

w04c — Image pyramids: solution

The Laplacian pyramid is built as

$$\mathbf{L}_i = \mathbf{g}_i - \text{expand}(\mathbf{g}_{i+1}),$$

where $\mathbf{g}_i$ is level i of the Gaussian pyramid and $\mathbf{g}_n$ (the coarsest level) is stored unmodified at the top.

Rearranging immediately gives the reconstruction:

$$\mathbf{g}_{i-1} = \mathbf{L}_{i-1} + \text{expand}(\mathbf{g}_i)$$

The scheme. Start at the top with $\mathbf{g}_n$, which you already have, and walk **downwards**:

```
g = g_n
for i = n down to 1:
    g = L_{i-1} + expand(g)
return g          # this is g_0 = I, the original image
```

Three things to get right:

1. **Direction.** Reconstruction runs coarse-to-fine. Trying to recover $\mathbf{g}_{i-1}$ from $\mathbf{g}_{i-1}$'s own level is a misreading.
2. **expand must match the reduce used when building.** If the analysis used blur-then-subsample, synthesis must upsample-then-blur with the same kernel, including the factor of 4 that compensates for the inserted zeros. Mismatch here is the commonest source of a reconstruction that is *nearly* but not exactly right.
3. **Losslessness.** The Laplacian pyramid is invertible, so the recovered $\mathbf{g}_0$ equals $\mathbf{I}$ up to floating-point error, provided no quantisation was applied to the levels.

This is Task 4 of Lab 5, where you verify it numerically.