w07b: The Hough transform, solution

Q1. Hough parameterisation.

Why (ρ, θ) rather than (m, c):

- **Vertical lines need $m = \infty$.** The (m, c) space is unbounded, so it cannot be discretised into a finite accumulator.
- Even for finite m, orientations are sampled very unevenly: a huge range of m covers near-vertical lines and a tiny range covers near-horizontal ones.

The (ρ, θ) form is bounded: $\theta \in [0°, 180°)$ and $|\rho| \leq \sqrt{H^2 + W^2}$, the image diagonal, with ρ signed. So a finite array suffices, and every orientation, including vertical, is treated equally.

What a single edge pixel traces. Fixing (x_0, y_0) and sweeping θ, $\rho(\theta) = x_0 \cos\theta + y_0 \sin\theta$ is a **sinusoid** in the (θ, ρ) plane; over 0° to 180° it is half a period. Edge pixels lying on one line give sinusoids that all pass through a single (ρ, θ) cell, the parameters of that line. A peak in the accumulator is therefore a set of collinear pixels, and its height is how many pixels voted for it.

Q2. One pixel's votes. For $(x, y) = (30, 40)$:

θ	$\rho = 30\cos\theta + 40\sin\theta$
0°	30.00
90°	40.00
143°	0.11

ρ is largest at $\theta = \text{atan2}(40, 30) = 53.1°$, where $\rho = \sqrt{30^2 + 40^2} = 50$. At that angle the line's normal points straight at the pixel, so ρ is the pixel's distance from the origin.

Q3. Sizing and indexing the accumulator.

(a) $\rho_{\max} = \lceil \sqrt{100^2 + 100^2} \rceil = \lceil 141.4 \rceil = 142$. Covering -142 to $+142$ in 1-pixel bins takes $2 \times 142 + 1 = 285$ rows. Sizing by $\pm W = \pm 100$ instead is the common bug: ρ can reach the image diagonal, so some votes would fall off the end of the array.

(b) ρ is signed but rows start at 0, so shift by $\rho_{\max}$: row $= \text{round}(\rho) + 142 = -29 + 142 = 113$.

(c) $\theta = \theta_g \bmod 180° = -30° \bmod 180° = 150°$. The gradient is the line's normal, so θ is the gradient angle itself; do **not** add 90°. Computing ρ from 150° rather than $-30°$ flips its sign, which is correct: (ρ, θ) and $(-\rho, \theta + 180°)$ are the same line.