

w03a — Linear filtering (FIR): solution

$$y[n] = \frac{1}{5} \sum_{k=-2}^2 x[n-k] = \frac{1}{5}(x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2])$$

— a 5-tap moving average. With $x = [2, -1, 0, 3, 1, 0]$ at $n = 0 \dots 5$ and zero outside:

n	window ($n+2$ to $n-2$)	$y[n]$
0	0, -1, 2, 0, 0	$1/5 = 0.2$
1	3, 0, -1, 2, 0	$4/5 = 0.8$
2	1, 3, 0, -1, 2	$5/5 = 1.0$
3	0, 1, 3, 0, -1	$3/5 = 0.6$
4	0, 0, 1, 3, 0	$4/5 = 0.8$
5	0, 0, 0, 1, 3	$4/5 = 0.8$

$$y[0 \dots 5] = [0.2, 0.8, 1.0, 0.6, 0.8, 0.8]$$

The output is also non-zero outside this range — $y[-2] = 0.4$, $y[-1] = 0.2$, $y[6] = 0.2$ — because convolution widens the support. Reporting only $n = 0 \dots 5$ is a fine answer; noticing the spread beyond it is better.

Note: the filter is centred, not causal

A causal filter may use only present and past samples. This one sums $k = -2 \dots 2$, so $y[n]$ depends on $x[n+1]$ and $x[n+2]$, future samples. That is exactly what image processing wants, because “future” there just means “to the right”.