

**w04a — Image gradients (Sobel): solution**

The  $3 \times 3$  neighbourhood centred on pixel (3, 3) (row 3, column 3, value 80) read off `8x8-marked.png`:

$$\begin{pmatrix} 82 & 83 & 83 \\ 79 & 80 & 80 \\ 81 & 76 & 83 \end{pmatrix}$$

With the filters **as printed on the sheet**,  $G_x = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix}$  and  $G_y = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$ , applied as convolution:

$$G_x = (83 - 82) + 2(80 - 79) + (83 - 81) = 1 + 2 + 2 = \boxed{+5}$$

$$G_y = (81 + 2 \cdot 76 + 83) - (82 + 2 \cdot 83 + 83) = 316 - 331 = \boxed{-15}$$

$$\text{magnitude} = \sqrt{(5)^2 + (-15)^2} = \sqrt{250} \approx \boxed{15.8}$$

$$\text{direction} = \text{atan2}(-15, 5) \approx \boxed{-71.6^\circ}$$

(Equivalently  $288.4^\circ$ , measured anticlockwise from the  $+x$  axis.)

**Sign conventions.** The course convention, stated in deck 04, is that **every printed derivative kernel is a convolution kernel and  $y$  increases downwards**. Convolution flips the kernel before summing, which is why these signs come out as the physical ones: intensity rises to the right along row 3, so  $G_x > 0$ , and intensity falls as you move downwards along column 3, so  $G_y < 0$ . Applying the same kernels as correlation flips both signs and gives  $(-5, +15)$  with direction  $108.4^\circ$ . Either is defensible provided the convention is stated; the magnitude is convention-free.

**Multiplication count**

Interior pixels of an  $8 \times 8$  image, ignoring a one-pixel border:  $6 \times 6 = 36$ .

Per pixel: two  $3 \times 3$  convolutions  $= 2 \times 9 = 18$  multiplications.

$$36 \times 18 = \boxed{648}$$

Other defensible counts:

- **720** — if the two squarings in  $\sqrt{G_x^2 + G_y^2}$  are counted ( $36 \times 2$  more).
- **432** — noticing that each Sobel kernel has three zero entries and skipping them leaves 6 useful taps per filter, so 12 per pixel. This is the sharpest count.
- Separability would cut it further still.

The square root is not a multiplication.