

w02b — Homogeneous coordinates: solution**Part 1: the proof.**

A line $\ell = (a, b, c)$ contains point $\mathbf{p} = (x, y, 1)$ iff $\ell \cdot \mathbf{p} = ax + by + c = 0$.

Let $\ell = \mathbf{p}_1 \times \mathbf{p}_2$. The cross product is perpendicular to both arguments, so

$$\ell \cdot \mathbf{p}_1 = (\mathbf{p}_1 \times \mathbf{p}_2) \cdot \mathbf{p}_1 = 0, \quad \ell \cdot \mathbf{p}_2 = 0.$$

Both points satisfy the line equation, so ℓ is the line through them.

(Equivalently: the scalar triple product $\det[\mathbf{p}_1 \ \mathbf{p}_2 \ \mathbf{p}_1] = 0$ because two columns are identical.)

Part 2: the computation.

$$(1, 1, 1) \times (2, 4, 1) = (1 \cdot 1 - 1 \cdot 4, \ 1 \cdot 2 - 1 \cdot 1, \ 1 \cdot 4 - 1 \cdot 2) = (-3, \ 1, \ 2).$$

So the line is $-3x + y + 2 = 0$, or equivalently

$$\boxed{3x - y - 2 = 0}$$

Check: $(1, 1) \Rightarrow 3 - 1 - 2 = 0 \checkmark$ and $(2, 4) \Rightarrow 6 - 4 - 2 = 0 \checkmark$. **Always do this check.** It costs one line and catches every sign slip.

Note the line is only defined up to scale: $(-3, 1, 2)$ and $(3, -1, -2)$ are the same line, so either sign is valid.

Common error: computing the middle component as $1 \cdot 1 - 1 \cdot 2 = -1$ by forgetting the minus sign in the cofactor expansion.