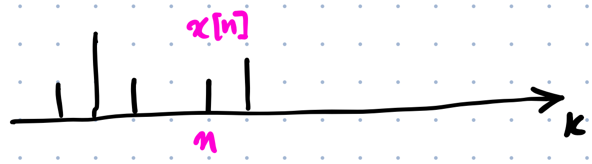
Jan 30, 2026

Discrete 1D signal: $x[k]$



(i) Average value of the signal of length N

(ii) Power

(iii) Compare two signals of length N

* Linear Systems

$$x[k] \longrightarrow \boxed{f} \longrightarrow y[k]$$

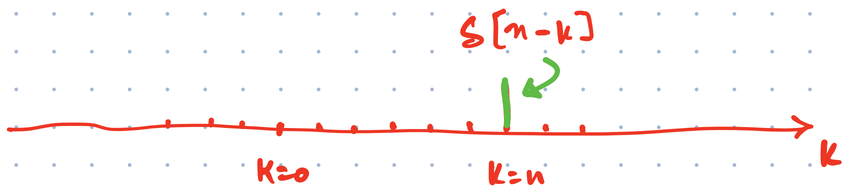
(i) $f(x_1 + x_2) = f(x_1) + f(x_2)$

(ii) $f(ax) = af(x)$ for some scalar a

* Convolutions

(i) Kronecker delta

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0 & \text{otherwise} \end{cases}$$



(ii) Multiply any signal $x[k]$ with impulse $\delta[n-k]$ we grab the value $x[n]$.

(iii) We can represent a signal as

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \, \underbrace{\delta[n-k]}_{\text{zero every where except at } k=n.}$$

Now say that we have linear system H.

$$H[x[n]] = H\left[\sum_{k=-\infty}^{\infty} x[k] \, \delta[n-k]\right]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, \underbrace{H\left[\delta[n-k]\right]}_{\text{Impulse response}} \quad \text{—— (1)}$$

* Linear Translation (Shift) Invariant System

- Response to shifted impulse is simply the shifted version of the response to impulse.

$\therefore$ if $H[\delta[k]] = h[k]$

$H[\delta[n-k]] = h[n-k]$

For LSI/LTI systems.

$$\underbrace{H[x[n]]}_{(x*h)_n} = \sum_{k=-\infty}^{\infty} \underset{\text{signal}}{x[k]}\; \underset{\text{Kernel/filter/mask}}{h[n-k]} \quad \leftarrow \text{Convolution}$$

(i) $x * h = h * x$

$$(x*h)_n = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Let $m = n - k$

$\Rightarrow k = n - m$

(i) As $k \to \infty$, $m \to -\infty$

$k \to -\infty$, $m \to \infty$

$$(x*h)_n = \sum_{m=\infty}^{-\infty} x[n-m]\, h[m]$$

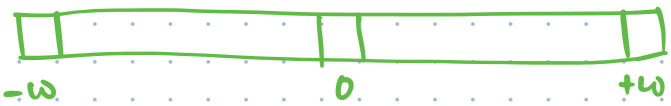
$$= \sum_{m=-\infty}^{\infty} x[n-m]\, h[m]$$

$$h = \begin{bmatrix} -3 & 1 & 2 & 0 & 7 \end{bmatrix}$$
$$\begin{matrix} -2 & -1 & 0 & 1 & 2 \end{matrix} \leftarrow \text{positions}$$

$$(x * h)_n = \sum_{m=-2}^{2} x[n-m]\, h[m]$$

Or more generally if you have a filter of width $2w+1$.

$$(x * h)_n = \sum_{m=-w}^{w} x[n-m]\, h[m]$$

$h =$ 

Example:

n

x: 0 1 1 3 4 5 7 8 1 1

0 1 2 3 4 5 6 7 8 9 ← pos

-7 3 1 0 2

h: -7 3 1 0 2

-2 -1 0 1 2 ← pos

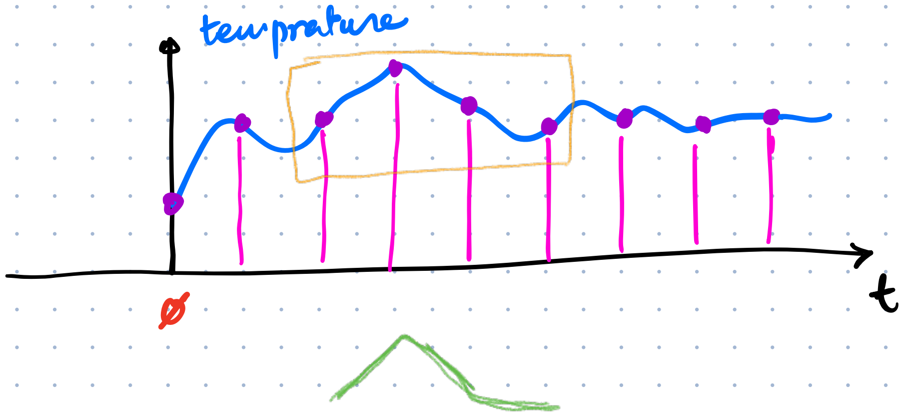
$$
\begin{aligned}
(x * h)_4 &= x[4-(-2)]\, h[-2] \\
&+ x[4-(-1)]\, h[-1] \\
&+ x[4-0]\, h[0] \\
&+ x[4-1]\, h[1] \\
&+ x[4-2]\, h[2] \\
&= (7)(-7) \\
&+ (5)(3) \\
&+ (4)(1) \\
&+ (3)(0) \\
&+ (1)(2) \\
&= ?
\end{aligned}
$$

Cross-correlation

$$cc_n = \sum_{m=-w}^{w} x[n+m]\, h[m]$$

no flip

Signal



Separability