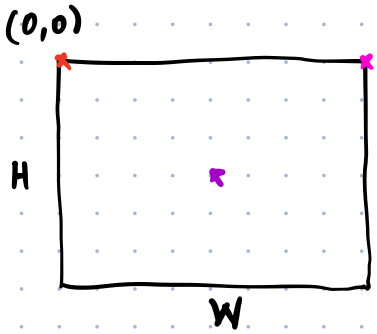
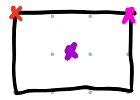
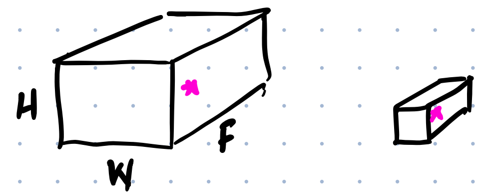
IMAGE INTERPOLATION





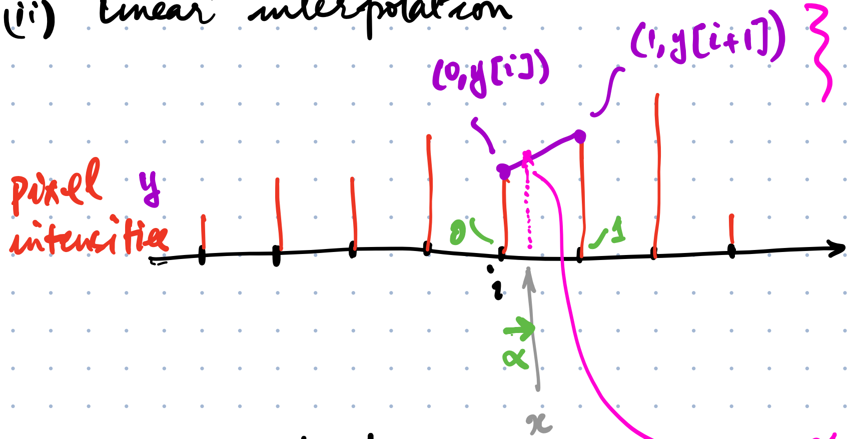


① mapping pixel locations

② computing pixel intensities

(i) Nearest-neighbour interpolation.

(ii) Linear interpolation



Fit a line between (0,y[i]) and (1,y[i+1])

$\to m, b$

pixel locations

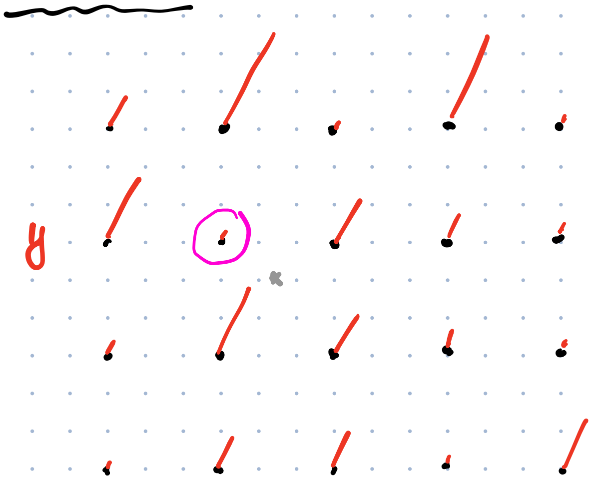
$i = int(x) = \lfloor x \rfloor$

$\boxed{\alpha = x - i}$

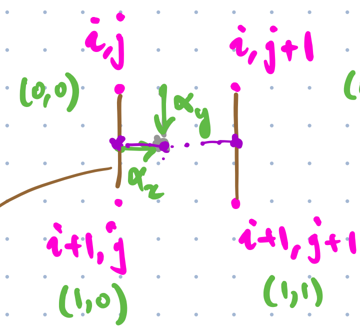
$\alpha m + b$

$\boxed{i,j = \lfloor x \rfloor, \lfloor y \rfloor}$

2D case



$\ast \to (x,y)$

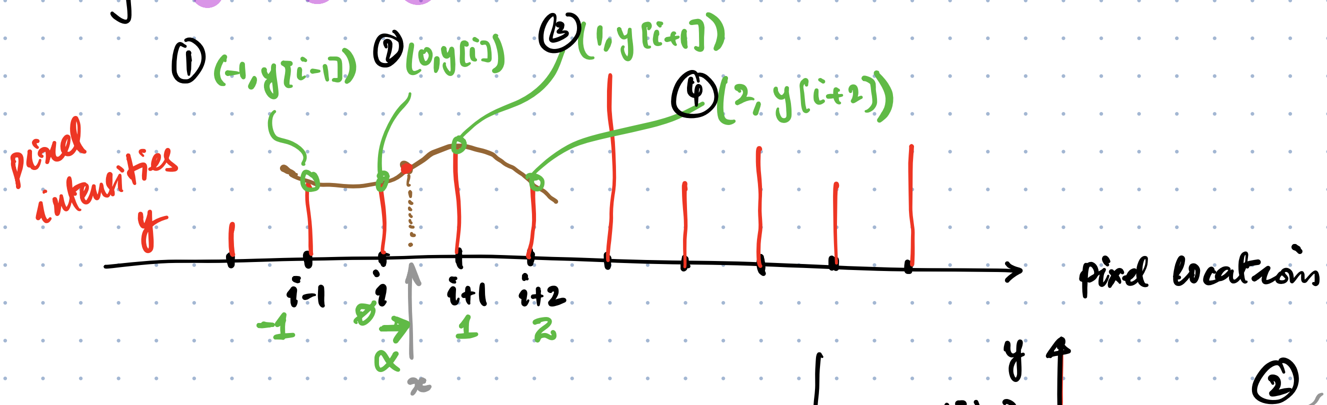


(0, y[0,0]), (1, y[1,0])

Cubic Interpolation

$$y = ax^3 + bx^2 + cx + d$$

$$y = mx + c$$



$$i = \lfloor x \rfloor$$

$$\alpha = x - i$$

② $y[i] = d$

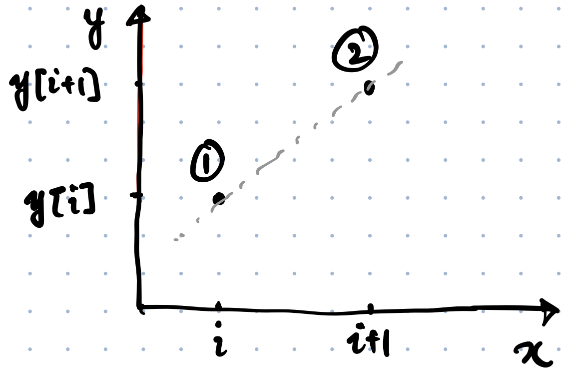
① $y[i-1] = -a + b - c + d$

③ $y[i+1] = a + b + c + d$

④ $y[i+2] = 8a + 4b + 2c + d$

$$\underbrace{\begin{bmatrix} y[i] \\ y[i-1] \\ y[i+1] \\ y[i+2] \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{X}$$

$$X = A^{-1} Y$$



$$y = mx + b$$

① $y[i] = mi + b$

② $y[i+1] = m(i+1)\, b$

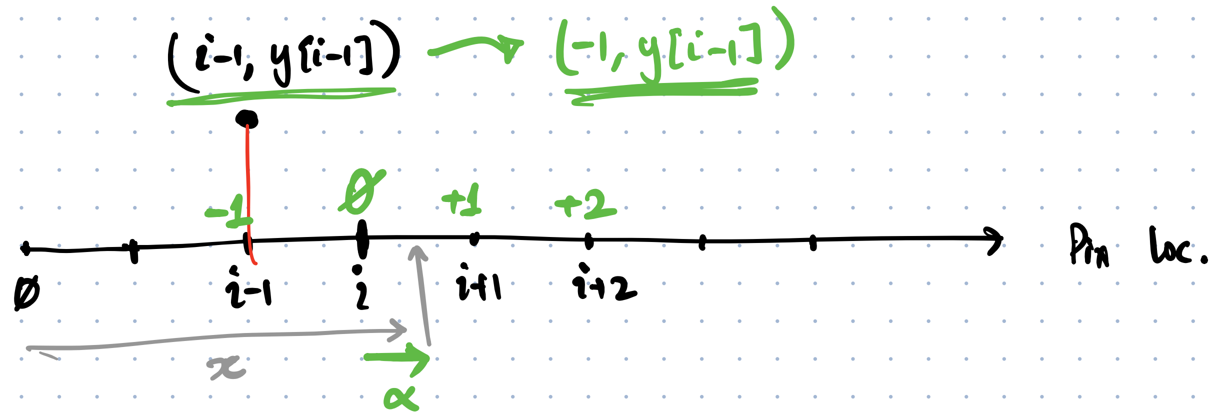
$$\Rightarrow \begin{bmatrix} y[i] \\ y[i+1] \end{bmatrix} = \begin{bmatrix} i & 1 \\ (i+1) & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} i & 1 \\ (i+1) & 1 \end{bmatrix}^{-1} \begin{bmatrix} y[i] \\ y[i+1] \end{bmatrix}$$

$$\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} y[i] \\ y[i+1] \end{bmatrix}$$

Once you have a, b, c and d.

Result is $\boxed{ax^3 + \cancel{bx^2 + cx} + d}$ $\boxed{a\alpha^3 + b\alpha^2 + c\alpha + d}$



$i = \lfloor x \rfloor$

$\alpha = x - i$

Bicubic Interpolation