w08a — Corner detection: solution

Q1. Gradient at the centre pixel

$$\begin{pmatrix} 10 & 10 & 10 \\ 50 & 50 & 50 \\ 90 & 90 & 90 \end{pmatrix}$$

With $G_x = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix}$ and $G_y = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$, applied as **convolution** (the course convention: the printed kernel is the convolution kernel, and y increases downwards):

$$G_x = (10 - 10) + 2(50 - 50) + (90 - 90) = \boxed{0}$$
$$G_y = (90 + 2 \cdot 90 + 90) - (10 + 2 \cdot 10 + 10) = 360 - 40 = \boxed{320}$$

So $\nabla I = (0, 320)$: a strong gradient in y, none in x.

Is it a corner? No. The intensity varies only vertically and is constant along each row — this is a **horizontal edge**. Every gradient vector in the window points the same way, so the structure tensor has one large eigenvalue and one essentially zero. A corner requires gradients in **two** independent directions.

Answering "yes, the gradient is large" misses the point: gradient *magnitude* detects edges; cornerness needs the *distribution* of gradient directions.

Q2. A scheme for a 5×5 window

Build the structure tensor over the window:

$$M = \sum_{(x,y) \in W} w(x, y) \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

then examine its eigenvalues $\lambda_1 \geq \lambda_2$:

- **Harris:** $R = \det(M) - \kappa \operatorname{tr}(M)^2$, $\kappa \approx 0.04$–0.06; corner if R is large and positive.
- **Shi-Tomasi:** $R = \lambda_2$; corner if λ_2 exceeds a threshold.

Then apply non-maximum suppression, or every pixel around a corner fires.

A complete scheme has two essentials: the sum over the window (not a single pixel), and a decision rule based on **both** eigenvalues.

Q3. Gradient distribution within a window

- **Flat region:** gradients are near zero in all directions; no dominant orientation. $\lambda_1 \approx \lambda_2 \approx 0$.
- **Edge:** gradients are large but all share **one** orientation (normal to the edge). $\lambda_1 \gg \lambda_2 \approx 0$.
- **Corner:** gradients take **two or more** distinct orientations. Both eigenvalues large.

This same table returns in `w08b` for optical flow and in Lab 7 Part 2; the structure tensor's role is the through-line.