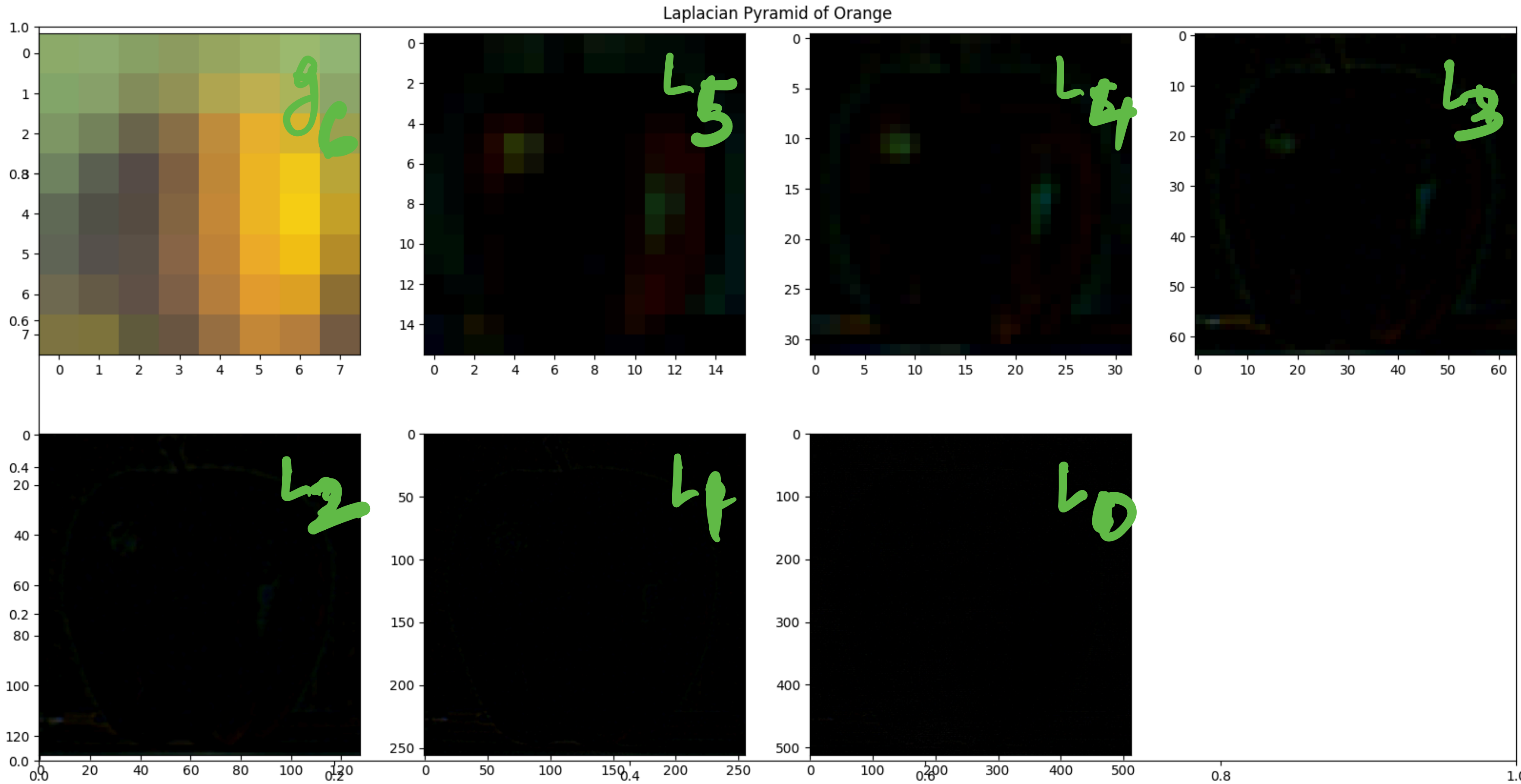
Constructing Laplacian Pyramid of First Image

In [23]:
```
plt.figure(figsize=(20,10))
plt.title('Laplacian Pyramid of Apple')
show_pyramid(lpA)
```



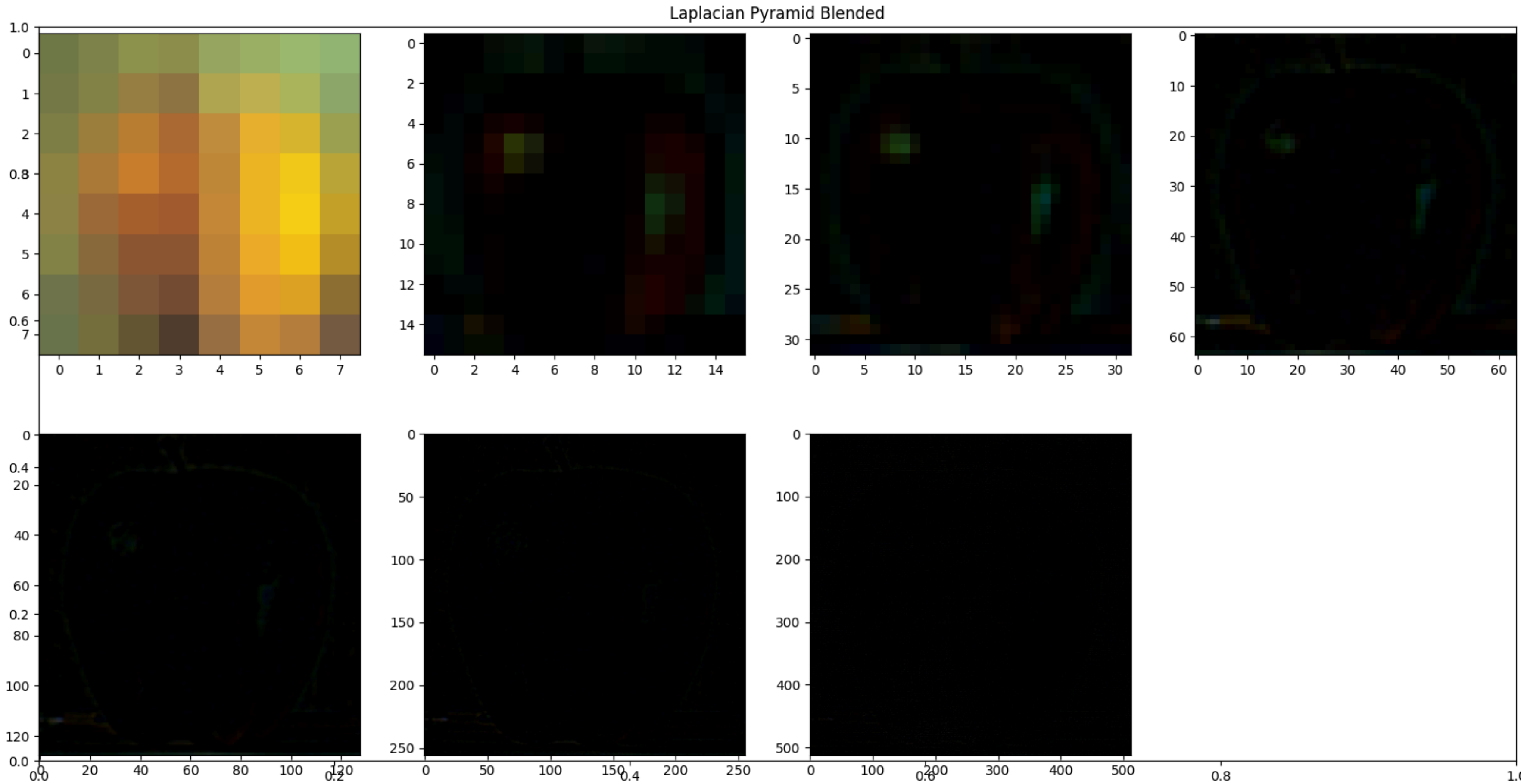
Constructing Laplacian Pyramid of Second Image

In [24]:
```
plt.figure(figsize=(20,10))
plt.title('Laplacian Pyramid of Orange')
show_pyramid(lpB)
```



Blending

In [25]:
```
# %load solutions/image-pyramids/laplacian_blending.py
LS = []
for la, lb in zip(lpA, lpB):
    rows, cols, dpt = la.shape
    ls = np.hstack((la[:,0:cols//2,:], lb[:,cols//2:,:]))
    LS.append(ls)

plt.figure(figsize=(20,10))
plt.title('Laplacian Pyramid Blended')
show_pyramid(LS)
```

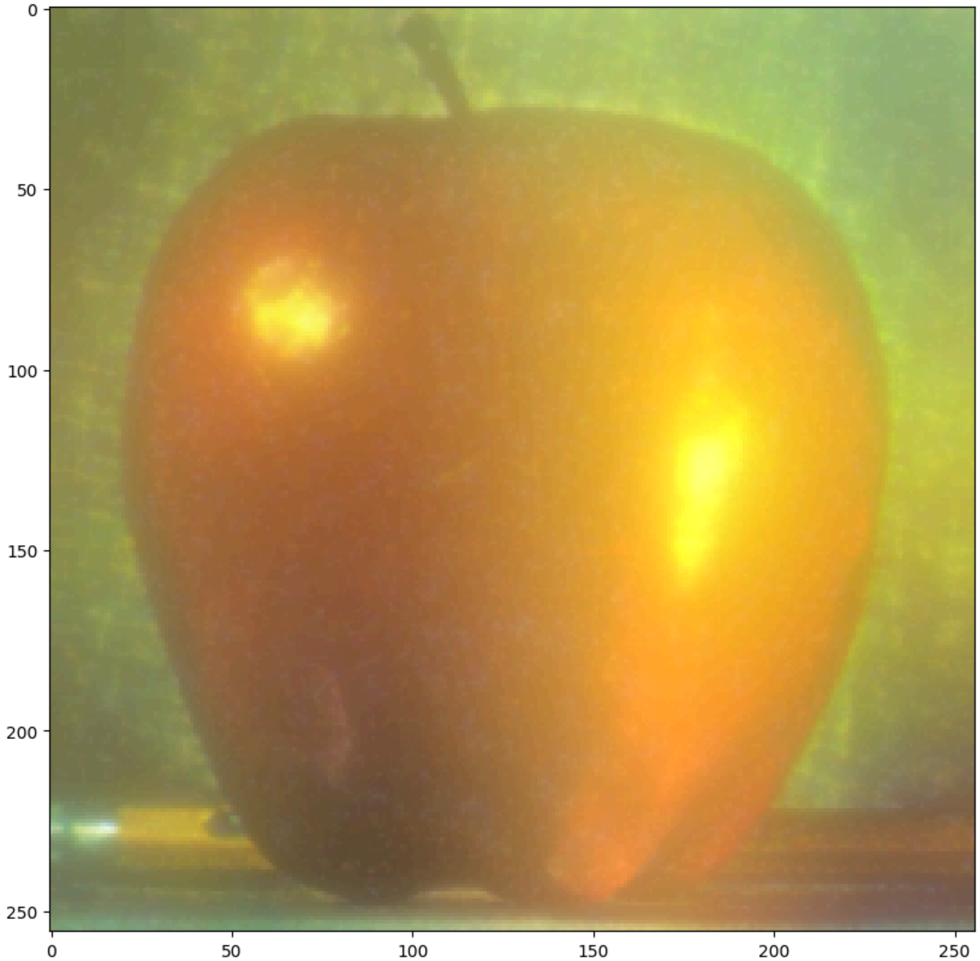


Reconstruction

In [26]:
```
# %load solutions/image-pyramids/reconstruct_from_laplacian.py
ls_ = LS[0]
for i in range(1,6):
    ls_ = cv.pyrUp(ls_)
    ls_ = cv.add(ls_, LS[i])
```

In [27]:
```
plt.figure(figsize=(10,10))
plt.imshow(ls_)
```

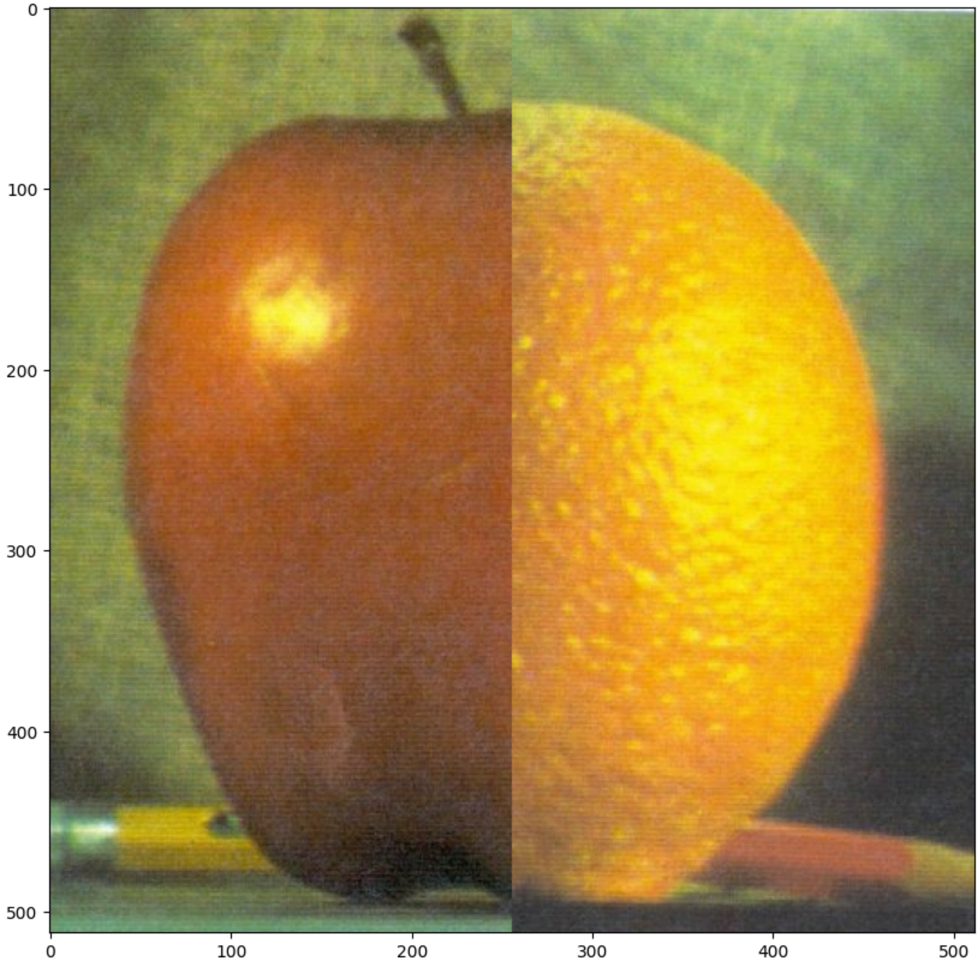
Out[27]: <matplotlib.image.AxesImage at 0x14a4c4350>



Blending without Laplacian Pyramids

In [28]:
```
real = np.hstack((A[:,:cols//2],B[:,cols//2:]))
```

In [29]:
```
plt.figure(figsize=(10,10))
plt.imshow(real);
```



Conclusions and Summary

- Gaussian pyramid
  - Coarse-to-fine search
  - Multi-scale image analysis (*hold this thought*)

- Laplacian pyramid
  - More compact image representation
  - Can be used for image compositing (computation photography)

- Downsampling
  - *Nyquist limit*: The Nyquist limit gives us a theoretical limit to what rate we have to sample a signal that contains data at a certain maximum frequency. Once we sample below that limit, not only can we not accurately sample the signal, but the data we get out has corrupting artifacts. These artifacts are "aliases".
  - Need to sufficiently low-pass before downsampling

Various image representations

- Pixels: great for spatial processing, poor access to frequency
- Fourier transform: great for frequency analysis, poor spatial info (soon)
- Pyramids: trade-off between spatial and frequency information

```
In [ ]:
```

1 2 3 4 1 1 3 2 1

1 1 2 2 2 2 1 1 3