

w04b — Taylor derivatives: solution

The marked cross in `8x8-marked.png` is exactly the stencil: two neighbours each side of pixel (3,3).

- Row 3, columns 1–5: 38, 79, **80**, 80, 75
- Column 3, rows 1–5: 41, 83, **80**, 76, 83

Deriving the stencils. Expand $I(x \pm 1)$ and $I(x \pm 2)$ about x to fourth order and eliminate the unwanted terms. This gives the standard five-point formulae:

$$\frac{\partial I}{\partial x} \approx \frac{-I_{x+2} + 8I_{x+1} - 8I_{x-1} + I_{x-2}}{12}, \quad \frac{\partial^2 I}{\partial x^2} \approx \frac{-I_{x+2} + 16I_{x+1} - 30I_x + 16I_{x-1} - I_{x-2}}{12}.$$

Results.

$$\begin{aligned} \frac{\partial I}{\partial x} &= \frac{-75 + 640 - 632 + 38}{12} = \frac{-29}{12} \approx \boxed{-2.42} \\ \frac{\partial I}{\partial y} &= \frac{-83 + 608 - 664 + 41}{12} = \frac{-98}{12} \approx \boxed{-8.17} \\ \frac{\partial^2 I}{\partial x^2} &= \frac{-75 + 1280 - 2400 + 1264 - 38}{12} = \frac{31}{12} \approx \boxed{2.58} \\ \frac{\partial^2 I}{\partial y^2} &= \frac{-83 + 1216 - 2400 + 1328 - 41}{12} = \frac{20}{12} \approx \boxed{1.67} \end{aligned}$$

(Here y increases downward, matching “top left is (0,0)”. A student using the opposite convention flips the sign of $\partial I/\partial y$ only.)

The three-point stencil is a different answer

Reading “two neighbours on each side” as *available* rather than *required*, and using plain central differences, gives

$$I_x = \frac{80-79}{2} = 0.5, \quad I_y = \frac{76-83}{2} = -3.5, \quad I_{xx} = 80 - 160 + 79 = -1, \quad I_{yy} = 76 - 160 + 83 = -1.$$

The question asks for Taylor series with two neighbours each side, so the five-point answer is the intended one. Note how different the two sets of numbers are: the choice of stencil matters a great deal on noisy data.