



IDEA: FIT A LINE.

$$y = mx + c$$

↑    ↑  
?    ?

At least two points.

Model Fitting: find the model parameters  $(m, c)$  that leads to smallest distance.

$x$	$y$	Model
$x_1$	$y_1$ ← ground truth.	$\hat{y}_1 = mx_1 + c$
$x_i$	$y_i$	$\hat{y}_i = mx_i + c$
input → $x_n$	$y_n$ ↑ Training data.	$\hat{y}_n = mx_n + c$



$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - mx_i - c)^2 = \epsilon$$

We need to compute derivatives w.r.t. both  $m$  &  $c$ .

$$\begin{aligned} \frac{\partial \epsilon}{\partial m} &= \frac{1}{n} \sum_{i=1}^n 2(y_i - mx_i - c)(-x_i) \\ &= -\frac{1}{n} \sum_{i=1}^n 2x_i(y_i - mx_i - c) \end{aligned}$$

$$\frac{\partial \epsilon}{\partial c} = -\frac{1}{n} \sum_{i=1}^n 2(y_i - mx_i - c)$$

To solve analytically set to 0 and solve for  $m$  &  $c$ .

Gradient of the error surface:

$$\begin{bmatrix} \frac{\partial \epsilon}{\partial m} \\ \frac{\partial \epsilon}{\partial c} \end{bmatrix}$$

Good news: For this problem, we are dealing with a quadratic surface.

So, we have a global minima (unique solution).