

w06b — Plane fitting: solution**Q1. Fitting $ax + by + cz + d = 0$ to n points.**

The plane equation is homogeneous in (a, b, c, d) , so we need a scale constraint; take $\|(a, b, c)\| = 1$, which makes $|ax_i + by_i + cz_i + d|$ the **perpendicular** distance from point i to the plane. Minimising the sum of squared perpendicular distances is total least squares:

1. Compute the centroid $\bar{\mathbf{p}} = \frac{1}{n} \sum_i \mathbf{p}_i$.
2. Form the $n \times 3$ matrix M whose rows are the centred points $\mathbf{p}_i - \bar{\mathbf{p}}$.
3. Take the SVD of M . The normal $\mathbf{n} = (a, b, c)$ is the right singular vector belonging to the **smallest** singular value — the direction of least variance.
4. Recover $d = -\mathbf{n} \cdot \bar{\mathbf{p}}$.

Equivalently, $\mathbf{n}$ is the eigenvector of the 3×3 scatter matrix $M^\top M$ with the smallest eigenvalue.

Points a complete answer covers.

- Take the *smallest* singular vector, not the largest. This is the usual slip.
- Subtract the centroid first. Without it the fit is wrong.
- Minimise **perpendicular** rather than vertical (z) distance. Writing $z = \alpha x + \beta y + \gamma$ and doing ordinary least squares works, but it cannot represent vertical planes and it weights errors anisotropically.
- Note that ≥ 3 non-collinear points are needed.

A stronger answer also notes that least squares has **zero breakdown point** and adds RANSAC on top, which is exactly Week 7.

Q2. Use in a driverless car.

The obvious one: fit a plane to the LiDAR or stereo point cloud to recover the **ground/road surface**. Then:

- Points close to the plane are drivable surface.
- Points significantly above it are obstacles — vehicles, pedestrians, kerbs. Segmenting them becomes trivial once the ground is removed.
- The plane normal gives the vehicle's pitch and roll relative to the road.
- Height above the plane gives a free sanity check on depth estimates (this is exactly the validation used in Lab 9, via KITTI's `Tr_cam_to_road`).

Any answer along these lines captures the point. The deeper question is *why the fit must be robust*: other vehicles occupy a large fraction of the points and are not part of the road, so plain least squares would be pulled off the true ground plane.