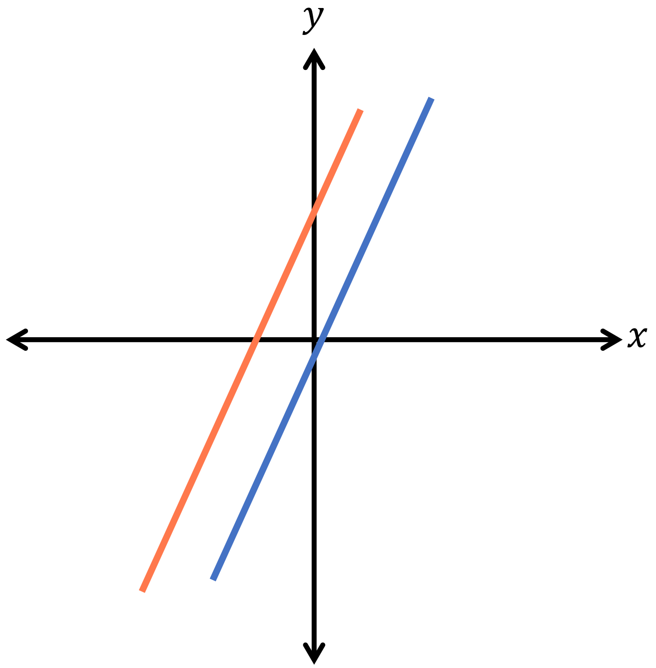
Intersecting two parallel lines



$y = 3x + 2 \longrightarrow$

$3x - y + 2 = \emptyset$ ①

$3x - y + 1 = \emptyset$ ②

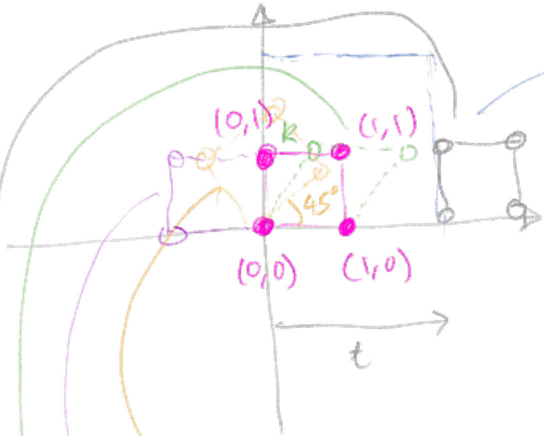
$$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 3 & -1 & 1 \end{vmatrix}$$

$$= i(-1+2) - j(3-6) + k(-3+3)$$

$$= \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$ Homogeneous Coordinate

Cartesian

$$\left(\frac{1}{0}, \frac{-3}{0} \right) ?$$



SCALE

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

ROTATION

$$\begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xC45 - yS45 \\ xS45 + yC45 \end{bmatrix}$$

REFLECT

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

SHEAR

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+ky \\ y \end{bmatrix}$$

TRANSLATION

$$\begin{bmatrix} t \\ 0 \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+t \\ y \end{bmatrix}$$

AFFINE TRANSFORMATION

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underset{\text{transformation matrix}}{A}\begin{bmatrix} x \\ y \end{bmatrix} + \underset{\text{translation}}{\begin{bmatrix} t_x \\ t_y \end{bmatrix}}$$

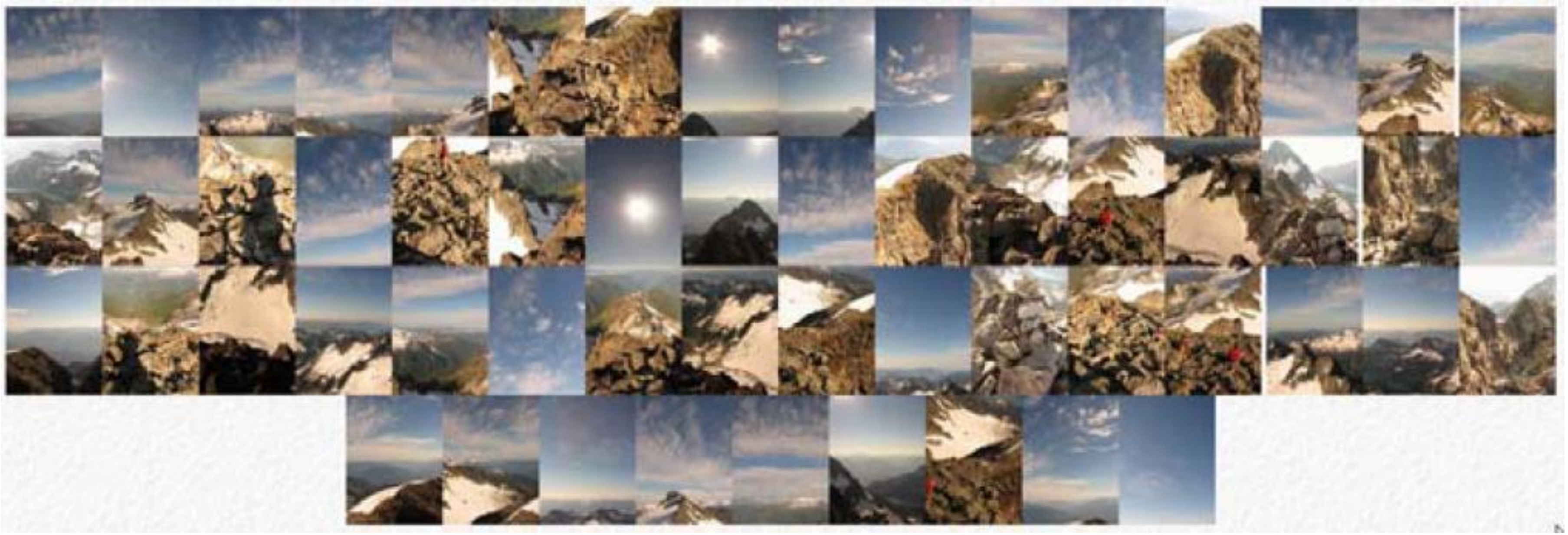
Use Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ \lambda \end{bmatrix} = \overset{A}{\begin{bmatrix} \boxed{2\times 2} & \begin{matrix} t_x \\ t_y \end{matrix} \\ 0 \quad 0 & 1 \end{bmatrix}}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\rightarrow \left(\frac{x'}{\lambda}, \frac{y'}{\lambda}\right)$$

$$\underset{4\times N}{\begin{bmatrix} | | | \cdots | \end{bmatrix}} = \underset{4\times 4}{\begin{bmatrix} A \end{bmatrix}}\underbrace{\begin{bmatrix} | | | \cdots | \end{bmatrix}}_{4\times N} \in \mathbb{R}^{4\times N}$$

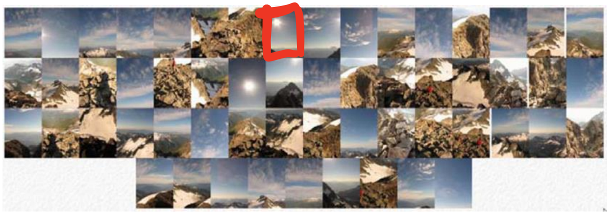
Image stitching



57 images

Camera should change orientation only, not position.
Keep camera settings (gain, focus, speed, aperture) fixed, if possible.

Image stitching



Using 28 out of 57 images

Image stitching

Using all 57 images

Image stitching (Autostitch)

Seams are not visible

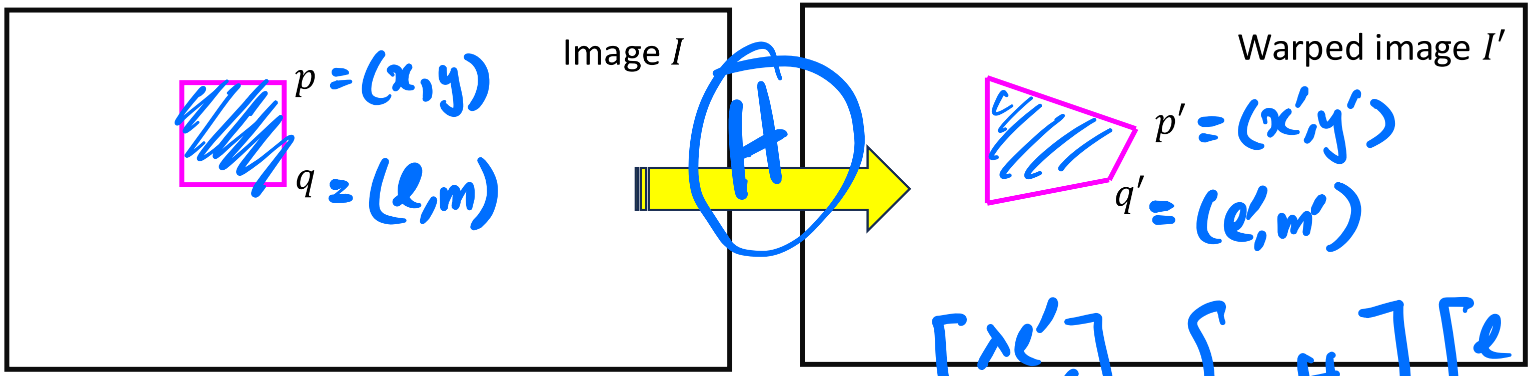
Using all 57 images. Laplacian blending.

Brown & Lowe; ICCV 2003

Linear image wraps

- To align multiple photos for image stitching, we must warp these images in such a way that all lines are preserved.
  - Lines before warping remain lines after warping
- Linear image wraps and *homographies*

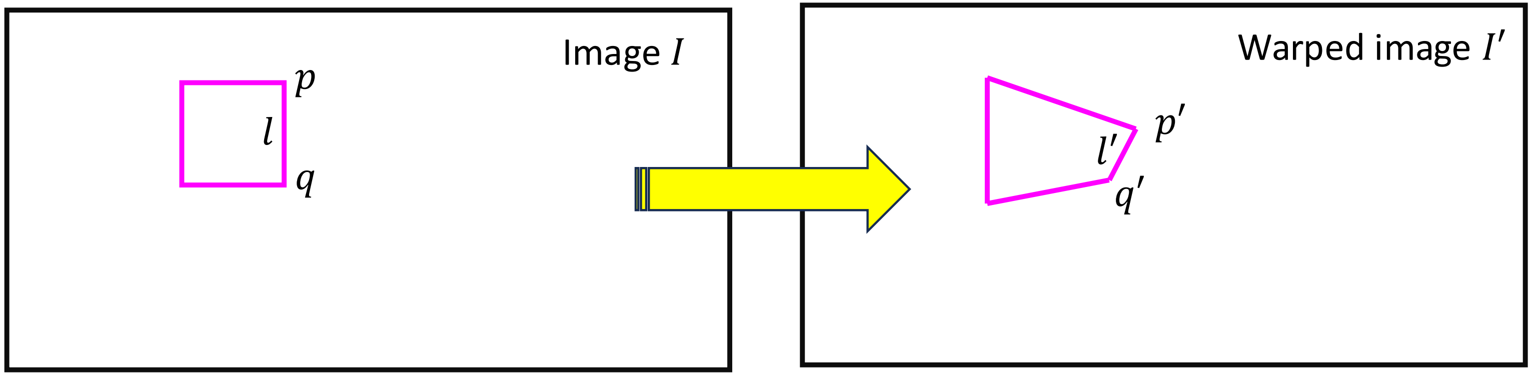
$$\begin{bmatrix} \lambda x' \\ \lambda y' \\ \lambda \end{bmatrix} = \underset{3\times 3}{\begin{bmatrix} & H & \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} \lambda l' \\ \lambda m' \\ \lambda \end{bmatrix} = \begin{bmatrix} & H & \end{bmatrix} \begin{bmatrix} l \\ m \\ 1 \end{bmatrix}$$

Linear image wraps

- Definition: an image warp is linear if every 2D line l in the original image is transformed into a line l' in the warped image
- Property: Every linear warp can be expressed as a 3×3 matrix H that transforms homogeneous image coordinates (we won't discuss the proof here)



Warping images using homography

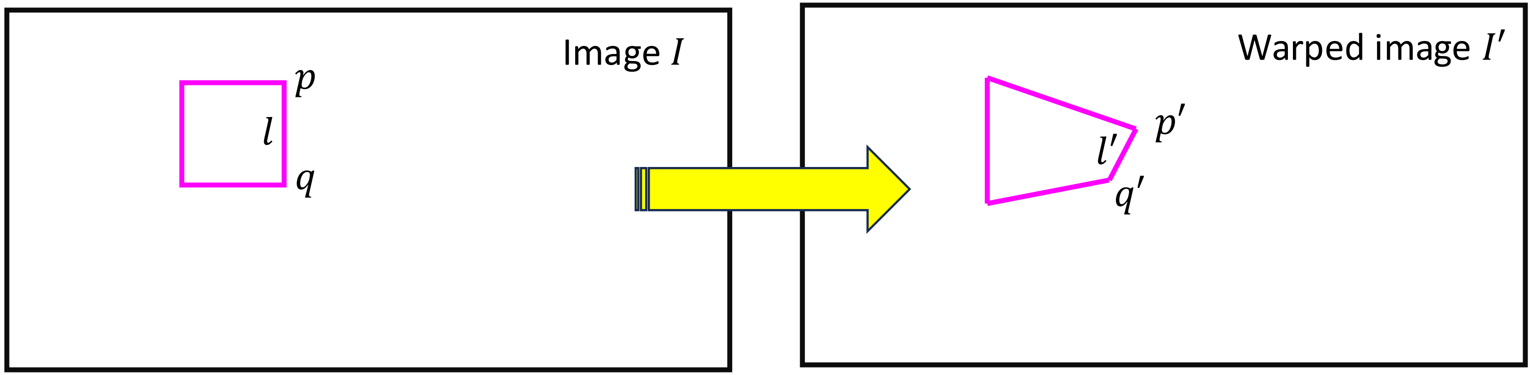
Linear warping equation: $I(\boldsymbol{p}) = I'(H\boldsymbol{p})$

Intensity at pixel in the source image I with homogeneous coordinates $\boldsymbol{p}$

Intensity at pixel in the warped image I' with homogeneous coordinates $H\boldsymbol{p}$

Matrix H is called homography

Scaling H by a factor $\lambda \neq 0$ does not change homography



$$H = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a/i & b/i & c/i \\ d/i & e/i & f/i \\ g/i & h/i & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a' & b' & c' \\ d' & e' & f' \\ g' & h' & 1 \end{bmatrix}$$

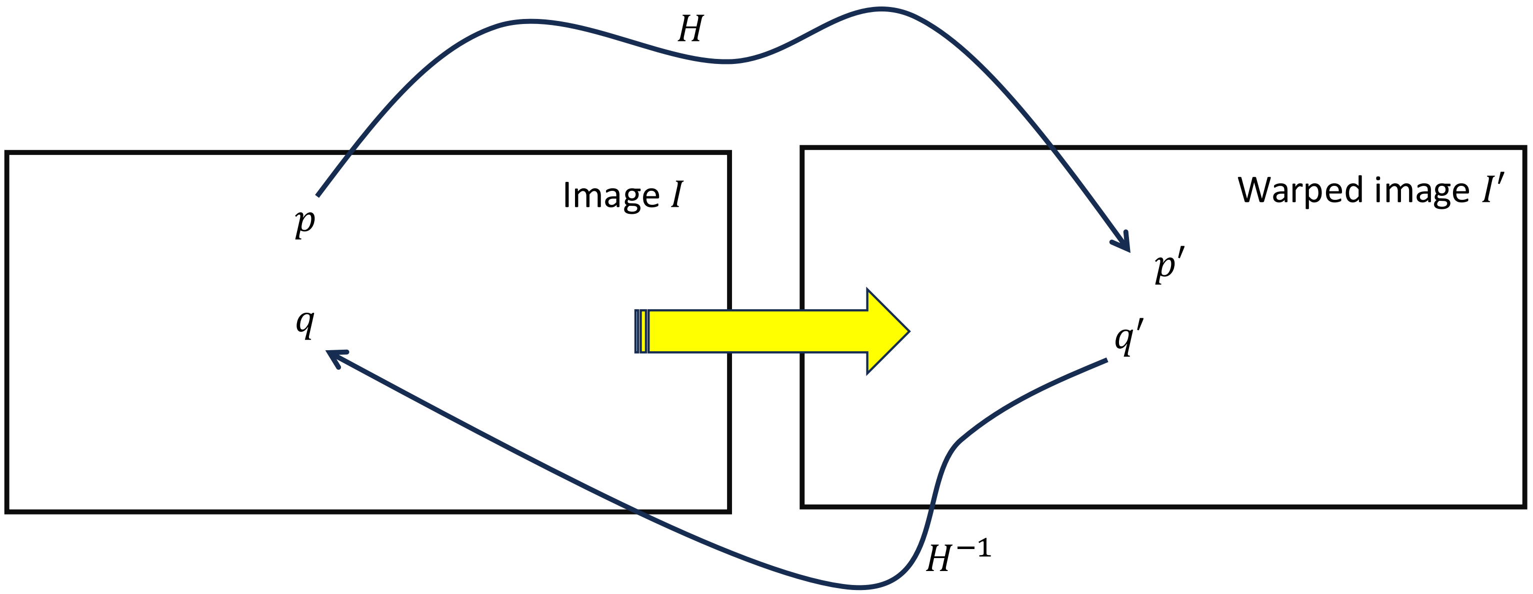
Pixel locations between the original image and its warped version.

8 unknowns.

Warping images using homography

Linear warping equation:

$$I(\boldsymbol{p}) = I'(H\boldsymbol{p}) \text{ and also } I'(\boldsymbol{q}') = I(H^{-1}\boldsymbol{q}')$$

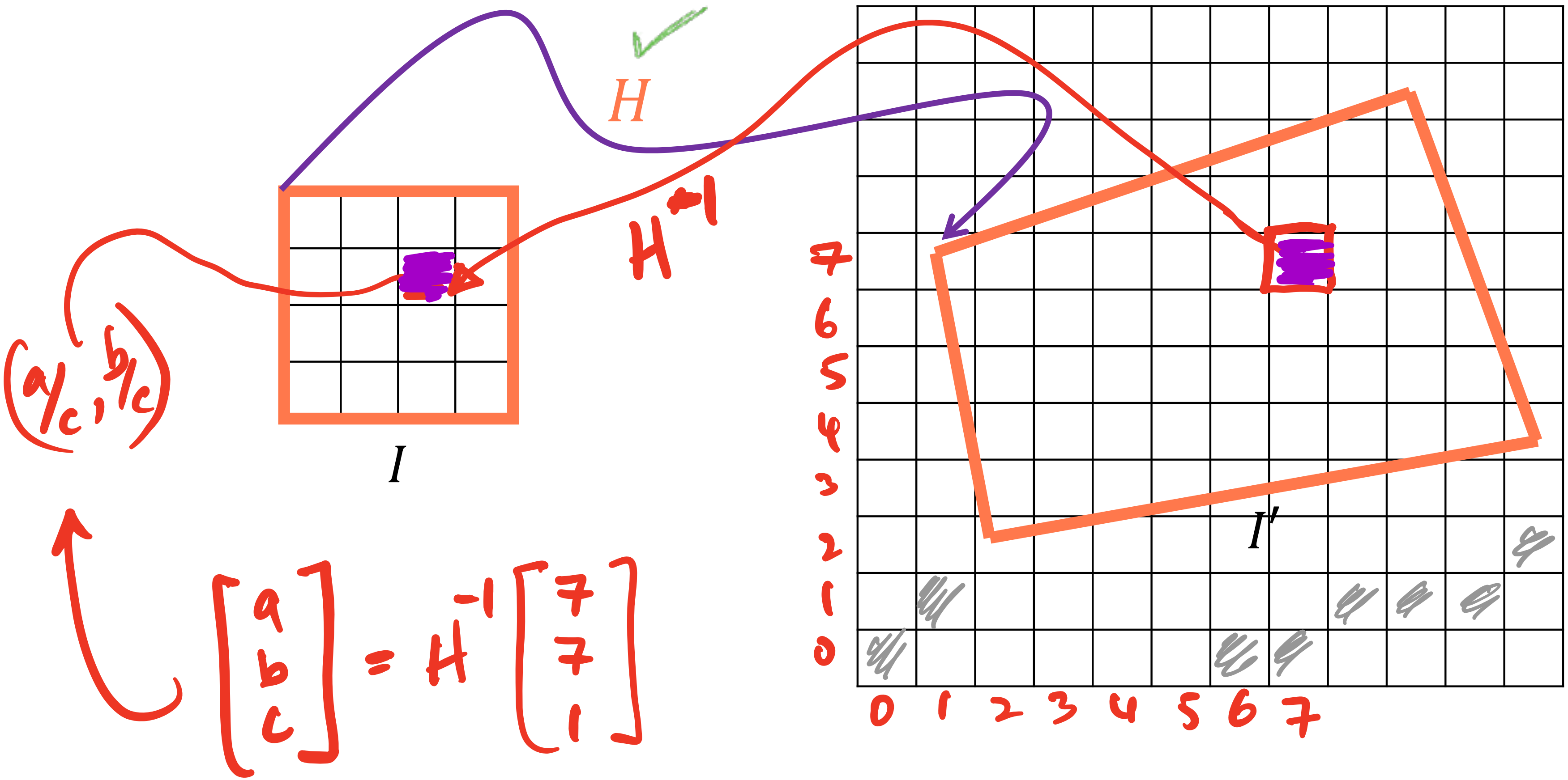


Computing warp I' from I and H

- Compute H^{-1}
- To compute the color of pixel (u, v) in the warped image
  - Compute $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = H^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$
  - Copy color from $I\left(\frac{a}{c}, \frac{b}{c}\right)$

What if location $\left(\frac{a}{c}, \frac{b}{c}\right)$ is not valid pixel locations?

Computing warp I' from I and H



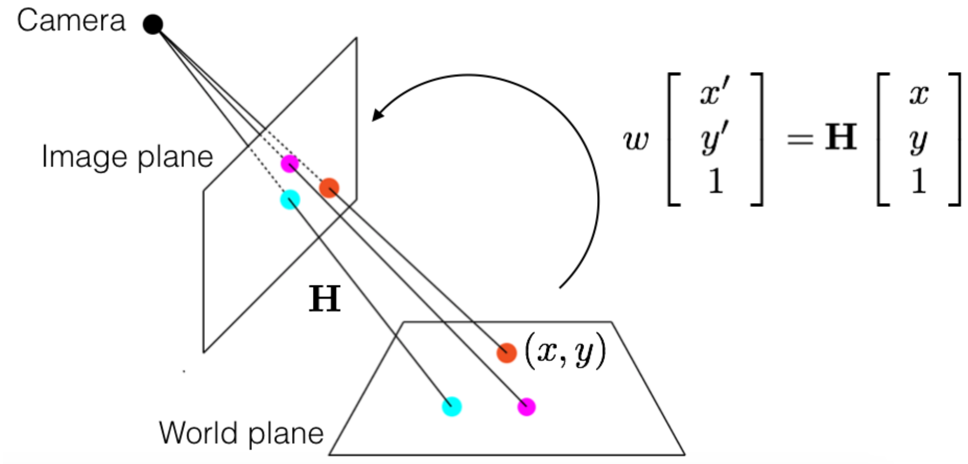
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = H^{-1} \begin{bmatrix} 7 \\ 7 \\ 1 \end{bmatrix}$$

Homography & image mosaicing

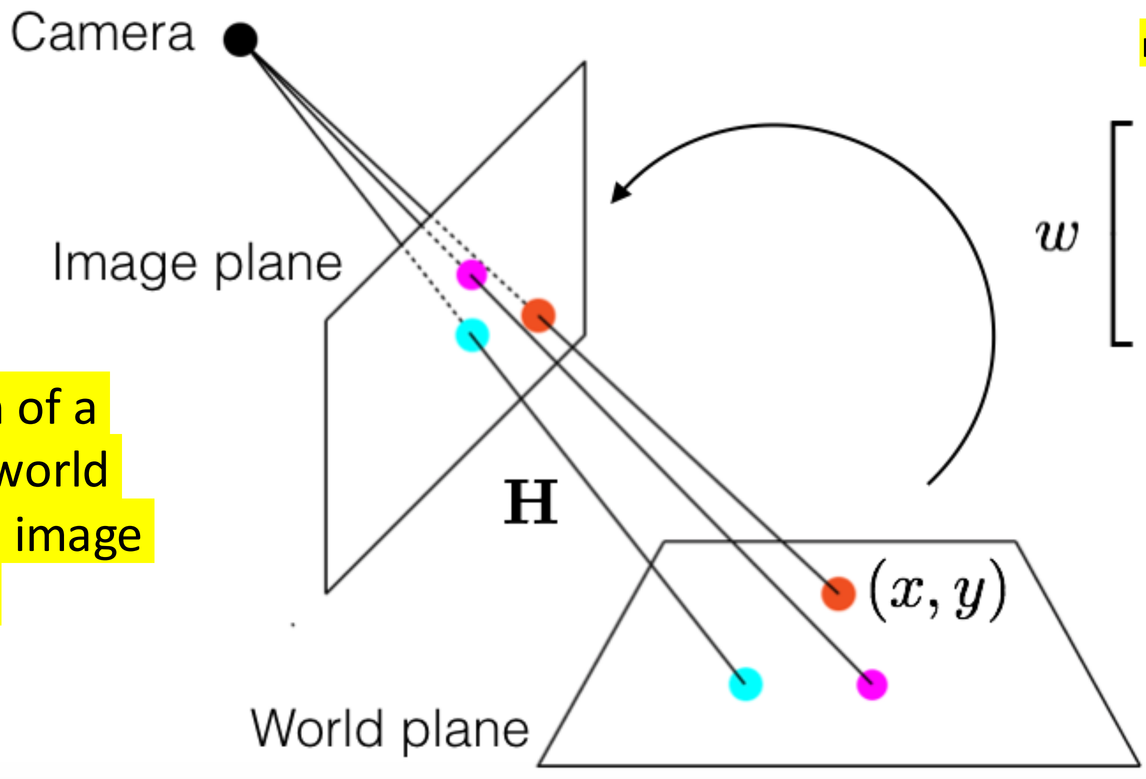
- Every photo taken from a tripod-mounted camera is related by a homography
- Assumptions
  - No lens distortion
  - Camera's center of projection does not move while camera is mounted on the tripod
- Problem
  - These homographies that relate photos taken from a tripod-mounted camera are *unknown*
    - We need to estimate them

Homography

- Generally speaking, points that lie on two planes are related via homography.



Homography



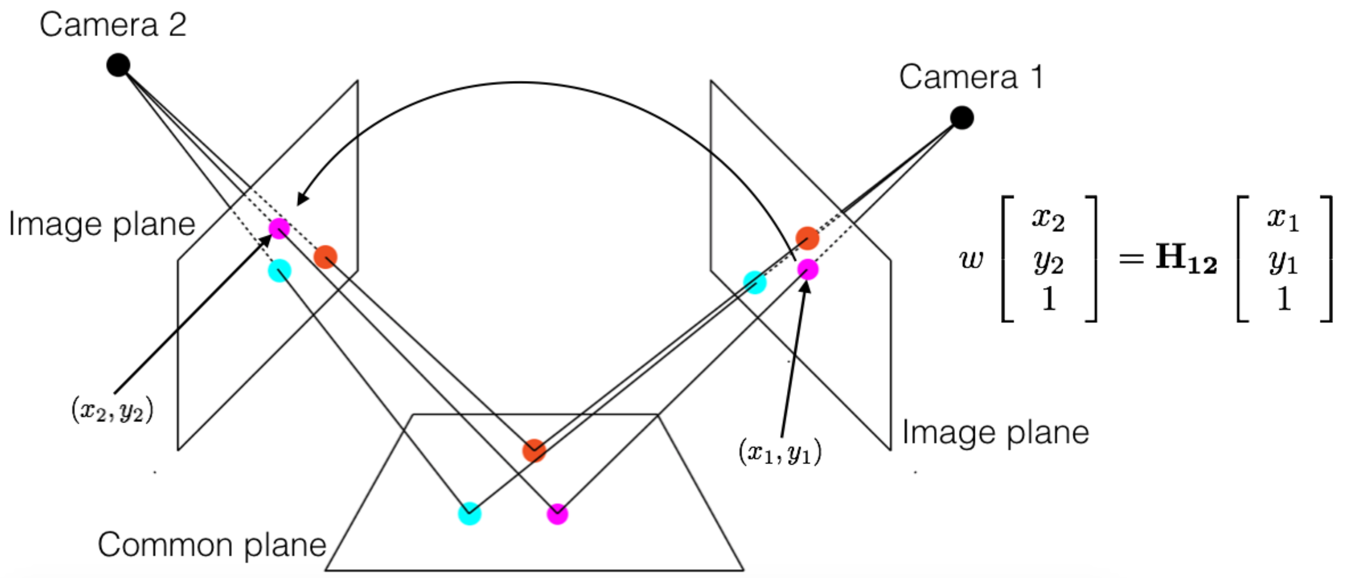
Projection of a point in a world plane to the image plane

The location of the point and that of its projection are related via a Homography.

$$w \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homography

- Generally speaking, points that lie on two planes are related via homography.
  - This also means that the projections of points (that lie on a common plane) in two cameras are related via homography.



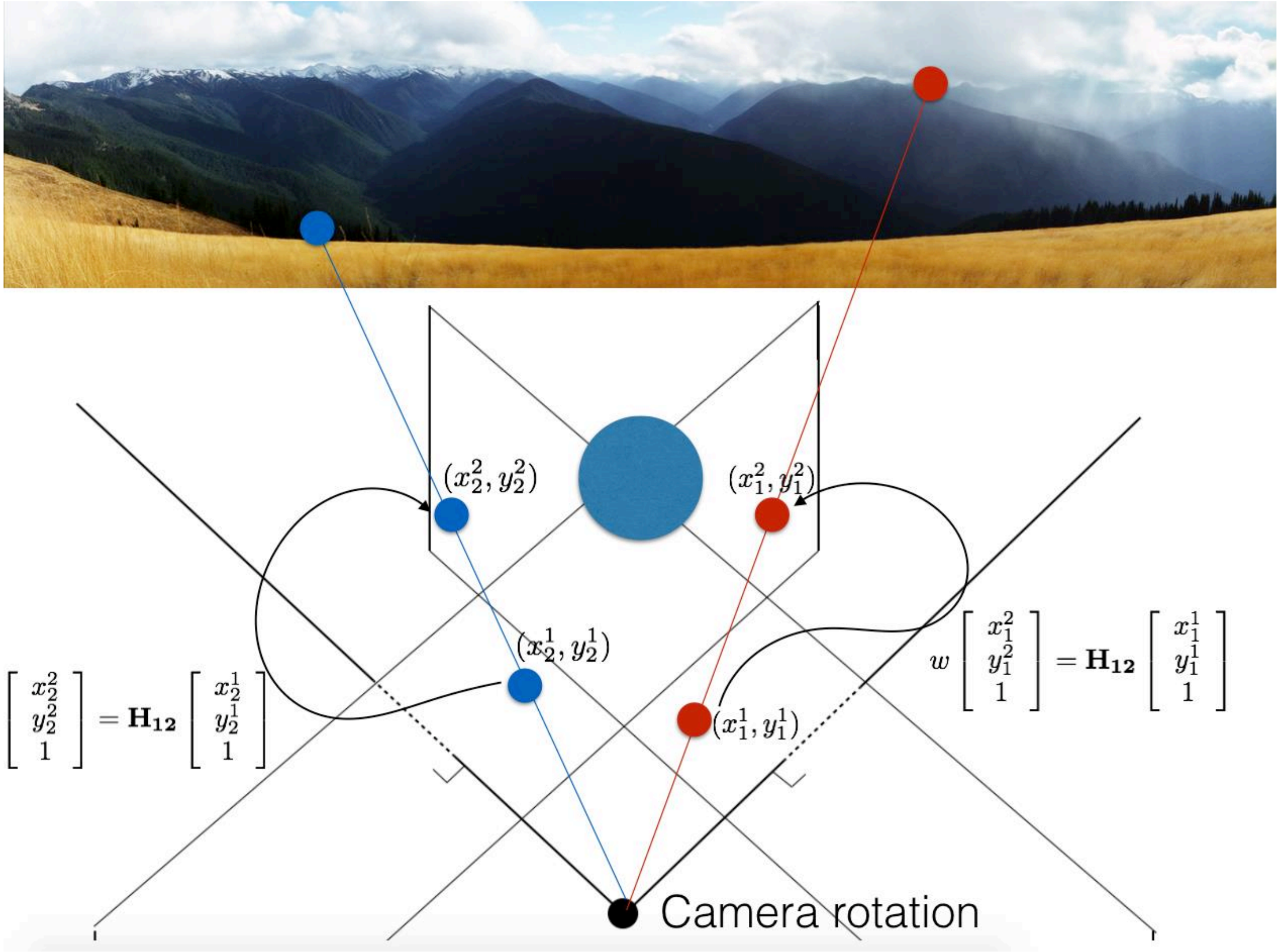
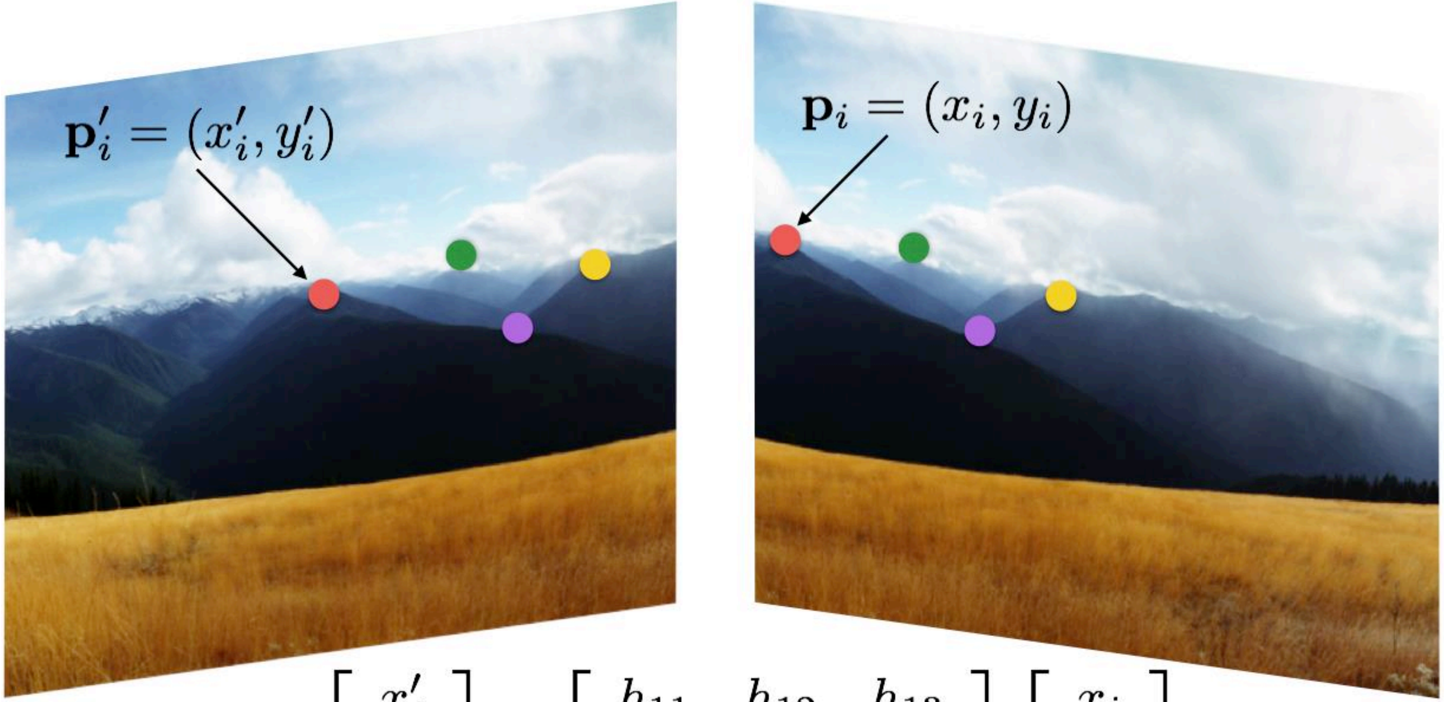
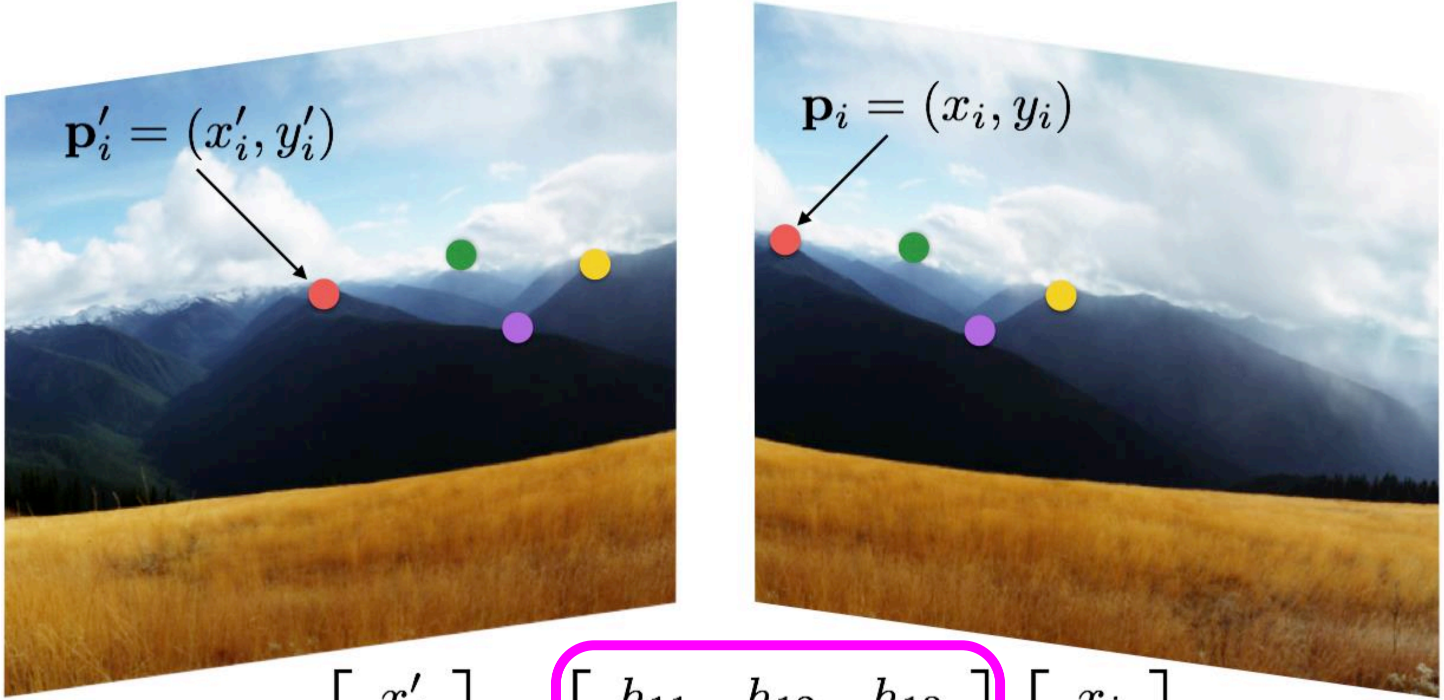

(x_2^2, y_2^2)
(x_1^2, y_1^2)
(x_2^1, y_2^1)
(x_1^1, y_1^1)
$\begin{bmatrix} x_2^2 \\ y_2^2 \\ 1 \end{bmatrix} = \mathbf{H_{12}} \begin{bmatrix} x_2^1 \\ y_2^1 \\ 1 \end{bmatrix}$
$w \begin{bmatrix} x_1^2 \\ y_1^2 \\ 1 \end{bmatrix} = \mathbf{H_{12}} \begin{bmatrix} x_1^1 \\ y_1^1 \\ 1 \end{bmatrix}$
Camera rotation

Image stitching



$$w \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Image stitching



$$w \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

How do we solve H?

Solving homography

$$w\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

How many degrees-of-freedom?

$$w x_i' = h_{11} x_i + h_{12} y_i + h_{13}$$
$$w y_i' = h_{21} x_i + h_{22} y_i + h_{23}$$
$$w = h_{31} x_i + h_{32} y_i + h_{33}$$

$$\begin{cases} x_i' = \dfrac{h_{11} x_i + h_{12} y_i' + h_{13}}{h_{31} x_i + h_{32} y_i + h_{33}} \\ \rule{8cm}{0.4pt} \\ y_i' = \rule{6cm}{0.4pt} \end{cases}$$

Solving for homography (Step 1)

- Re-write homography relationship as homogeneous equations

Solving for homography (Step 2)

- We can then write these as matrix-vector product

Solving for homography (Step 3)

- Given n correspondences between two images, setup $A\boldsymbol{x} = 0$ and solve for $\boldsymbol{x}$.

Solving $A\boldsymbol{x} = 0$

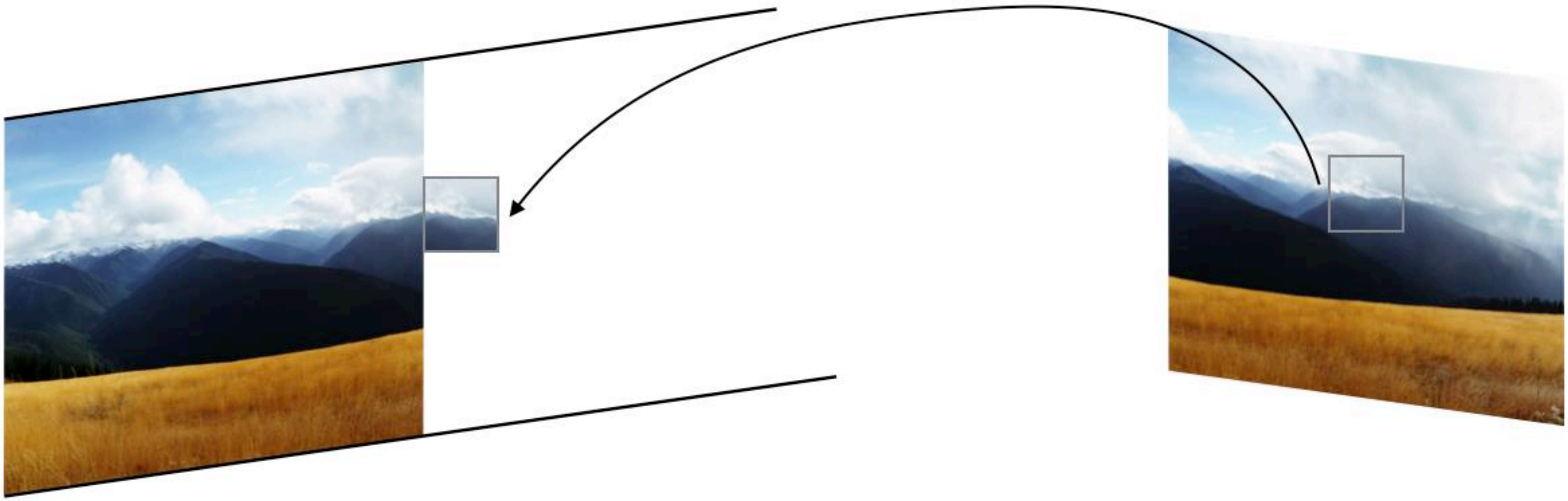
- Estimate using least-square fitting

$$\boldsymbol{x}^* = \underset{\boldsymbol{x}}{\operatorname{argmax}} \; \|A\boldsymbol{x}\|^2 \quad \text{s.t.} \quad \|x\| = 1$$

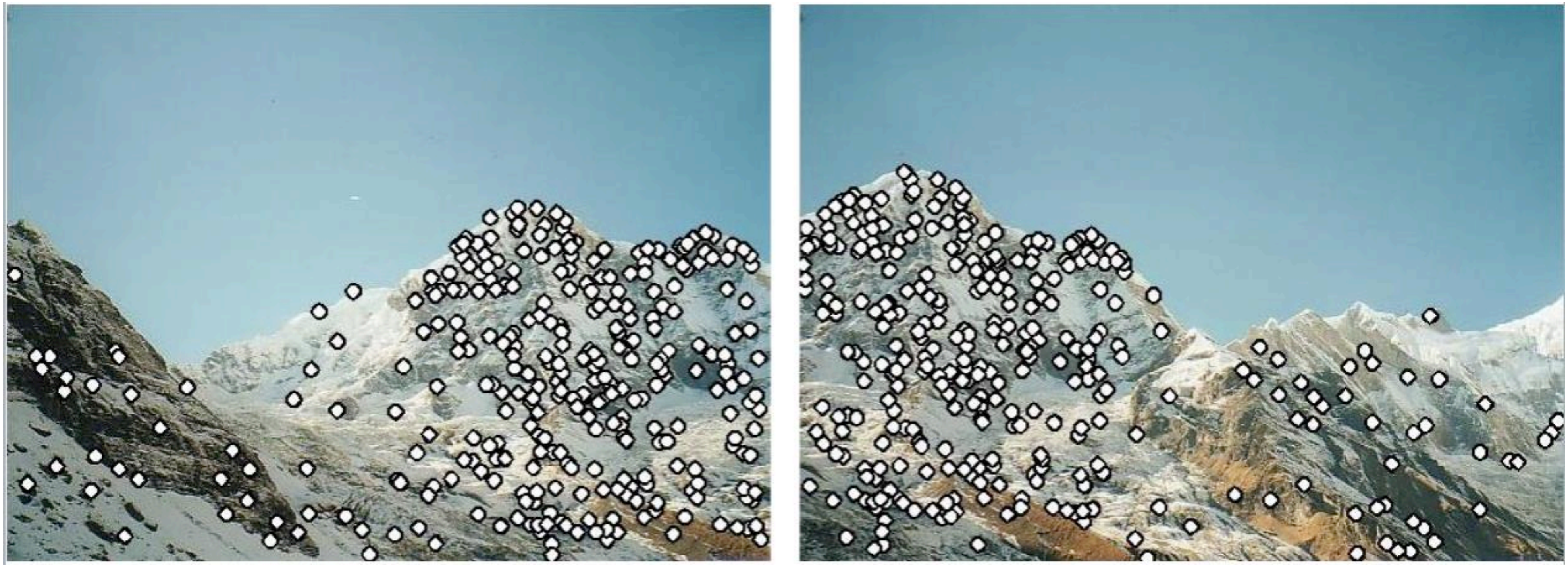
- The solution is the right *null-space* of A; therefore, the solution is the eigenvector corresponding to the smallest eigenvalue of $A^T A$

Image stitching

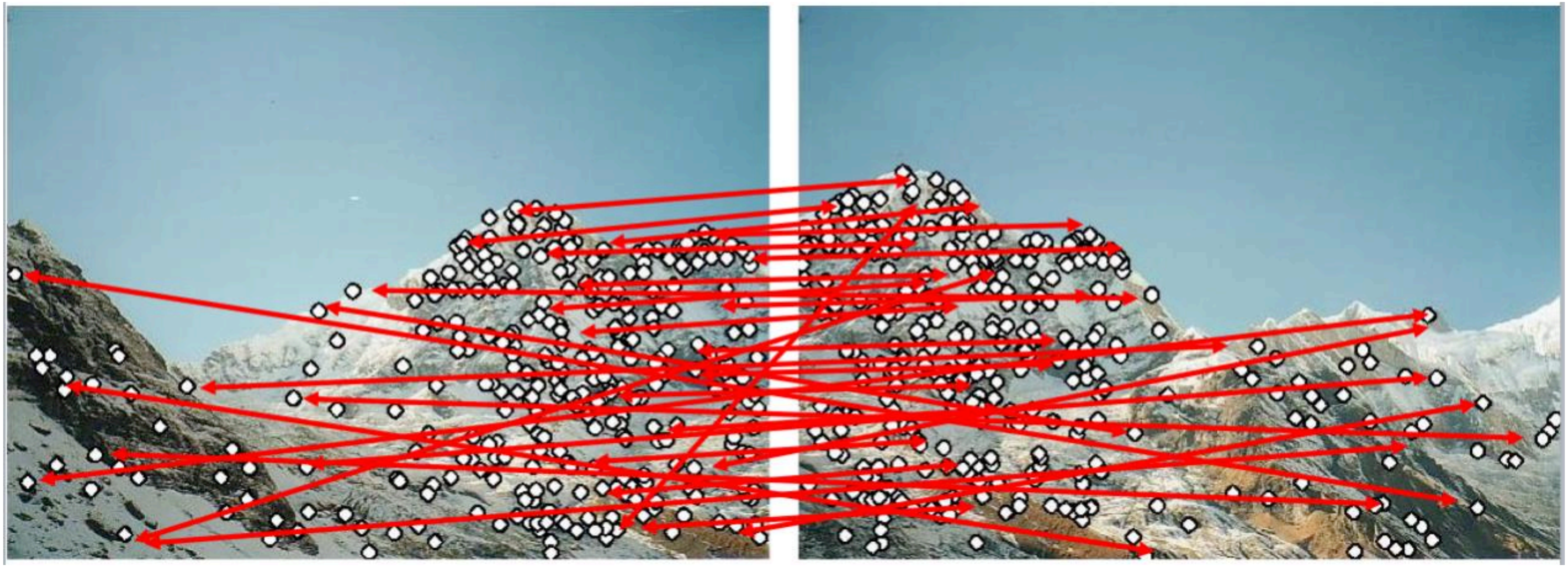
- Estimate homography
- Use it to fill the colors from the “other” image



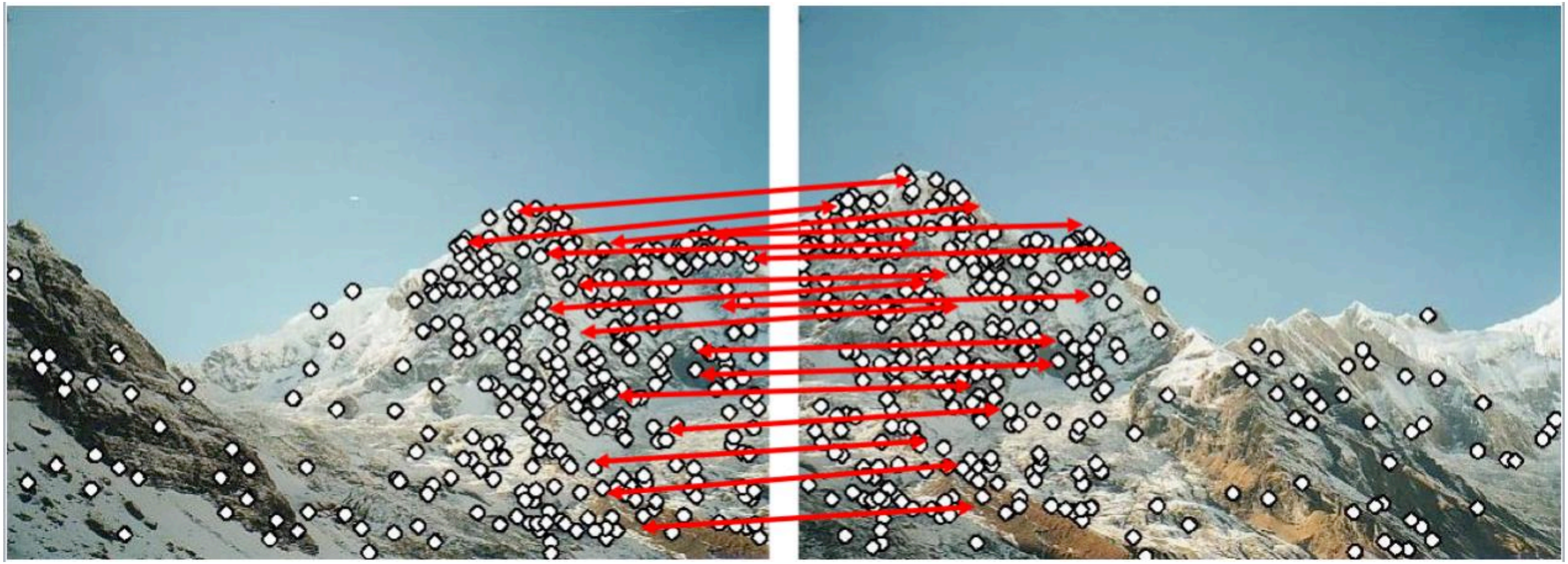
Extract features



Find matches



Use RANSAC to estimate homography



Perform image stitching