Computational Photography (CSCI 3240U) — In-class Exercise

Please hand in this paper to the instructor before the end of the lecture.

Lastname (PRINT):	____________	Firstname:	____________
Student number:	____________	Date:	____________

Q. Cubic Interpolation in 1D

Consider a discrete signal

indices

$$\begin{array}{c} 0\ 1\ 2\ 3\ 4\ 5 \\ y[0..5] = [3, 2, 0, 4, 3, 5] \end{array}$$

Setup the cubic interpolant to compute $y[2.2]$ and $y[1.5]$.

Hint: you don't need to actually solve for the value; however, you are required to set up the equations that we an subsequently use to compute the value.

Cubic polynomial is $p(x) = ax^3 + bx^2 + cx + d$, lets write it for $x = -1, 0, 1, 2$.

$$\begin{bmatrix} p(-1) \\ p(0) \\ p(1) \\ p(2) \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

For $y[2.2]$,

$$\begin{array}{cccccc} \cancel{0} & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 0 & 4 & 3 & 5 \\ & -1 & 0 & 1 & 2 & \end{array}$$

$$\begin{bmatrix} 2 \\ 0 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \rightarrow$$

Solve and evaluate at $p[0.2]$.

For $y[1.5]$,

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 0 & 4 & 3 & 5 \\ -1 & 0 & 1 & 2 & & \end{array}$$

$$\begin{bmatrix} 3 \\ 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \rightarrow$$

Solve and evaluate at $p[0.5]$

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Q. Homogeneous Coordinates

Show that that line between two 2D points (x_1, y_1) and (x_2, y_2) is given by $(x_1, y_1, 1) \times (x_2, y_2, 1)$. Now use this fact to compute the line $ax + by + c = 0$ between $(1, 1)$ and $(2, 4)$.

Equation of a line is $ax+by+c=0$.

Because the line passes through point (x_1,y_1) therefore

$$ax_1+by_1+c=0. \quad \text{—} (1)$$

Similarly, the line passes through (x_2,y_2). therefore

$$ax_2+by_2+c=0. \quad \text{—} (2)$$

From (1) $(a,b,c)\cdot(x_1,y_1,1)=0$ and from (2) $(a,b,c)\cdot(x_2,y_2,1)=0$. Therefore, (a,b,c) is $\perp$ to both $(x_1,y_1,1)$ and $(x_2,y_2,1)$. Therefore

$$(a,b,c)=(x_1,y_1,1)\times(x_2,y_2,1).$$

Line between $(1,1)$ and $(2,4)$ is

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & 1 \\ 2 & 4 & 1 \end{vmatrix} = \hat{i}(1-4) - \hat{j}(1-2) + k(4-2)$$

$$= -3\hat{i} + \hat{j} + 2k$$

$\therefore$ line is $-3x+y+2=\cancel{0}$

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Q. Consider the following image, use Sobel filters to compute gradient direction and magnitude at pixel $(3, 3)$

Top left corresponds to pixel location $(0, 0)$

7	13	12	10	10	13	7	12
8	7	36	41	44	35	11	14
12	44	82	83	83	77	44	13
12	38	79	80	80	75	41	13
11	36	81	76	83	82	43	8
13	42	85	83	79	85	38	10
11	9	35	45	39	44	8	14
13	12	10	13	11	10	10	8

Sobel filters in x and y direction are $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}$, respectively. Now imagine that you are asked to compute gradient magnitude for each pixel (ignoring the boundaries), how many multiplications would you need to compute it?

$$\begin{bmatrix} 82 & 83 & 83 \\ 79 & 80 & 80 \\ 81 & 76 & 83 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{matrix} -82 + 0 + 83 & 1 \\ -158 + 0 + 160 & = +2 \\ -81 + 0 + 83 & +2 \end{matrix} = 5 \; I_x$$

$$\xrightarrow{}$$
I_x

$*$

$$I_y \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{matrix} -82 & -166 & -83 \\ +0 & +0 & +0 \\ +81 & +152 & +83 \end{matrix}$$

$$= -1 \quad -14 \quad +0$$

$$= -15 \; I_y$$

gradient : $\begin{bmatrix} 5 \\ 15 \end{bmatrix}$

gradient magnitude : $\sqrt{5^2 + 15^2}$

gradient direction : $\arctan\left(\frac{15}{5}\right)$

#multiplications for each pixel

$= 9 + 9 + 2 = 20$

total multiplications $= (6)(6)(20)$.

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Q. Consider an image $\mathbf{I}$ whose Laplacian pyramid is $\{\mathbf{L}_0, \mathbf{L}_1, \mathbf{L}_2, \cdots, \mathbf{g}_n\}$. Provide a scheme for computing $\mathbf{g}_{i-1}$ from $\mathbf{L}_{i-1}$ and $\mathbf{g}_i$ where $i \in [n, 1]$.

Let $g_i \in \mathbb{R}^{H \times W}$

Then $L_{i-1} \in \mathbb{R}^{2H \times 2W}$

Upsample g_i by adding $\emptyset$ rows and columns at alternate places and then convolving with ω to fill in the $\emptyset$ values.

$$g_i \xrightarrow[\text{upsample}]{} \hat{g}_{i-1} \in \mathbb{R}^{2H \times 2W}$$

$$g_{i-1} = L_{i-1} + \hat{g}_{i-1}$$

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Q. Image Interpolation (1D & 2D)

(a) 1D linear interpolation. Let $y[n]$ be discrete samples. For $x = i + t$ with $i \in \mathbb{Z}$ and $t \in [0, 1)$, the linear interpolant is

$$\hat{y}(x) = (1 - t)\, y[i] + t\, y[i + 1].$$

0 1 2 3 4

Given $y = [3,\ -1,\ 4,\ 2,\ 0]$ and $i = 2$ (0-based indexing), compute $\hat{y}(2.3)$ and $\hat{y}(2.75)$.

2.3

$x = i + t$

$t = x - i$

$t = 2.3 - 2$

$= 0.3$

$$\hat{y}(x) = (1 - \underset{0.3}{t})\, \underset{4}{y[i]} + \underset{0.3}{t}\, \underset{2}{y[i+1]}$$

$$= (0.7)(4) + (0.3)(2)$$

$$= 2.8 + 0.6$$

$$= 3.4$$

(b) 2D bilinear interpolation. Consider the 2×2 pixel block at integer coordinates

$$I(0, 0) = 100, \quad I(1, 0) = 120, \quad I(0, 1) = 80, \quad I(1, 1) = 140.$$

For a query point (u, v) with $u, v \in [0, 1]$ (relative to the top-left pixel), the bilinear value is

$$\hat{I}(u, v) = (1 - u)(1 - v)\, I(0, 0) + u(1 - v)\, I(1, 0) + (1 - u)v\, I(0, 1) + uv\, I(1, 1).$$

Compute $\hat{I}(0.25,\ 0.60)$.

$$\hat{I}(0.25, 0.6) = (1 - 0.25)(1 - 0.6)(100) + (0.25)(1 - 0.6)(120)$$

$$+ (1 - 0.25)(0.6)(80) + (0.25)(0.6)(140)$$

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Q. Write down the two assumptions made by the *Thin Lens Model*.

Q. Write down the *Focusing Property* in the context of the *Thin Lens Model*.

Assumptions of Thin Lens Model.

1. Lines pass through the center do not bend.
2. Parallel lines meet at the focal plane.

Focusing property.

1. Rays emitted from one point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.

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Q. Consider a rectangular object that is 10 m tall and 5 m wide. This object sits at a distance of 50 meters from a pinhole camera. The focal length of this camera is 5 cm. What is the area of the image of this object. To simplify the calculations, we assume that the object sits facing the camera.

Q. Consider an ideal pinhole camera. Now assume that the size of the pinhole is small, as small as can be without exhibiting into diffraction effects. This pinhole is used to image a distant object. Under what conditions do you think the image of this object will be blurry?

$Y = 10\ m, \quad X = 5\ m, \quad Z = 50\ m, \quad f = 5\ cm = \frac{5}{100}\ m$

$y = f\frac{Y}{Z} = (0.05)\frac{1\cancel{0}}{5\cancel{0}} = (0.05)(0.2)$

$x = f\frac{X}{Z} = (0.05)\frac{\cancel{5}^{1}}{\cancel{5}0} = (0.05)(0.1)$

Area of the imaged object $= (x)(y) = (0.05)^2(0.2)(0.1)$

The image will never be blurry.

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Q. Consider the following 8×8 grayscale image that stores each pixel using 4 bits.

0	5	7	3	14	4
1	4	8	4	15	3
2	5	9	5		
3	2	10	4		
4	4	11	3		
5	4	12	4		
6	5	13	5		

64

8

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 2 & 2 & 2 \\ 0 & 0 & 1 & 1 & 2 & 2 & 3 & 3 \\ 4 & 4 & 5 & 5 & 5 & 6 & 6 & 7 \\ 4 & 4 & 5 & 6 & 6 & 6 & 7 & 7 \\ 8 & 8 & 8 & 9 & 9 & 10 & 10 & 11 \\ 8 & 9 & 9 & 9 & 10 & 10 & 11 & 11 \\ 12 & 12 & 12 & 13 & 13 & 14 & 14 & 14 \\ 12 & 13 & 13 & 13 & 14 & 15 & 15 & 15 \end{bmatrix}$$

8

pixels = 64

Complete the following tasks:

1. **Compute the histogram**: count how many times each intensity value (0–15) appears in the image.
2. **Plot the histogram** as a bar chart (x-axis: intensity values, y-axis: frequency).
3. **Answer the following**:
   - Which intensity values occur most frequently?
   - Which intensity values occur least frequently?
   - What is the total number of pixels? Does the sum of the histogram counts match this total?

HISTOGRAM

Is this a distribution? NO

HISTOGRAM ÷ 64

$$\int_{64}$$

$$\frac{5+4+5+2+4+4+5+3+4+5+4+3+4+5+4+3}{64}$$

$$\frac{5}{64} + \frac{4}{64} \cdots + \frac{3}{64}$$

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Q. Consider the following image, use Taylor's Series to compute $(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y})$ and $(\frac{\partial^2 I}{\partial x^2}, \frac{\partial^2 I}{\partial y^2})$ at pixel $(3,3)$. We use two neighbours on each side to compute the required derivatives.

Top left corresponds to pixel location $(0,0)$

7	13	12	10	10	13	7	12
8	7	36	41	44	35	11	14
12	44	82	83	83	77	44	13
12	38	79	80	80	75	41	13
11	36	81	76	83	82	43	8
13	42	85	83	79	85	38	10
11	9	35	45	39	44	8	14
13	12	10	13	11	10	10	8

Consider row 38, 79, 80, 80, 75.

$$\begin{bmatrix} 38 \\ 79 \\ 80 \\ 80 \\ 75 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -1 & 1/2 \\ 1 & 0 & 0 \\ 1 & 1 & 1/2 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} I(0) \\ I'(0) \\ I''(0) \end{bmatrix} \rightarrow \text{Solving this yields } \frac{\partial I}{\partial x} \text{ and } \frac{\partial^2 I}{\partial x^2}.$$

Now consider column 41, 83, 80, 76, 83

$$\begin{bmatrix} 41 \\ 83 \\ 80 \\ 76 \\ 83 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -1 & 1/2 \\ 1 & 0 & 0 \\ 1 & 1 & 1/2 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} I(0) \\ I'(0) \\ I''(0) \end{bmatrix} \rightarrow \text{Solving this yields } \frac{\partial I}{\partial y} \text{ and } \frac{\partial^2 I}{\partial y^2}.$$

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Q. A causal Finite Impulse Response (FIR) filter is defined by

$$y[n] = \cancel{\frac{1}{5}} \sum_{k=-2}^{2} x[n-k]\, f[k].$$

$f[k] = [1, 1, 1, 1, 1]\frac{1}{5}$

ave. filter

Compute $y[n]$ for input $x[n] = [2, -1, 0, 3, 1, 0]$. Assume $x[n]$ is 0 outside given samples.

n 0 1 2 3 4 5

$x[3] = 3$ ✓

$x[5] = 0$ ✓

$$y[0] = \frac{1}{5}\left(x[0+2] + x[0+1] + x[0+0] + x[0-1] + x[0-2]\right) =$$

$$= \frac{1}{5}\left(0 + (-1) + 2 + 0 + 0\right) = \frac{1}{5}$$