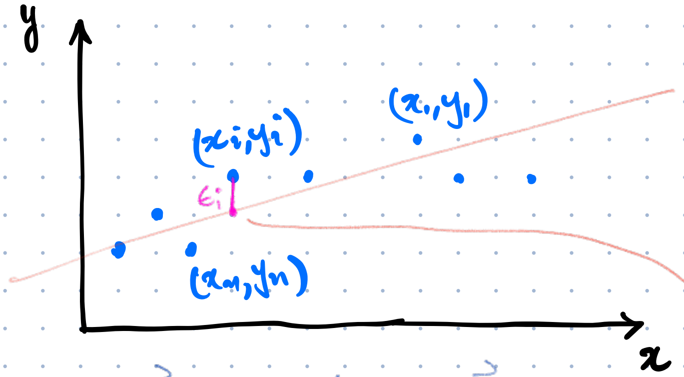
March 4, 2026.



$$y = mx + b$$

$$y = \theta_1 x + \theta_0$$

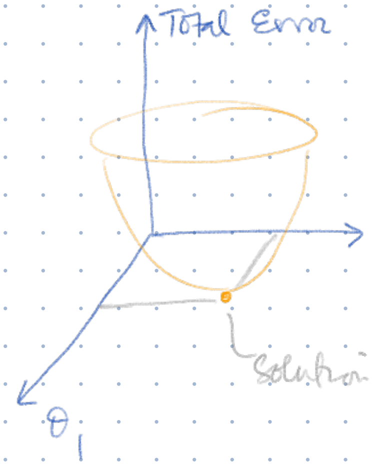
$$y = \begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix}$$

$$\epsilon_i = y_i - (\theta_1 x + \theta_0)$$

Total loss: ~~$\epsilon_1 + \cdots + \epsilon_n$~~

-//- : $\epsilon_1^2 + \cdots + \epsilon_n^2$

$$\begin{array}{ccccc} \vec{y} & & A & & \vec{p} \\ \begin{bmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix} & = & \begin{bmatrix} x_1 & 1 \\ \vdots & \\ x_i & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix} & & \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix} \\ \in \mathbb{R}^{n\times 1} & & \in \mathbb{R}^{n\times 2} & & \in \mathbb{R}^{2\times 1} \end{array}$$



$$A\vec{p} = \vec{y}$$

$$\Rightarrow A^T A \vec{p} = A^T y$$

$$\Rightarrow \vec{p} = (A^T A)^{-1} A^T y$$

θ_1 & θ_0

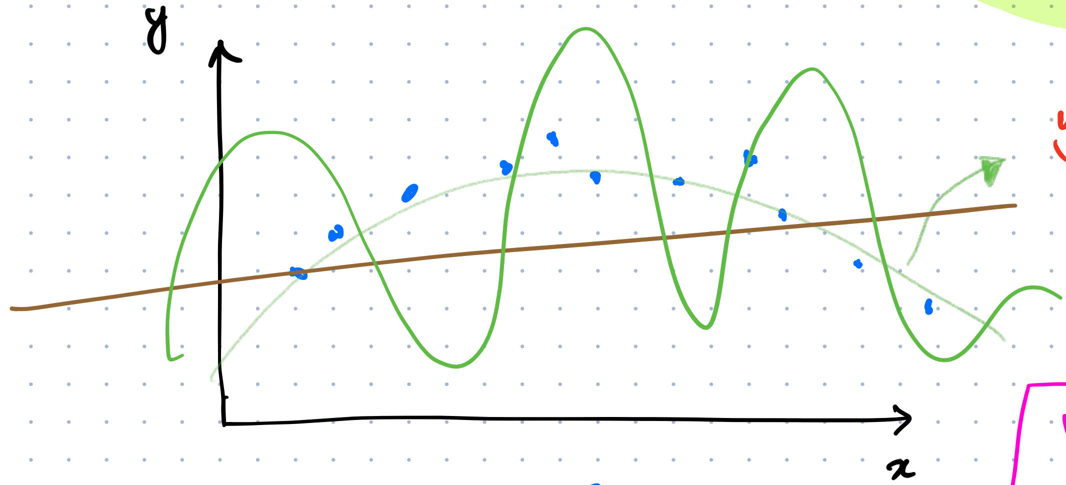
Total error: $E(\theta_0, \theta_1)$

$$\frac{\partial E}{\partial \theta_0} = 0$$

$$\frac{\partial E}{\partial \theta_1} = 0$$

gradient

$$\nabla E = \begin{bmatrix} \frac{\partial E}{\partial \theta_0} \\ \frac{\partial E}{\partial \theta_1} \end{bmatrix}$$



$$y = x^2\theta_2 + x\theta_1 + \theta_0$$

$$\boxed{y = x\theta_0\theta_1 + \theta_1}$$

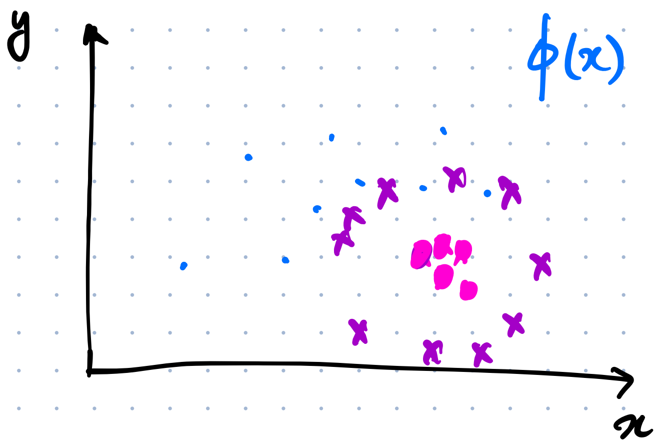
non-linear model.

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ & & \\ x_n^2 & x_n & 1 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_1 \\ \theta_0 \end{bmatrix}$$

$$\vec{y} = A \vec{p}$$

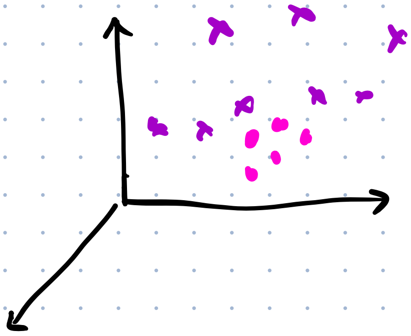
$$\Rightarrow \vec{p} = (A^T A)^{-1} A^T \vec{y}$$

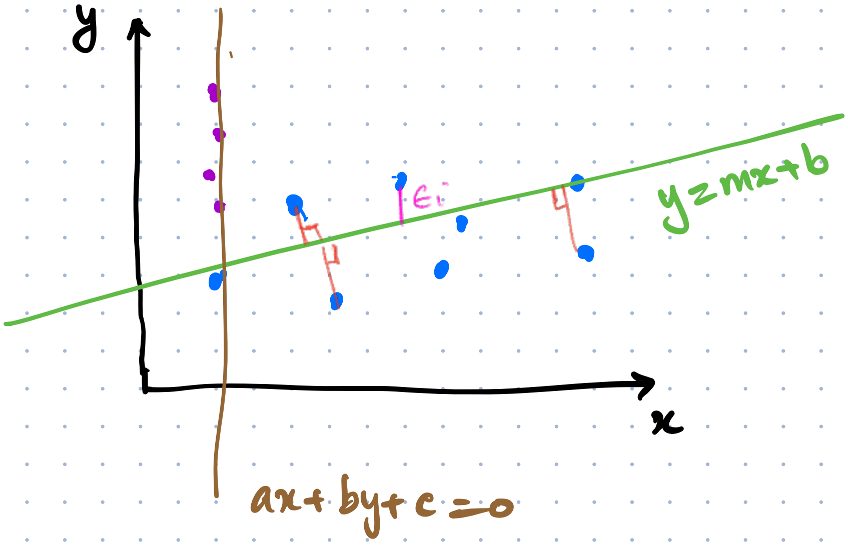
$$\vec{p} = A \backslash \vec{y}$$ matlab.



$\phi(x)$

$$y = \theta_1 \phi_1(x) + \theta_0 \phi_0(x)$$





$$A\vec{p} = \vec{y}$$

$$A\vec{p} = \vec{0} \qquad \vec{p}^{T}p = 1$$

$$\vec{p} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$A = \begin{bmatrix} x_1 - \langle x \rangle & y_1 - \langle y \rangle \\ x_n - \langle x \rangle & y_n - \langle y \rangle \end{bmatrix}$$