

w02a — Image formation: solution

The rows of R are the camera axes expressed in world coordinates:

$$\mathbf{x}_c = (0, 0, 1), \quad \mathbf{y}_c = (0, 1, 0), \quad \mathbf{z}_c = (-1, 0, 0).$$

So the camera looks along the **negative world x direction**.

Treat $(11, 2, 1)$ as the camera centre C in world coordinates. Then

$$\mathbf{P} - C = (4, 3, 0) - (11, 2, 1) = (-7, 1, -1).$$

Project onto the camera axes:

$$X_c = (-7, 1, -1) \cdot (0, 0, 1) = -1, \quad Y_c = (-7, 1, -1) \cdot (0, 1, 0) = 1, \quad Z_c = (-7, 1, -1) \cdot (-1, 0, 0) = 7.$$

Pinhole projection with $f = 3$:

$$x = f \frac{X_c}{Z_c} = \frac{-3}{7} \approx -0.43, \quad y = f \frac{Y_c}{Z_c} = \frac{3}{7} \approx 0.43$$

$Z_c = 7 > 0$, so the point is in front of the camera. Always do this sanity check: it costs one line and catches every sign slip.

Note on an ambiguity in the question

The question says “a camera whose rotation and translation is R and $(11, 2, 1)$ ” without saying whether that vector is the camera **centre** C (giving $\mathbf{X}_c = R(\mathbf{X}_w - C)$, used above) or the translation $\mathbf{t}$ in $\mathbf{X}_c = R\mathbf{X}_w + \mathbf{t}$.

Under the second reading, $\mathbf{X}_c = (0, 3, -4) + (11, 2, 1) = (11, 5, -3)$, which puts the point **behind** the camera ($Z_c < 0$) and yields $(-11, -5)$. That is almost certainly not intended.

Either reading is valid provided the convention is stated and the sign of Z_c is checked.