February 4, 2026.

Signal : 3 2 4 5 6

filter : 1 0 -1

Q. How to deal with boundaries?

(i) 3 2 4 5 6

-1 0 1

1 □ □

$3 = 2w + 1$

$\Rightarrow 2w = 3 - 1$

$\Rightarrow w = 1$

(ii) 0 3 2 4 5 6 0

-1 0 1

2 □ □ □ □

(iii) 1 3 2 4 5 6 1

-1 0 1

1 □ □ □ □

(iv) 6 3 2 4 5 6 3

-1 0 1

4 □ □ □ □

(v) 2 3 2 4 5 6 5

-1 0 1

0 □ □ □ □

* Cost of computing convolutions?

Example: $4000 \times 4000 \times 3 \leftarrow$ image

$515 \times 515 \leftarrow$ filter

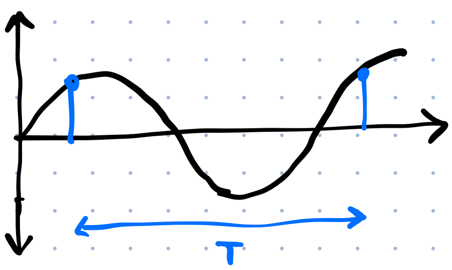
* Fourier Transforms

(i) Periodic functions: $f(t+T) = f(t)$ for $T > 0$

T → Fundamental period

$f(t + nT) = f(t)$

→ every integer multiple of a fundamental period is also a period.



Examples: What is the period of $\cos(4\pi t)$?

$$\cos(4\pi t) = \cos(4\pi t + 2\pi)$$
$$= \cos(4\pi(t + 1/2))$$

↓ 1/2

* Harmonic oscillator

$$A \sin(\omega t + \phi) = A \sin(2\pi f t + \phi)$$

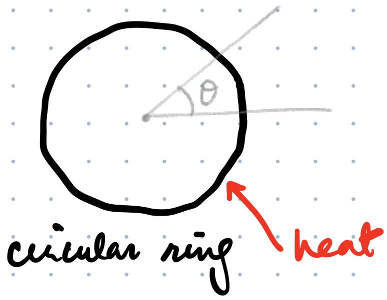
ω → angular freq.

A → amplitude

f → frequency

ϕ → phase

* Sum of sinusoids



$$\text{Temperature} = \sum_{n=1}^{N} A_n \underbrace{\sin(n\theta + \phi)}_{\text{harmonies}}$$

- Jean Baptiste Joseph Fourier

  Any univariate function can be re-written as a sum of sines and cosines of different frequencies (1807)

- Fourier series

$$\sum_{n=1}^{N} A_n \sin(2\pi n t + \phi_n)$$

$$A_n \sin(2\pi n t + \phi_n) = A_n \{\sin(2\pi n t)\cos\phi_n + \cos(2\pi n t)\sin\phi_n\}$$

$$= \underbrace{a_n}_{A_n \sin\phi_n} \cos(2\pi n t) + \underbrace{b_n}_{A_n \cos\phi_n} \sin(2\pi n t)$$

$$\Rightarrow \sum_{n=1}^{N} A_n \sin(2\pi n t + \phi_n) = \sum_{n=1}^{N} a_n \cos(2\pi n t) + b_n \sin(2\pi n t)$$

Hint: $\cos t = \dfrac{e^{it} - e^{-it}}{2}$

$\sin t = \dfrac{e^{it} + e^{-it}}{2i}$, $i = \sqrt{-1}$

$$\Rightarrow \frac{a_0}{2} + \sum_{n=1}^{N} a_n \cos(2\pi n t) + b_n \sin(2\pi n t) = \boxed{\sum_{n=-N}^{N} c_n e^{2\pi i n t}}$$

any periodic fn.

$$c_n = \frac{a_n + i b_n}{2}$$

$$c_{-n} = \frac{a_n - i b_n}{2}$$

$$c_0 = \frac{a_0}{2}$$

$\ast$ How do we solve for c_n?

$$f(t) = \sum_{n=-N}^{N} c_n e^{2\pi i n t}$$

. we scale $f(t)$ s.t. its period is 1.

$$e^{-2\pi i k t} f(t) = \sum_{n=-N}^{N} c_n e^{-2\pi i k t} e^{2\pi i n t}$$

$$= \cdots + c_k + \cdots$$

$$\Rightarrow \quad c_k = e^{-2\pi i k t} f(t) - \sum_{n=-N,\, n\neq k}^{N} c_n e^{2\pi i (n-k) t}$$

$$\int_0^1 c_k \, dt = \int_0^1 e^{-2\pi i k t} f(t)\, dt - \underbrace{\int_0^1 \sum_{n=-N,\, n\neq k}^{N} c_n e^{2\pi i (n-k) t}\, dt}_{\phi}$$

$$c_k = \int_0^1 e^{-2\pi i k t} f(t)\, dt$$

$\uparrow$

Fourier co-efficients

Often:

$$\hat{f}(n) = \int_{-1/2}^{1/2} e^{-2\pi i n t} f(t)\, dt$$

$\uparrow$
coeff

$$f(t) = \sum_{n=-N}^{N} \hat{f}(n) \underbrace{e^{2\pi i n t}}$$

represents oscillations at a freq. proportional to n

- Fundamental frequency f_0 corresponds to $n=1$
- Higher frequencies are multiples of this fundamental frequency.

Harmonies

(i) First harmonic $n=1$.

(ii) $n=1,2,3,\ldots$ represents harmonics. $\underbrace{\hat{f}(n)}$ represent amplitude and phase.

don't forget its a complex number.

* Exercise

Is
$$\begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$
separable ?

Decompose M using SVD.

$$M = U \Sigma V^T$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix} =$$