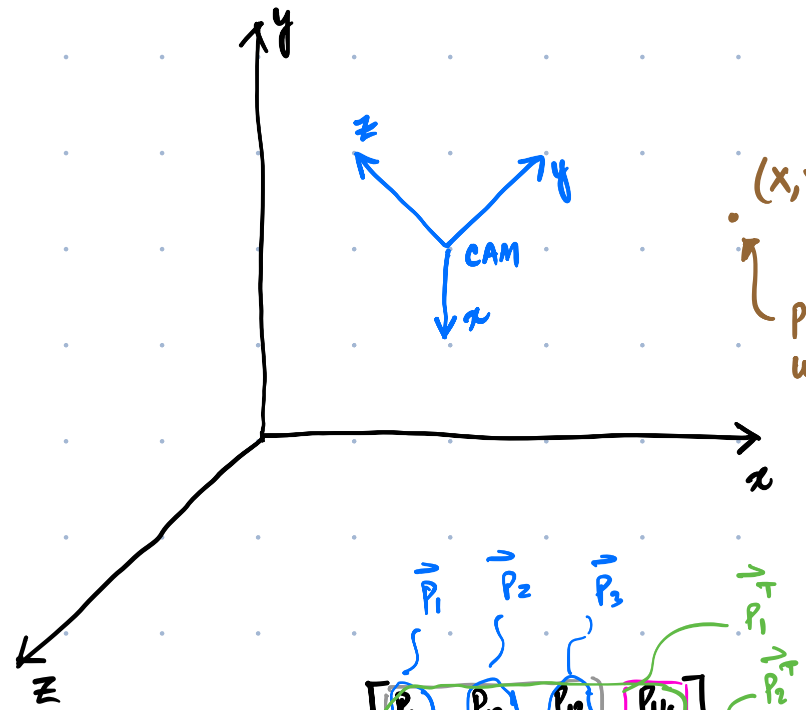
Jan 23, 2026




$\vec{X} = (X, Y, Z, 1)$

$(X, Y, Z)$ point in the world coordinate system.

$\vec{x} = P\vec{X}$

$$P = \begin{array}{c} \begin{array}{cccc} \vec{p}_1 & \vec{p}_2 & \vec{p}_3 & \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \end{array} \begin{array}{l} \vec{p}_1^{\,T} \\ \vec{p}_2^{\,T} \\ \vec{p}_3^{\,T} \end{array}$$

$$M = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \qquad \vec{p}_4 = \begin{bmatrix} p_{14} \\ p_{24} \\ p_{34} \end{bmatrix}$$

* Camera center:

(i) Right null-space of $P$. $\because$ $P\vec{C} = \phi$.

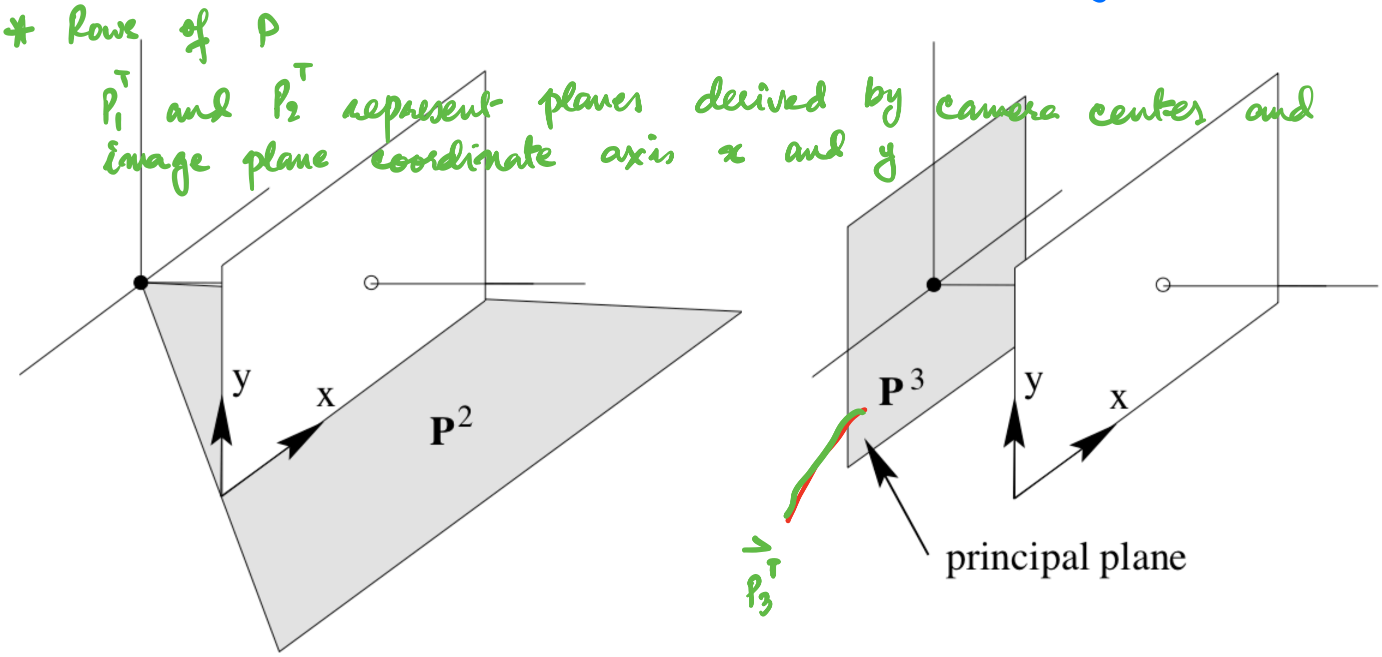
(ii) For finite cameras: $\tilde{C} = -M^{-1}\vec{p}_4$ where $C = (\tilde{C}, 1)^T$.

* Columns of $P$

$p_1, p_2$, and $p_3$ represent vanishing points in $x$, $y$, and $z$ directions

* Rows of $P$

$p_1^T$ and $p_2^T$ represent planes derived by camera centers and image plane coordinate axis $x$ and $y$



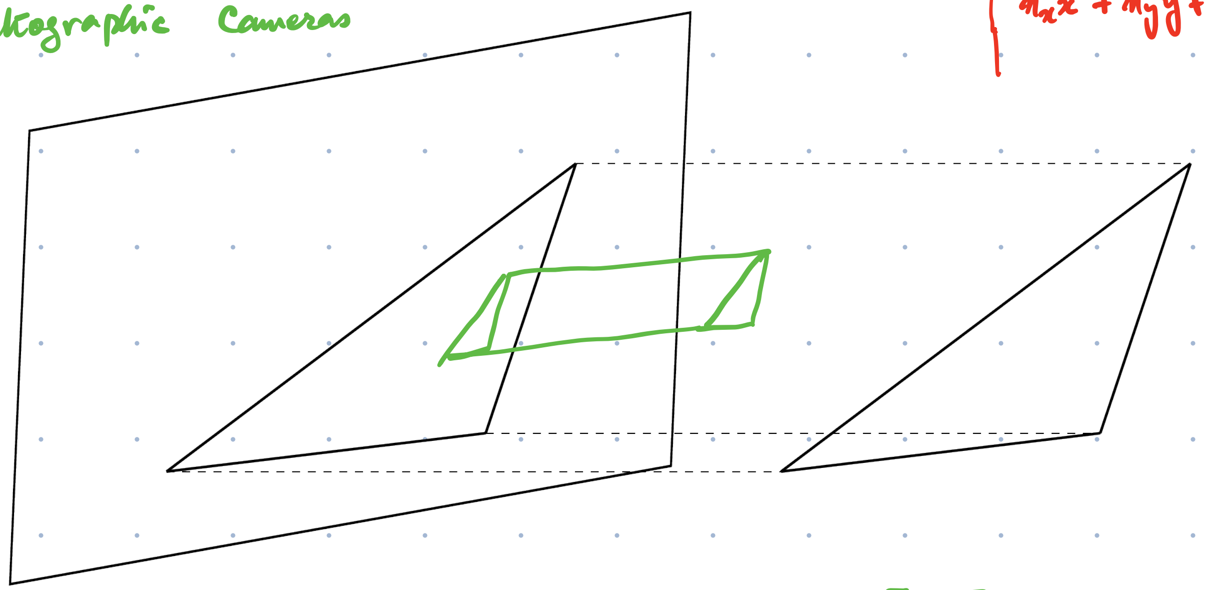
Eq. of a plane:

$$ax + by + cz + d = 0$$

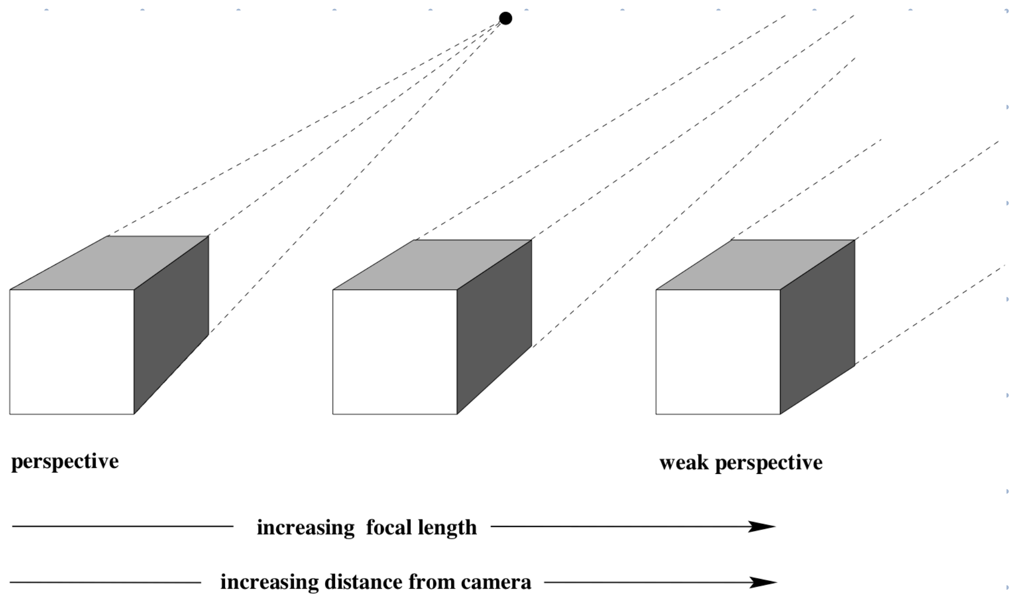
Eq. of a plane passing through $\vec{p}$ and normal $\hat{n}$.

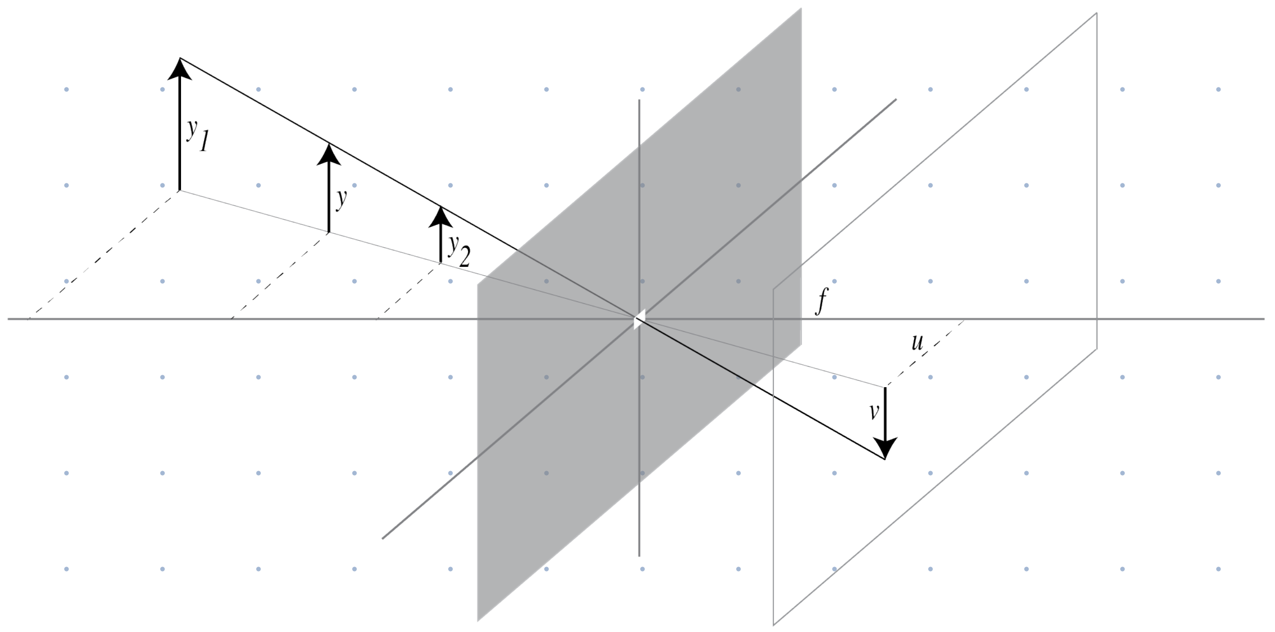
$$n_x x + n_y y + n_z z + \vec{n}\cdot\vec{p} = 0$$

## Orthographic Cameras

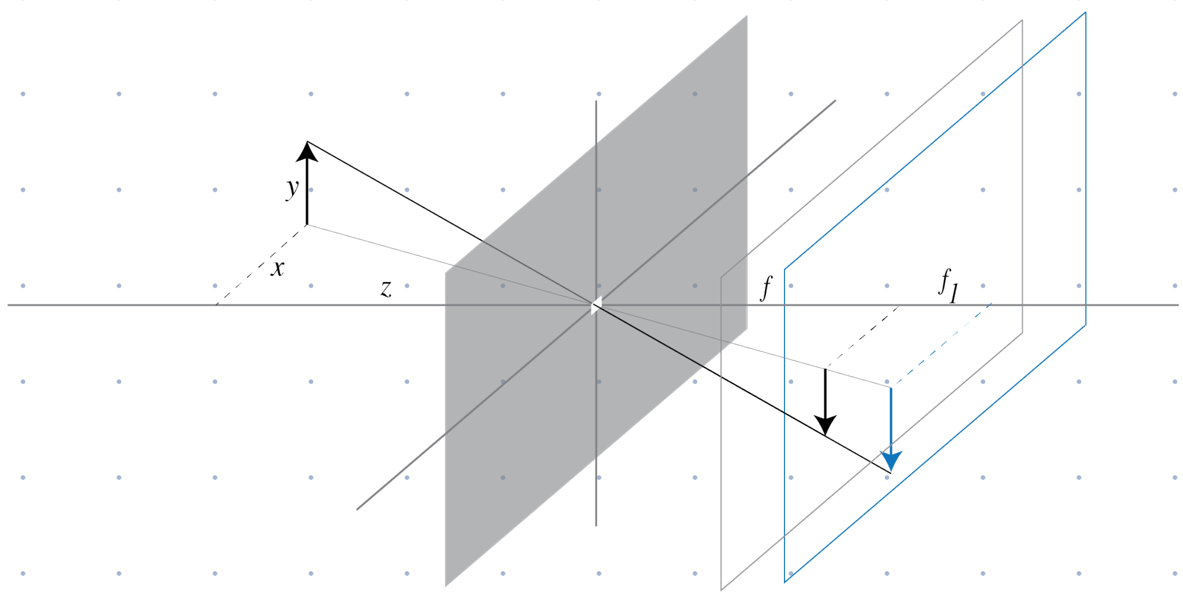


$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

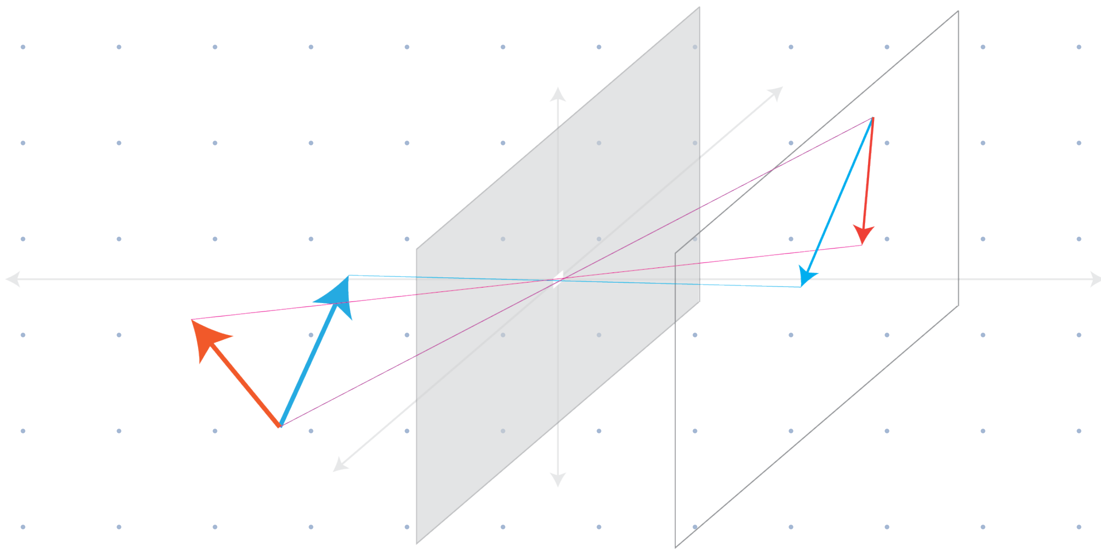


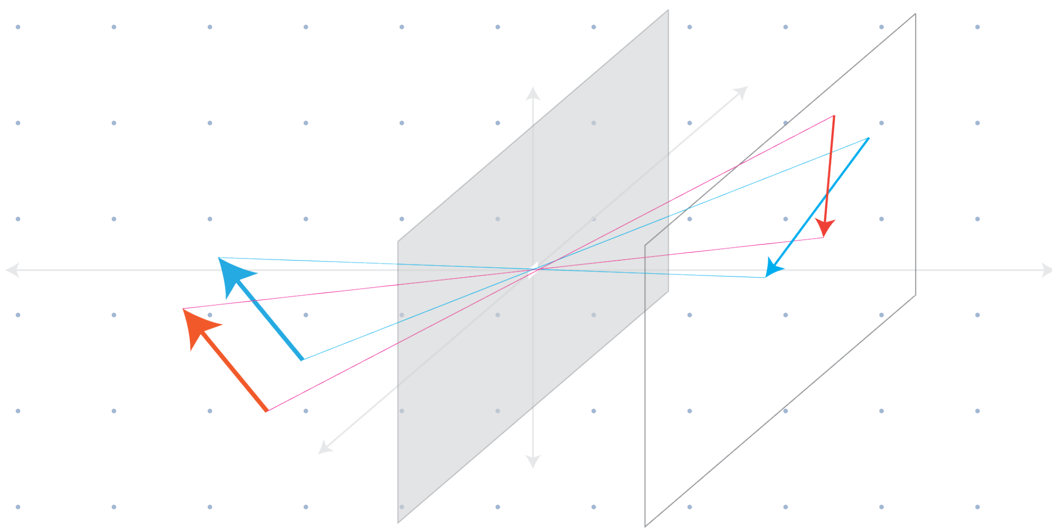


$y_1$
$y$
$y_2$
$f$
$u$
$v$

$y$
$x$
$z$
$f$
$f_1$





* Forward Projection

$$\vec{x} = P\vec{X}$$

* Backprojection

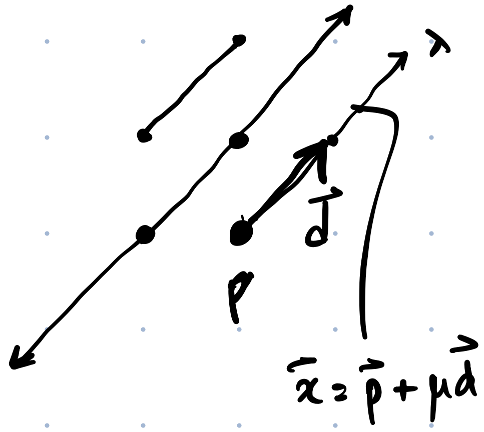
Maps image points to rays in the world

$$\vec{X}(\lambda) = P^{+}\vec{x} + \lambda\vec{C}$$

image point

pseudo inverse of $P$

Camera center




$$P^+ = P^T (P P^T)^{-1}$$

For finite cameras:

$$\vec X(\mu) = \mu \begin{pmatrix} M^{-1}\vec x \\ \emptyset \end{pmatrix} + \begin{pmatrix} -M\vec p_4 \\ 1 \end{pmatrix}$$

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$$P \begin{cases} M \\ \vec p_4 \end{cases}$$

Center of projection: $\begin{bmatrix} -M\vec p_4 \\ 1 \end{bmatrix}$

$$\vec X(\mu) = \mu \begin{pmatrix} M^{-1}\vec x \\ 0 \end{pmatrix} + \begin{pmatrix} -M\vec p_4 \\ 1 \end{pmatrix}$$