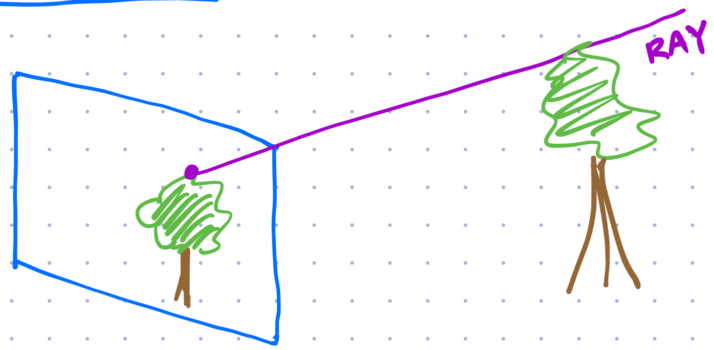
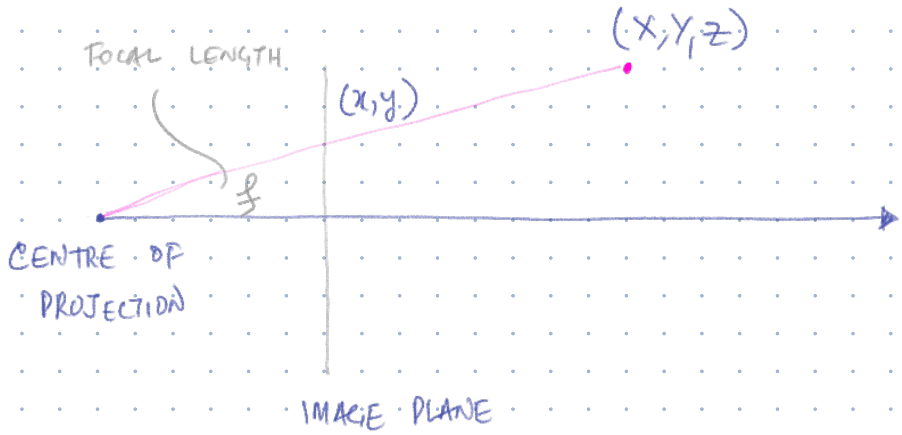
IMAGE FORMATION



PINHOLE CAMERA MODEL

Relates points in the world (X, Y, Z) to points in the image plane (x, y).



$$x = f\frac{X}{Z}$$

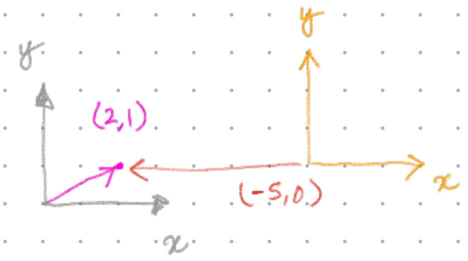
$$y = f\frac{Y}{Z}$$

$$\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{CAMERA}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\text{WORLD}}$$

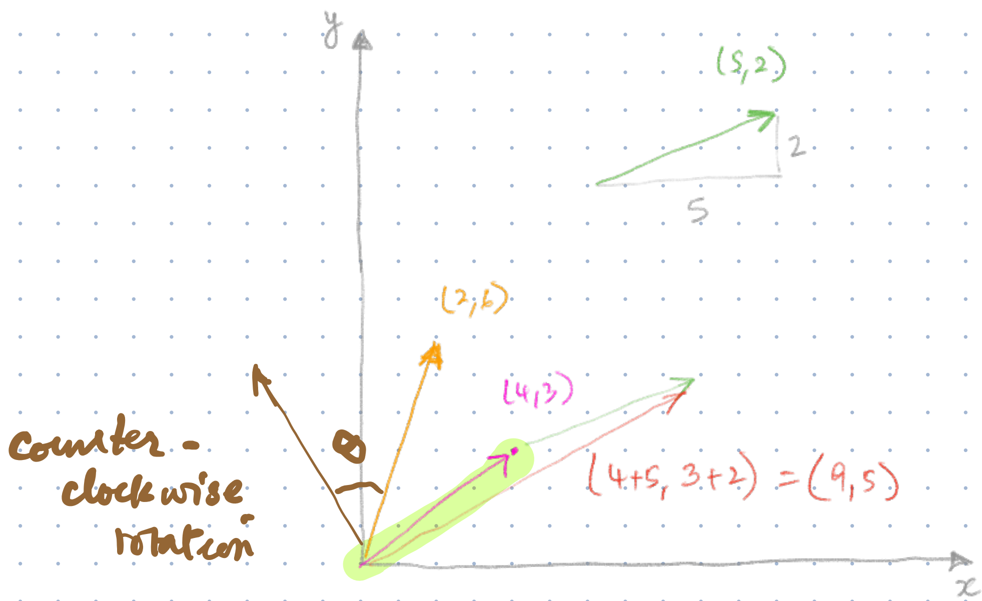
$$\Big\downarrow \ \text{Cartesian Space}$$

$$\underbrace{\left(\frac{fX}{Z}, \frac{fY}{Z} \right)}_{\text{IMAGE}}$$

MULTIPLE COORDINATE SYSTEMS



TRANSLATION & ROTATION



ROTATION $\quad \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

TRANSLATION $\quad \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

TRANSLATING & ROTATING

vector : $\vec{v}$

rotate : R

translate : $\vec{t}$

new vector : $\vec{v}' = R\vec{v} + \vec{t}$ — (1)

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\vec{t} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \cdot \\ \cdot \\ 1 \end{bmatrix} \quad \vec{v}_h' = \begin{bmatrix} \cos\theta & -\sin\theta & 5 \\ \sin\theta & \cos\theta & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} \qquad 2$$

Cartesian

$$\vec{v}'$$

ORTHOGONAL MATRIX

$$\begin{bmatrix} \vdots & \vdots & & \vdots \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

$$\vec{v}_i^T \vec{v}_j = 0 \quad \forall\, i \neq j$$
$$= 1 \quad i = j$$

3D ROTATIONS

3x3 Matrix

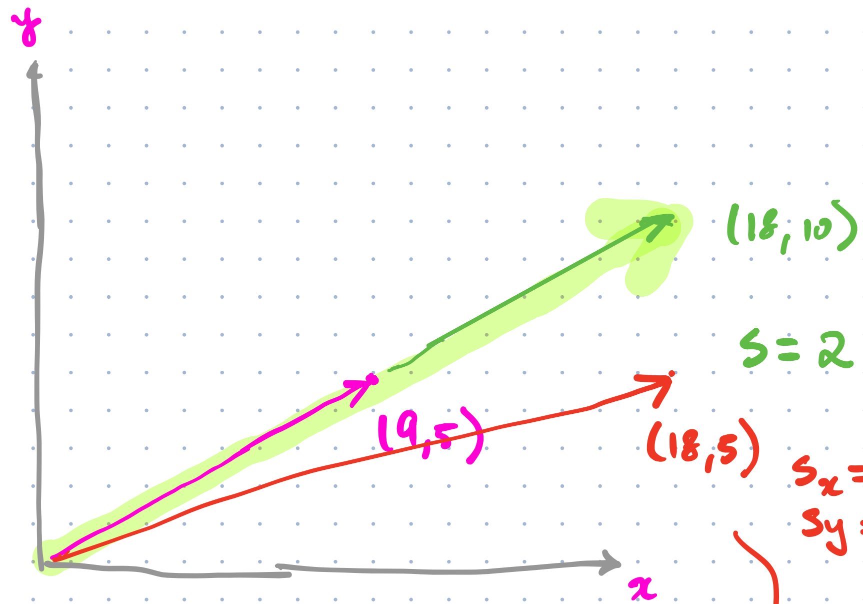
3D TRANSLATIONS

3D vector

EXAMPLE:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

SCALE



$S_x = 2$
$S_y = 1$

$$\begin{bmatrix} s9 \\ s5 \end{bmatrix} = s \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

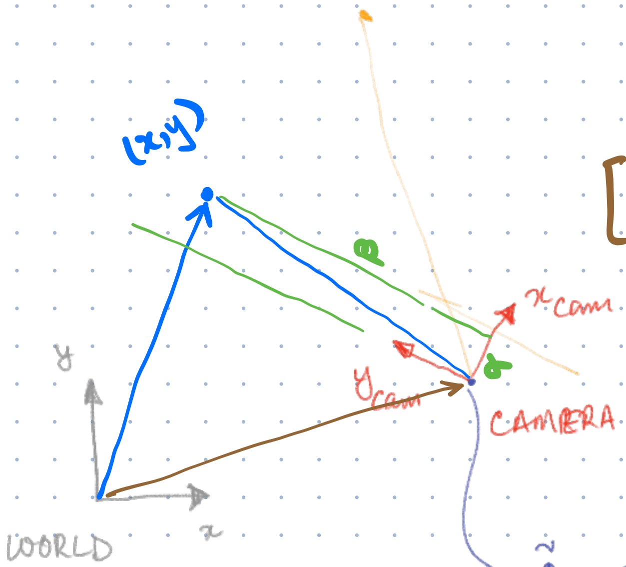
$$\begin{bmatrix} S_x \\ S_y \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix} \quad \downarrow \text{ BAD}$$

$$\begin{bmatrix} S_x & S_y \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix} \quad \downarrow \text{ BAD}$$

$$\begin{bmatrix} S_x \\ S_y \end{bmatrix} \begin{bmatrix} 9 & 5 \end{bmatrix} = \begin{bmatrix} 9S_x & 5S_x \\ 9S_y & 5S_y \end{bmatrix} \quad \downarrow \text{ V. BAD}$$

$$\begin{bmatrix} S_x 9 \\ S_y 5 \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} 9 \\ 5 \end{bmatrix} \quad \uparrow \text{ GOOD}$$

CHANGE OF COORDINATE SYSTEM



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \alpha \tilde{R} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \tilde{R} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \tilde{R} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\Rightarrow \tilde{R}^{-1} \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} t_x \\ t_y \end{bmatrix} \right) = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\tilde{C}$: center of projection

$\tilde{R}$: rotation of the camera

1. $\tilde{C}$ in cam is $\vec{0}$
2. x_{cam} in cam is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
3. y_{cam} in cam is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
4. $\tilde{C} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$ in world
5. $x_{cam} = \tilde{R} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in world
6. $y_{cam} = \tilde{R} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in world

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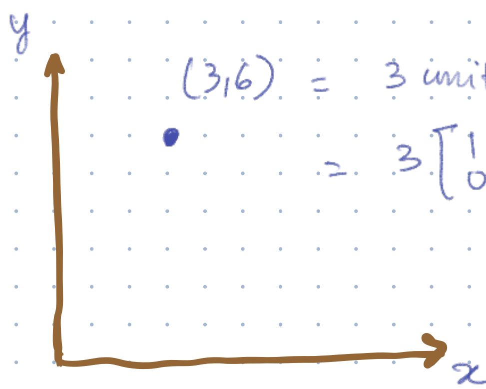
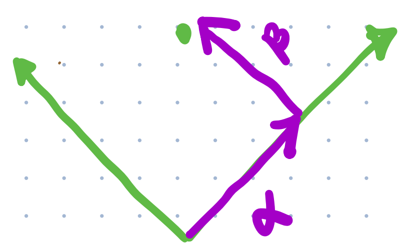


(3,6) = 3 units along x and 6 units along y

$= 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 6 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$$\begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{CAMERA}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

~~WORLD~~ CAMERA

$\downarrow$ Cartesian Space

$$\left(\frac{fX}{Z}, \frac{fY}{Z} \right)$$

IMAGE

$\tilde{C}$: centre

$\tilde{R}$: camera rotation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} & \tilde{R} & \end{bmatrix}^{-1} \left(\begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} - \begin{bmatrix} \tilde{C} \end{bmatrix} \right)$$