Image Gradients

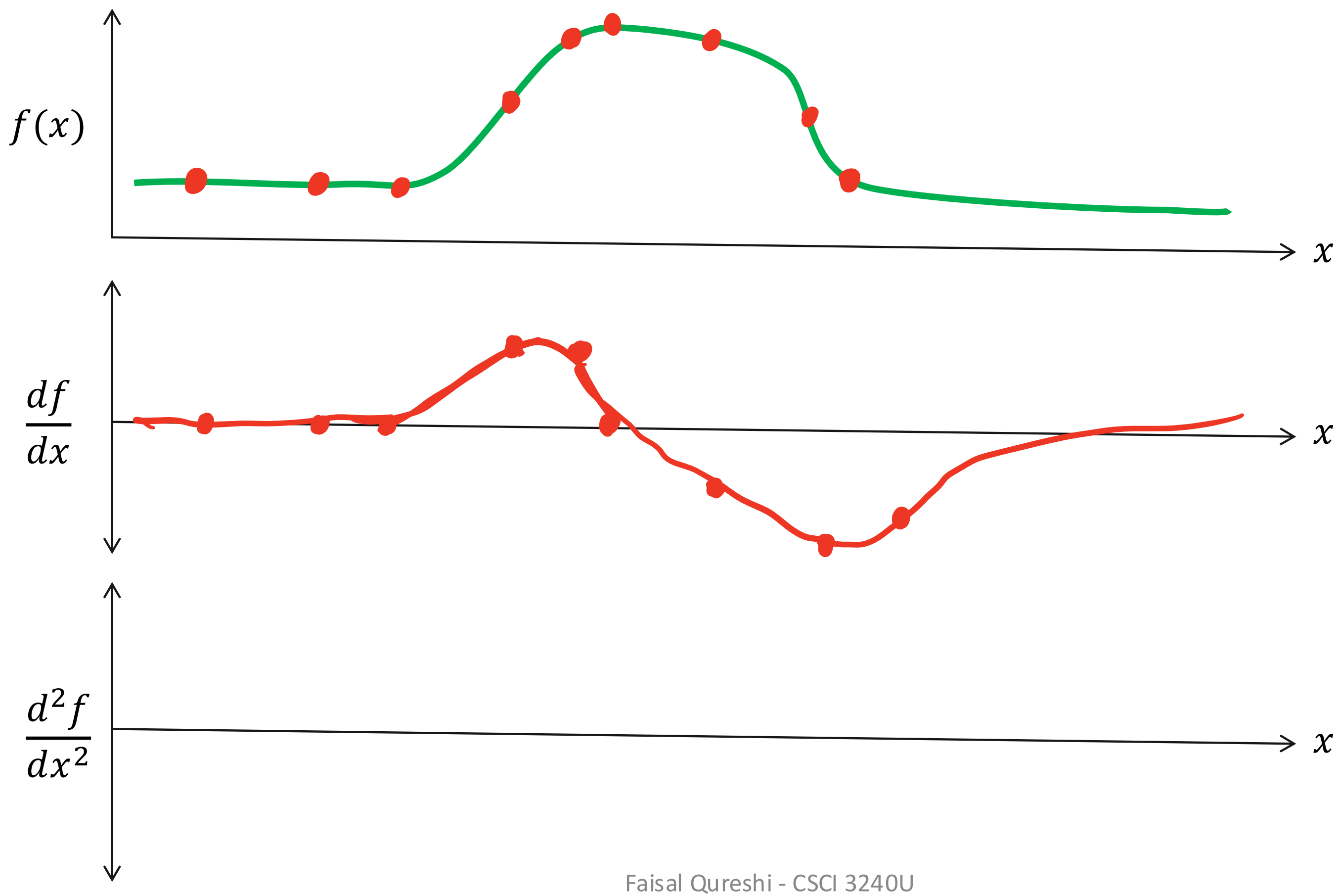
Computational Photography (CSCI 3240U)

Faisal Z. Qureshi
http://vclab.science.ontariotechu.ca
Ontario Tech University

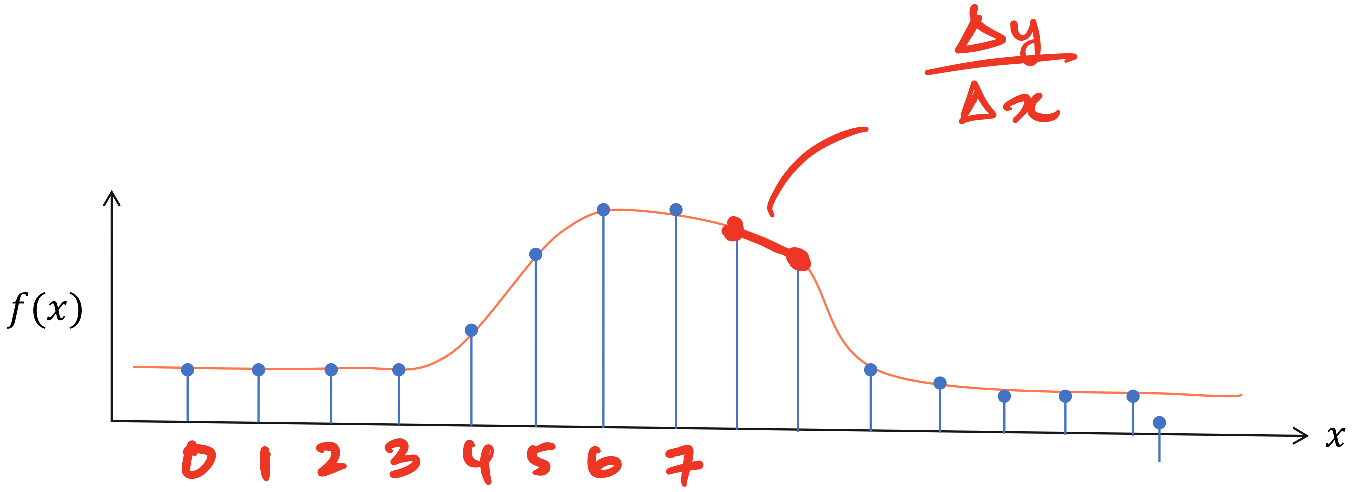
Today’s lecture

- Why do we care about image gradients?
- Computing image gradients
- Sobel filters
- Gradient magnitude and directions
- Visualizing image gradients

Derivative: $\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$



Derivative: $\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$



Finite-difference approximation

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

↳ SLOPE

Use finite difference approximation to compute image derivatives

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

$f =$

1	1	9	8	6	0	0
0	1	2	3	4	5	6

$f' =$

0	8	-				

$f'' =$

Use finite difference approximation to compute image derivatives

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

$f =$

1	1	9	8	6	0	0
0	1	2	3	4	5	6

-1 | 1

$f' =$

0	8	-1	-2	-6	0	?

$f * [1, -1] =$

0						

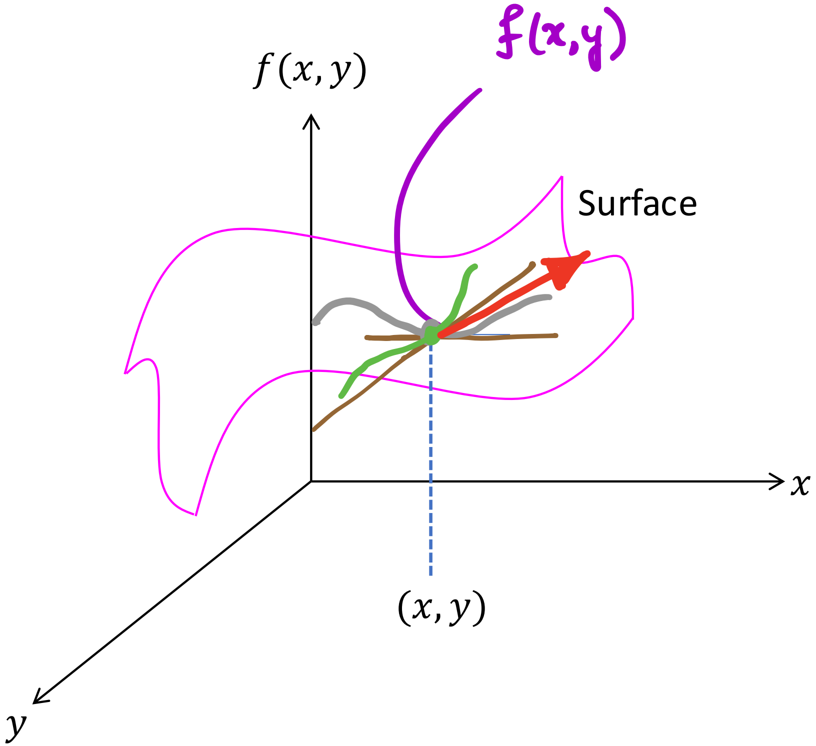
Convolution (linear filtering)

Partial derivatives

$$\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

↓

GRADIENT



How to compute image derivatives?

- Option 1: reconstruct a continuous function $f(x, y)$, then compute partial derivatives as f

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x, y)}{\epsilon}$$

$$\frac{\partial f(x,y)}{\partial y} = \lim_{\epsilon \to 0} \frac{f(x, y+\epsilon) - f(x, y)}{\epsilon}$$

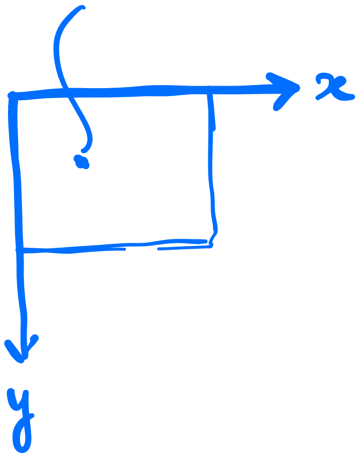
How to compute image derivatives?

- Option 2: use finite differences to take a discrete derivative as

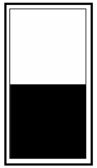
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f[x+1,y]-f[x,y]}{1}$$

$$\frac{\partial f(x,y)}{\partial y} \approx \frac{f[x,y+1]-f[x,y]}{1}$$

① $\frac{\partial f}{\partial x}$ ② $\frac{\partial f}{\partial y}$



- We can achieve this using *convolution*

$H_x =$  $H_y =$ 

[+1, -1] ①

② $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Image derivatives in x and y directions

Image

$f =$

1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

$f(x,y) \rightarrow$ intensity; (x,y): Pixel Locations

GRADIENT $(0,0) = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$

Derivative along x

$f_x = f * [1, -1] =$ ($\frac{\partial f}{\partial x}$)

Boundary Conditions

0	8	-1		

Derivative along y

$f_y = f * [1, -1]^T =$ ($\frac{\partial f}{\partial y}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$)

7				
-7				
4				

Image gradient: $\nabla f = \left[\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y}\right]$

$f_x =$

0	8	-1	-7	?
0	0	0	0	
2	2	3	-7	
-2	-1	6	-2	

$f_y =$

7	7	-1	0	7
-7	-5	-3	0	7
4	0	-3	0	5

What is gradient at pixel location $(0,2)$? $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$(3,2)$? $\begin{bmatrix} -7 \\ 0 \end{bmatrix}$

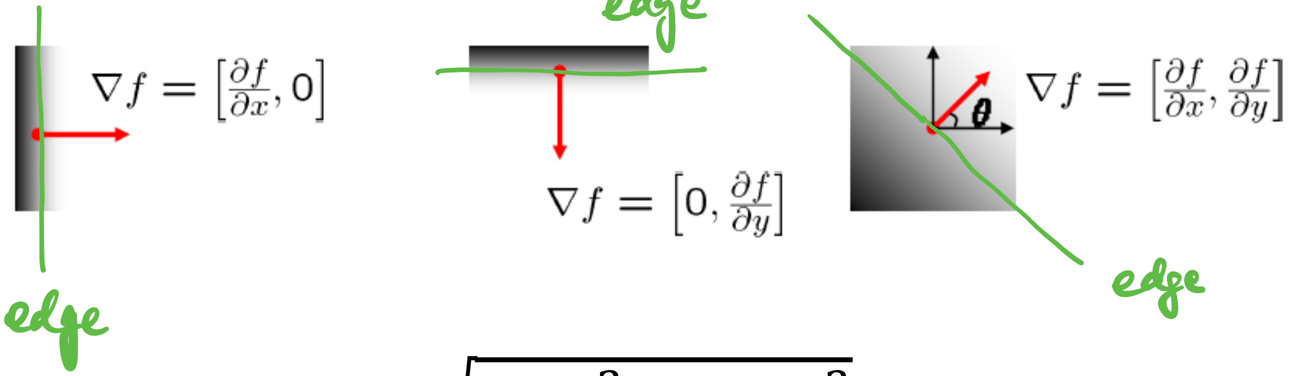
$(4,0)$? UNDEFINED

Image Gradient: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

∇ \nabla

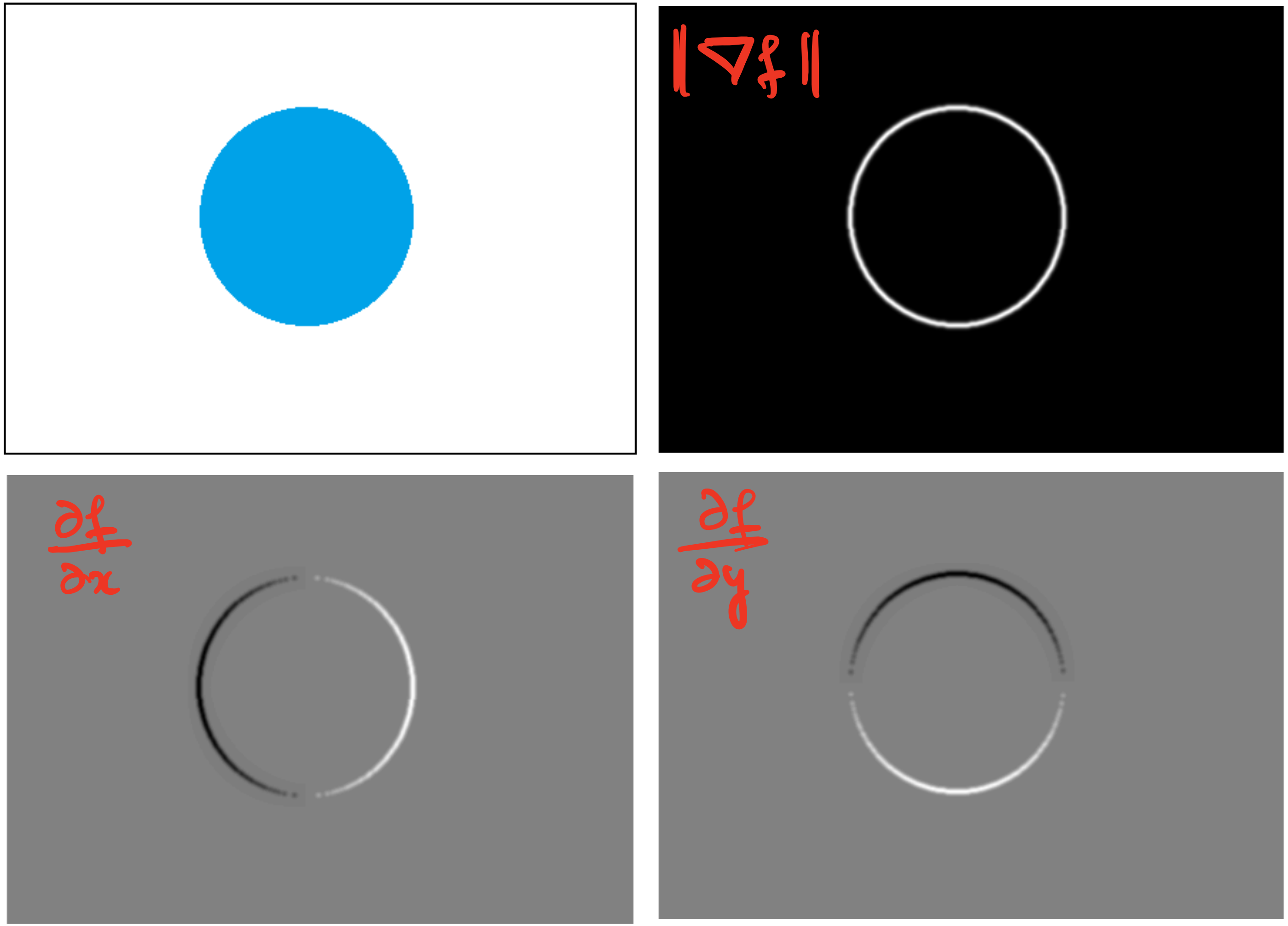
Δ \delta

- The gradient of an image ∇f points to the direction of most rapid change in intensity



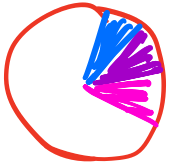
- Gradient magnitude: $\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$

- Gradient direction: $\theta = \tan^{-1}\left(\frac{\partial I}{\partial y} / \frac{\partial I}{\partial x}\right)$



$\|\nabla f\|$
$\frac{\partial f}{\partial x}$
$\frac{\partial f}{\partial y}$

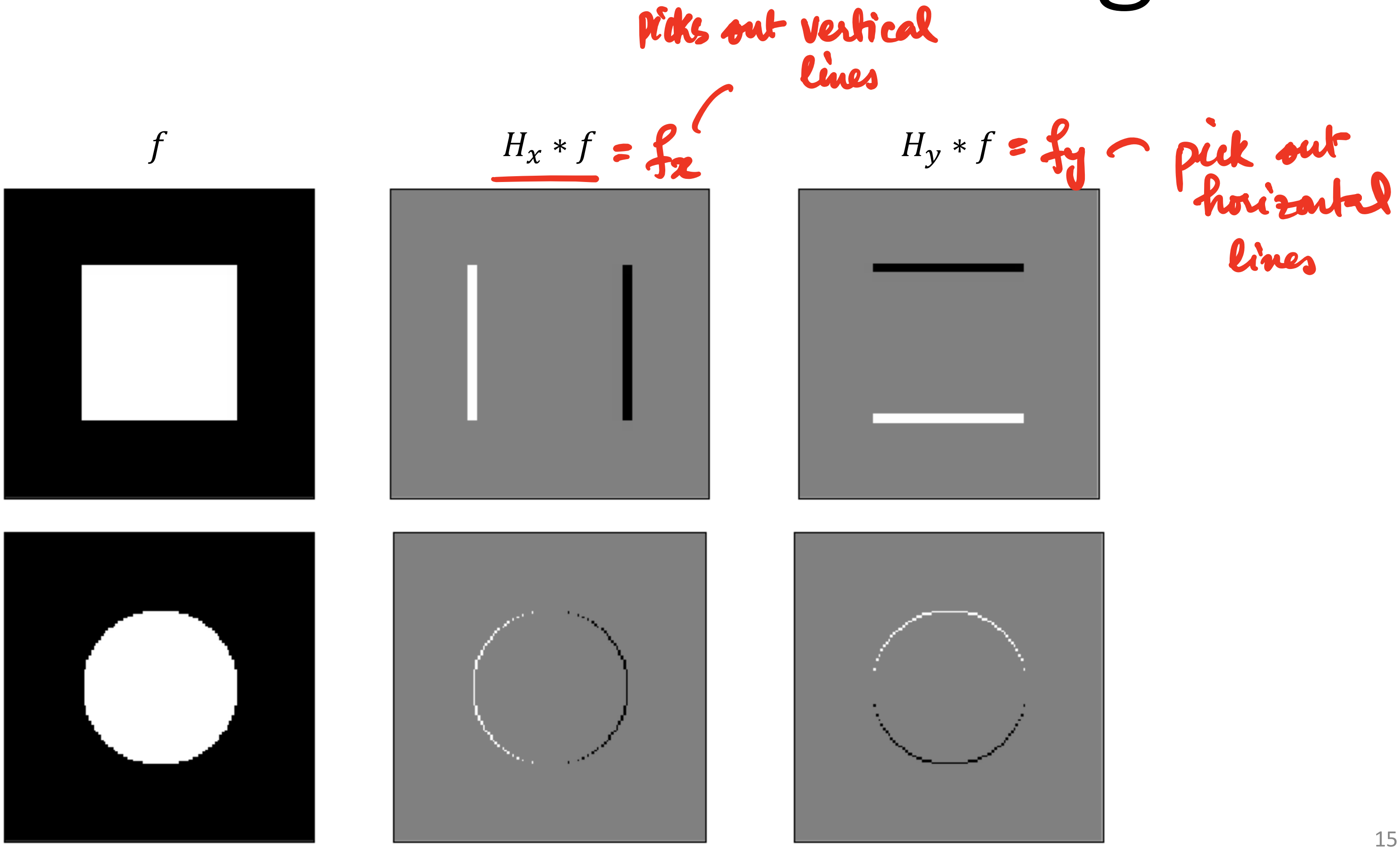
Gradient direction and magnitude



$$\|\nabla I\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

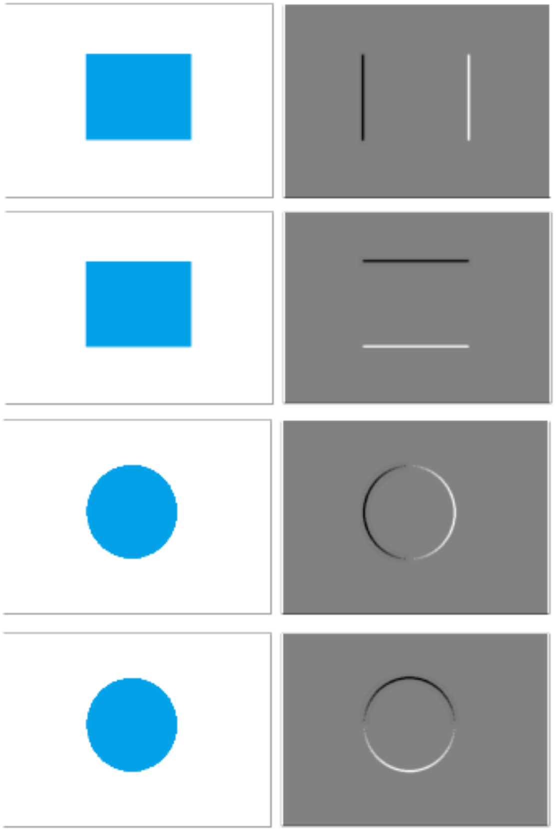
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

Partial Derivative of an Image



Partial Derivative of an Image

f



$H_x * f$

$H_y * f$

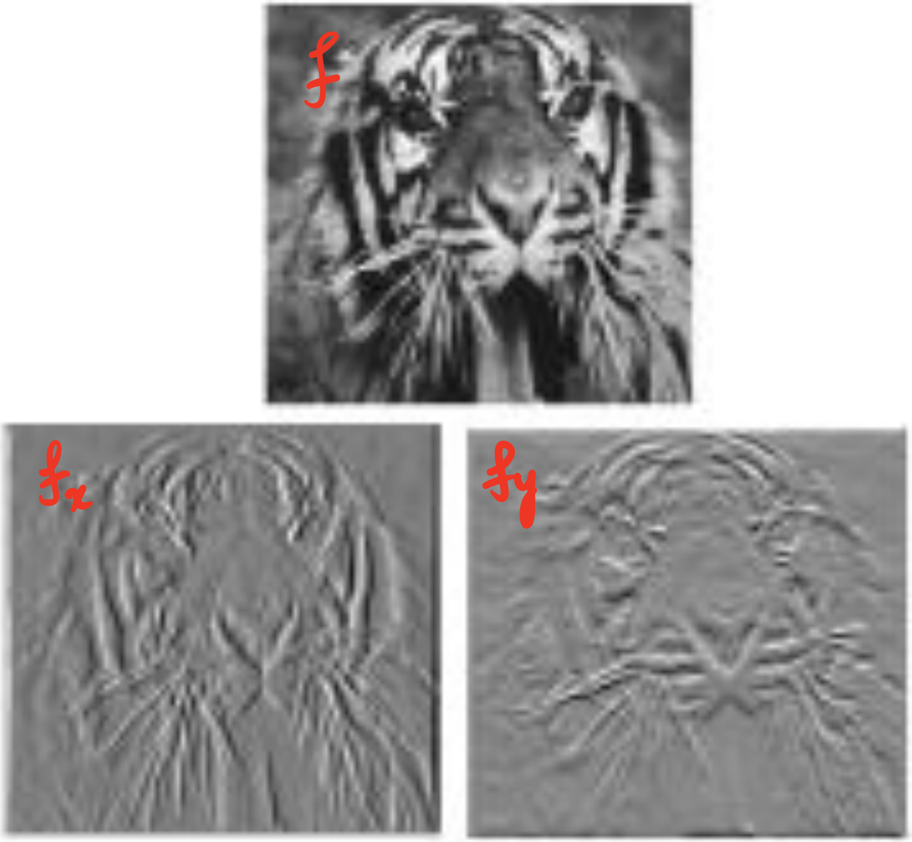
$H_x * f$

$H_y * f$

$$H_x = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$H_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Partial Derivatives of an Image



Filters for computing image derivatives

Finite Difference Filters

Sobel

$H_x =$

1	0	-1
2	0	-2
1	0	-1

$H_y =$

1	2	1
0	0	0
-1	-2	-1

Prewire

$H_x =$

1	0	-1
1	0	-1
1	0	-1

$H_y =$

1	1	1
0	0	0
-1	-1	-1

Roberts

$H_x =$

0	-1
1	0

$H_y =$

1	0
0	-1

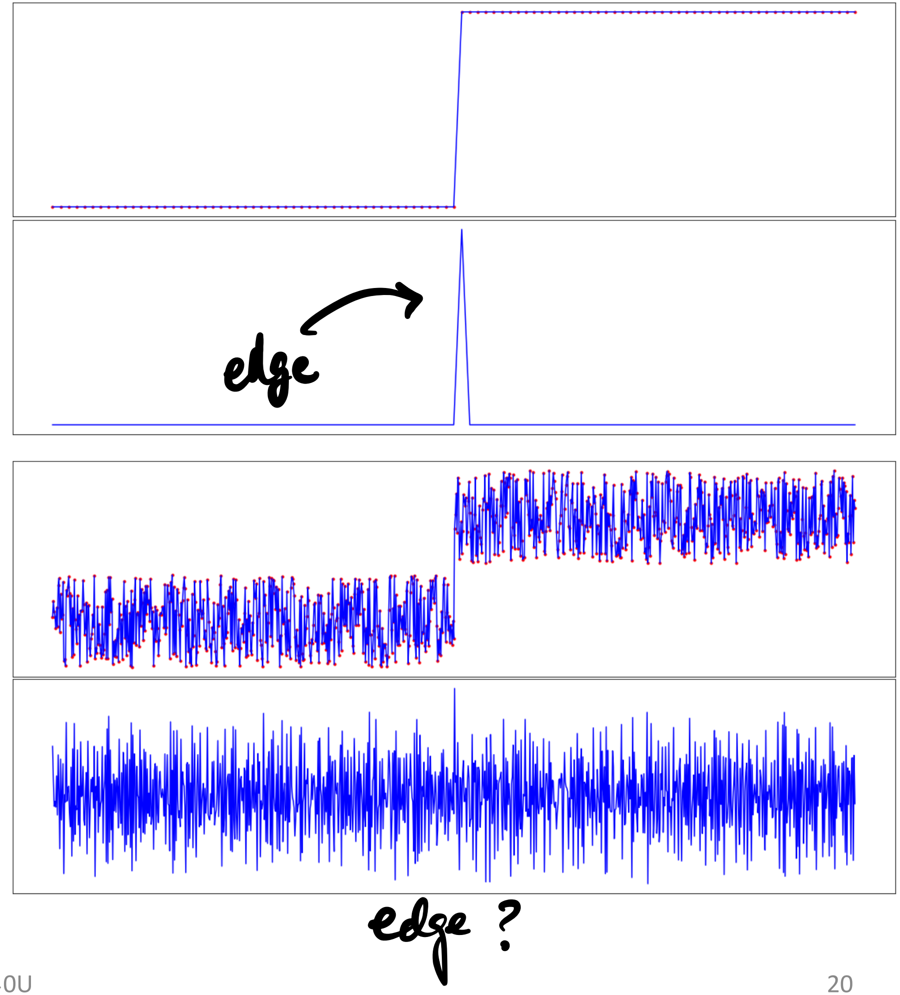
Image noise and gradients

Clean signal f

$\frac{df}{dx}$

Noisy signal g

$\frac{dg}{dx}$



Effects of Noise

- Gradient is highly sensitive to noise
- Difference filters (that we can use for gradient computation) respond strongly to noise
  - The larger the noise, the stronger the response
- How to handle it?
  - Smooth first. Get rid of high-frequency component.

1. Average $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

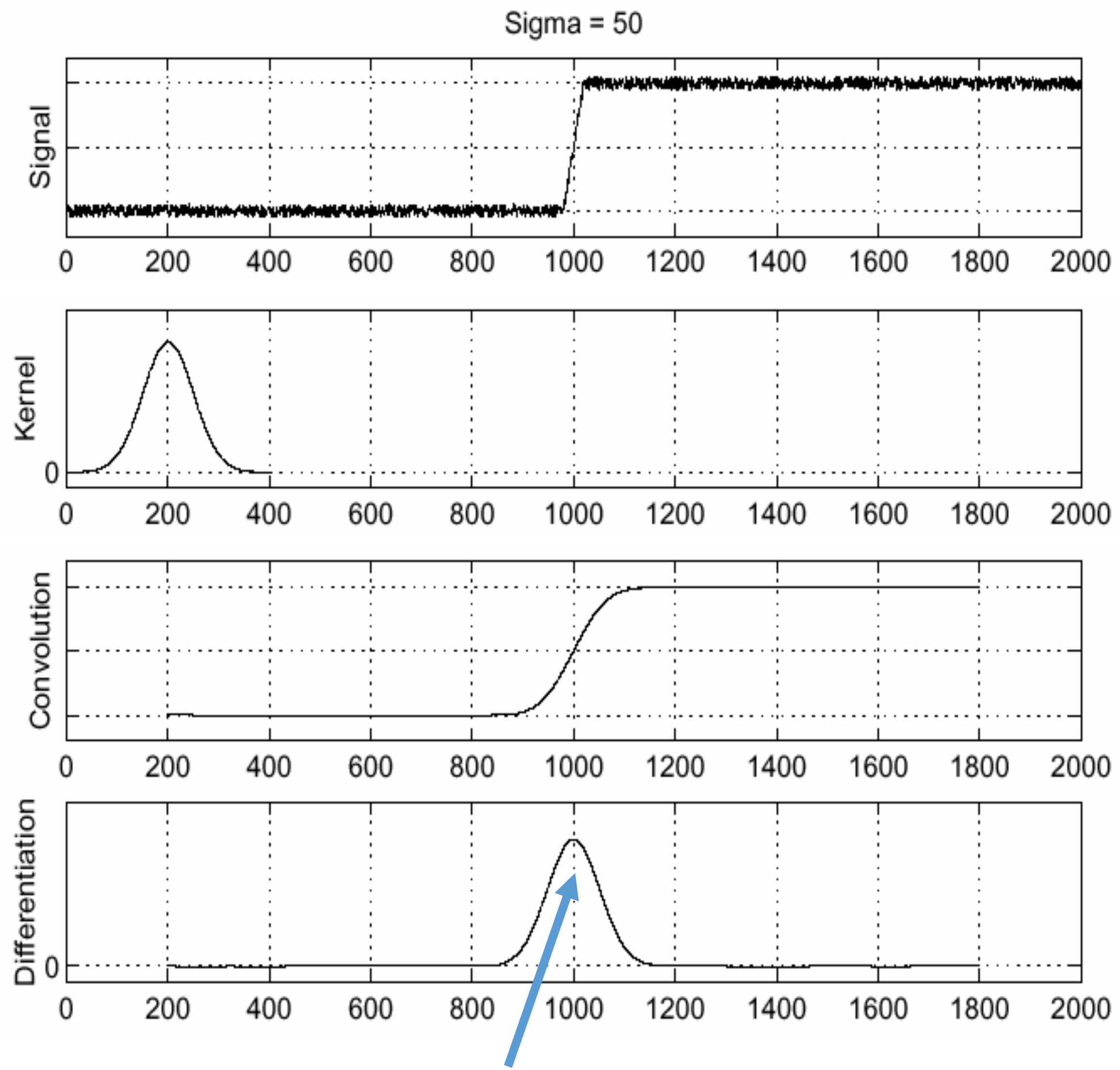
2. Gaussian

f

g

Smoothed signal $\quad f * g$

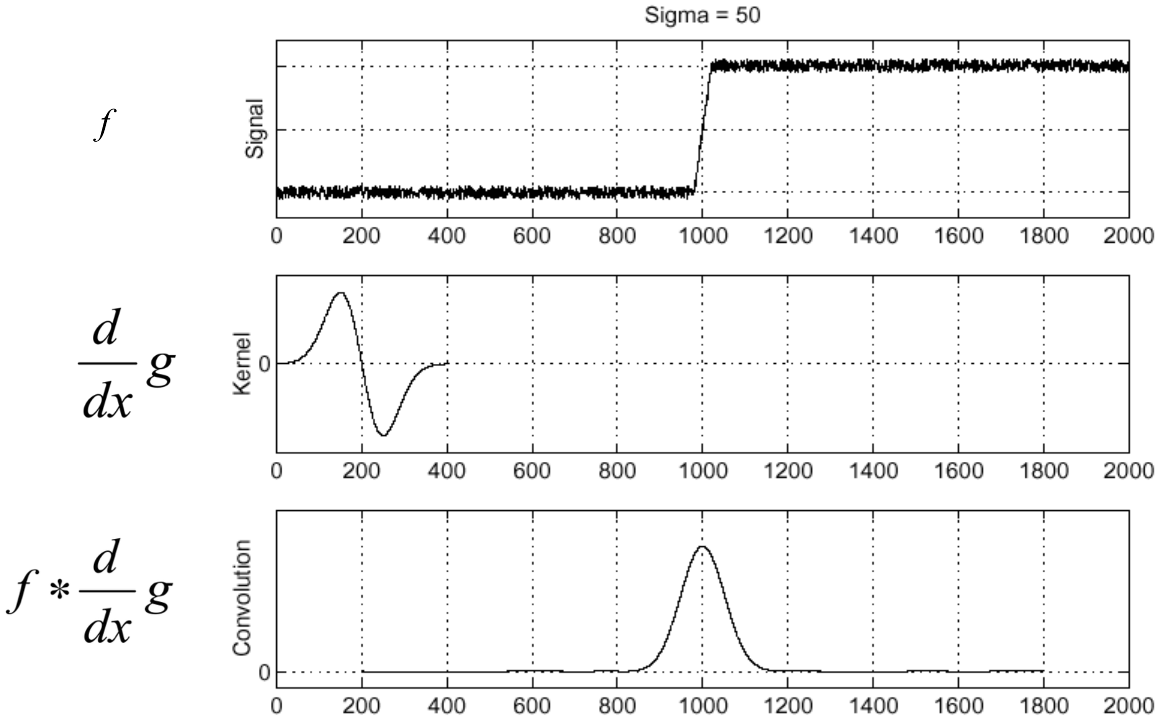
Derivative $\quad \frac{d}{dx}(f * g)$



Peak corresponds to edge

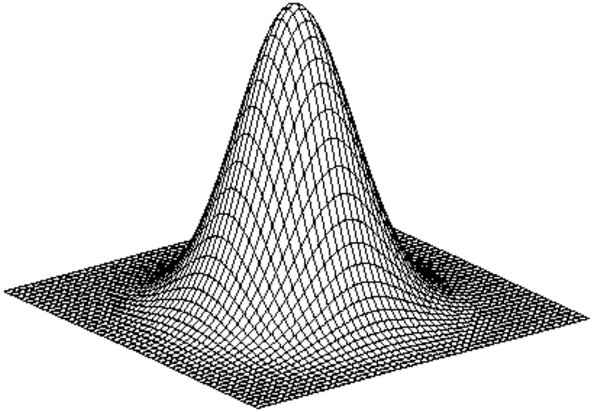
Differentiation via Convolution

- Convolution is associative: $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$



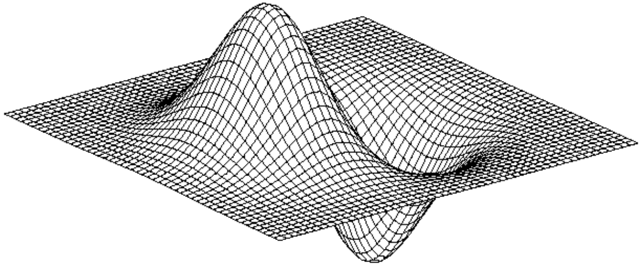
We saved one convolutional operation

Derivative of Gaussian filter



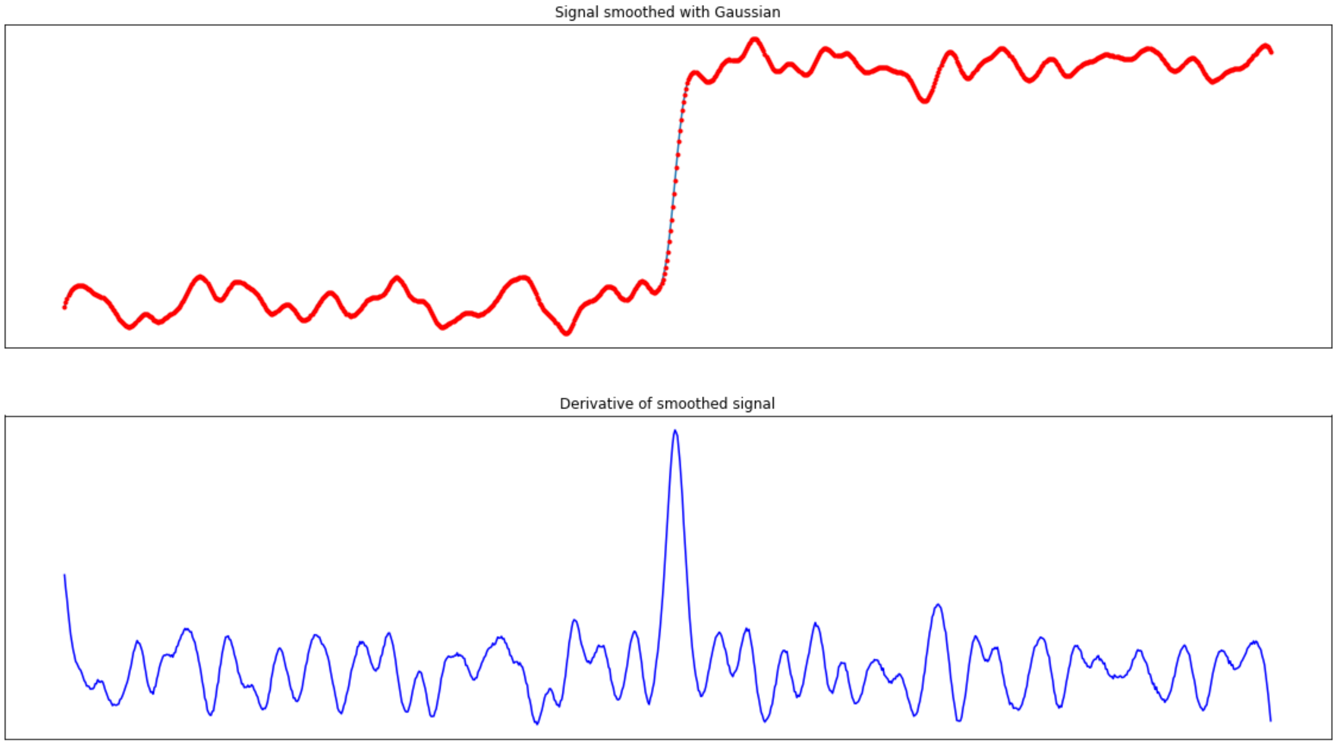
2D

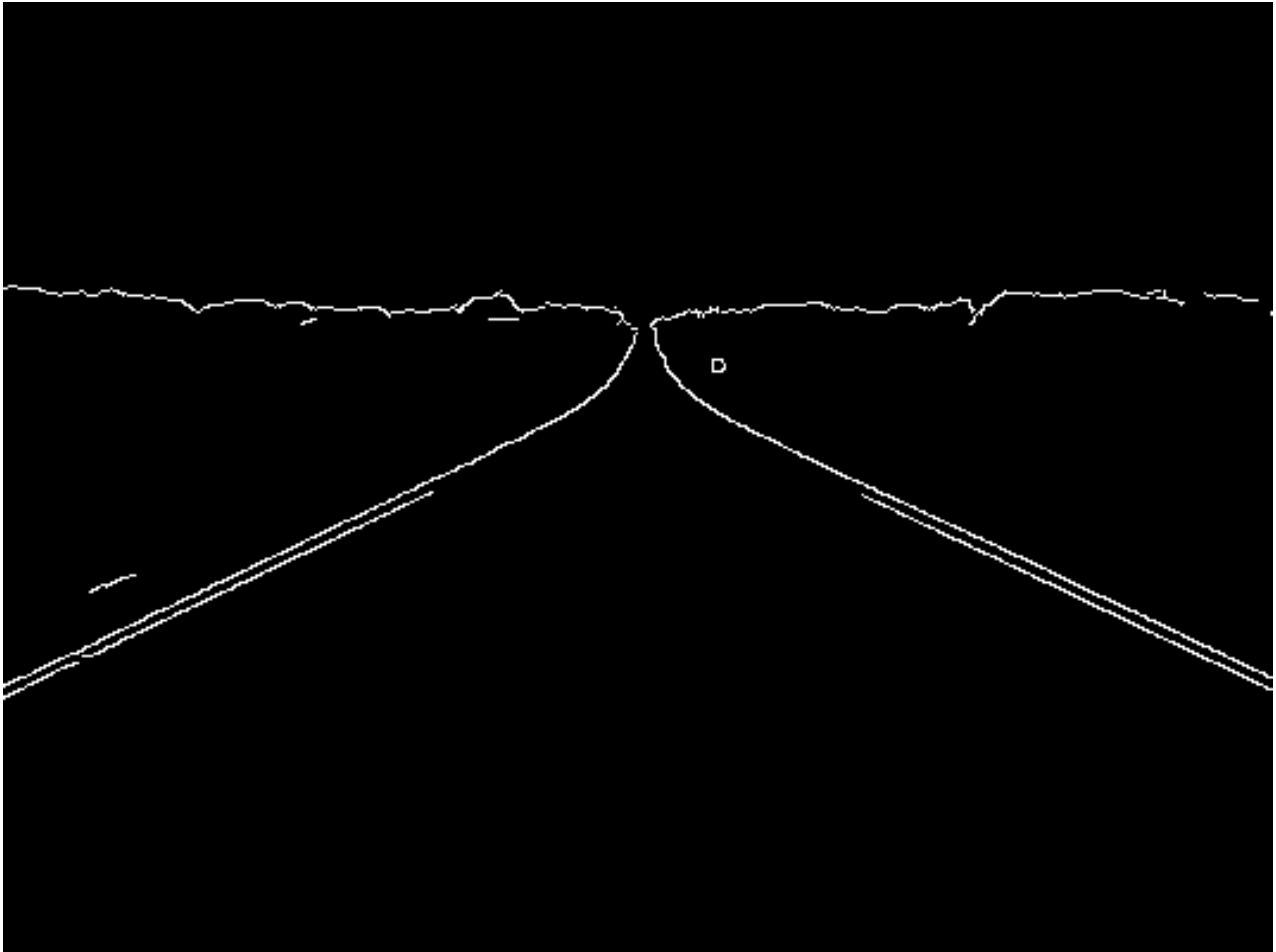
$* [1, -1] \; =$



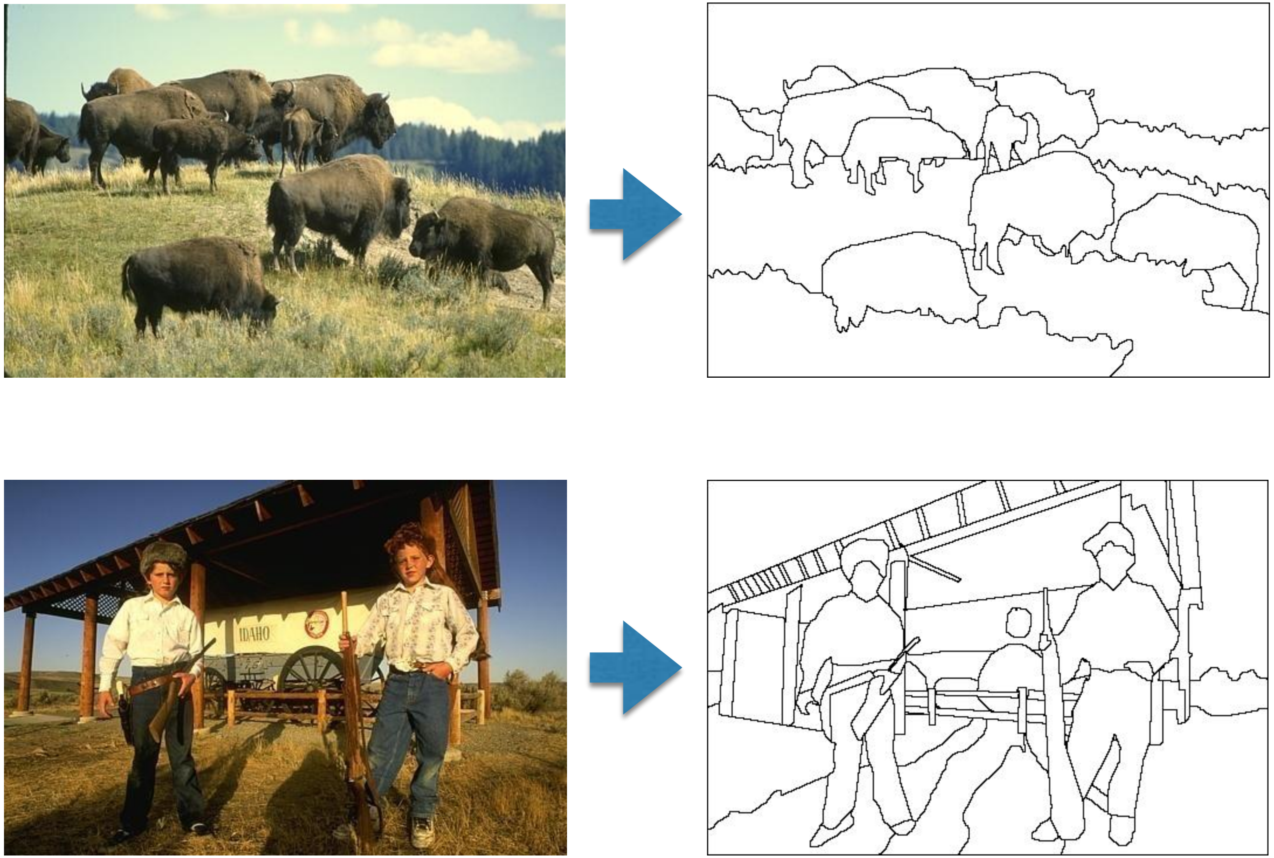
2D

Image noise and gradients





Edge Detection



Edge Detection

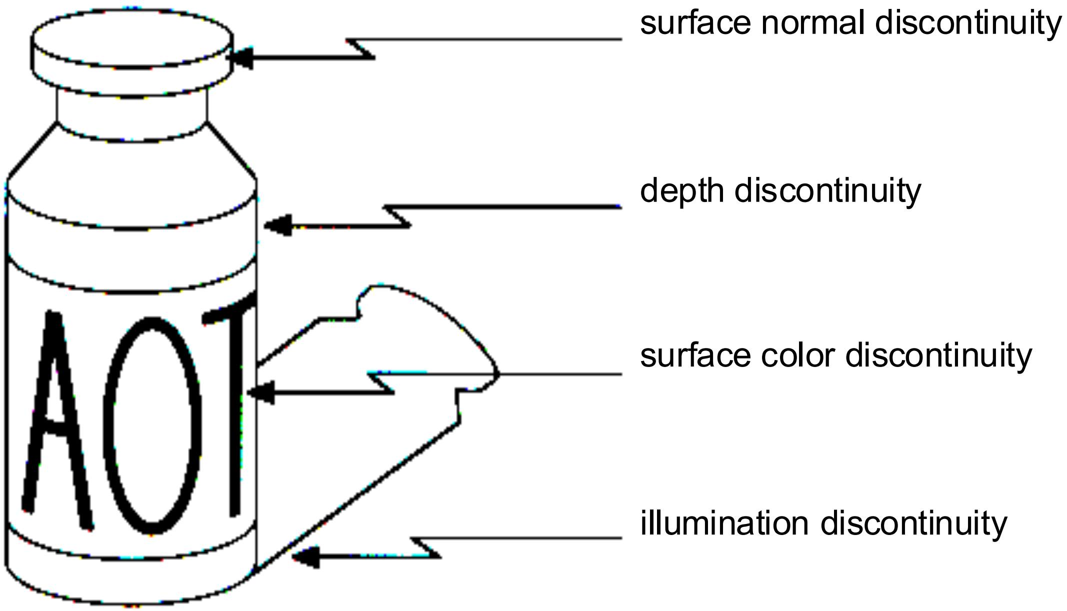
- Identify sudden changes (discontinuities) in an image
- Most semantic and shape information seen in an image can be encoded using the edges
- Edges are more compact than pixels

An artist's line drawing

[Source: D. Lowe]

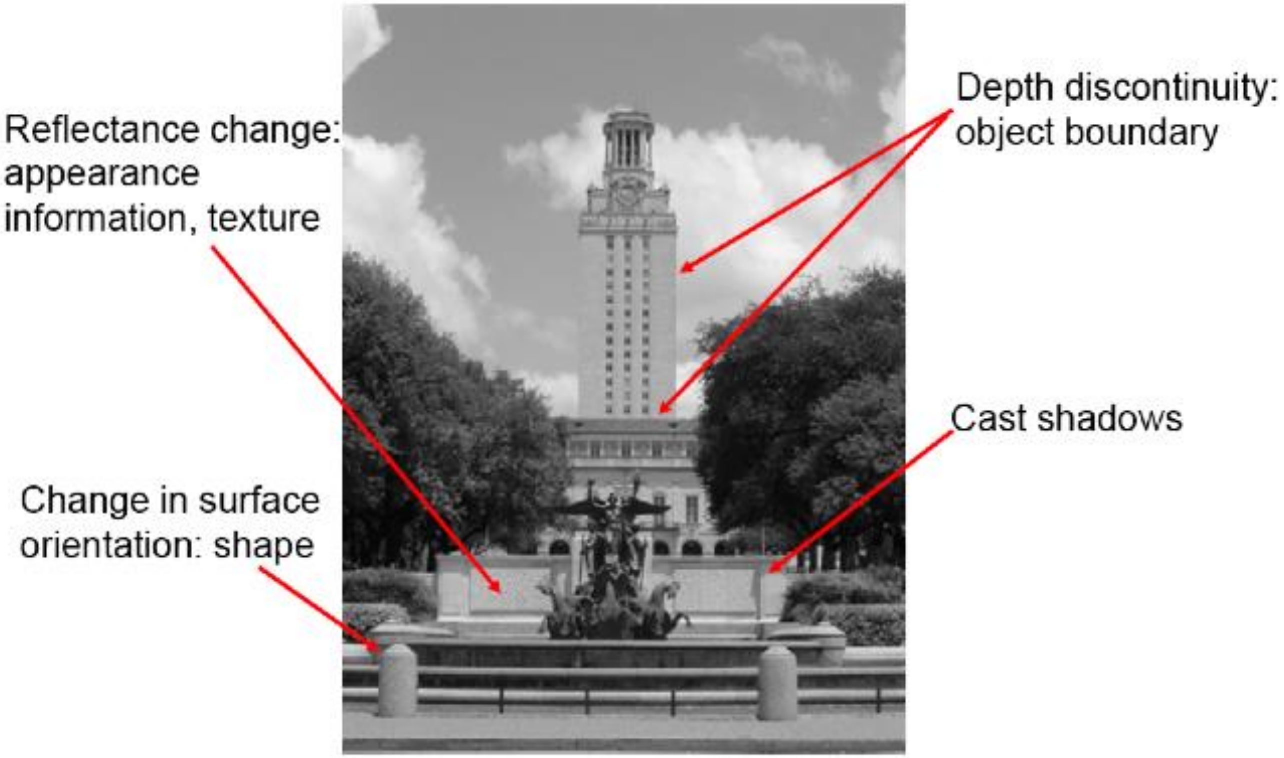
Origin of Edges

- Edges are caused by



[Source: Steve Seitz]

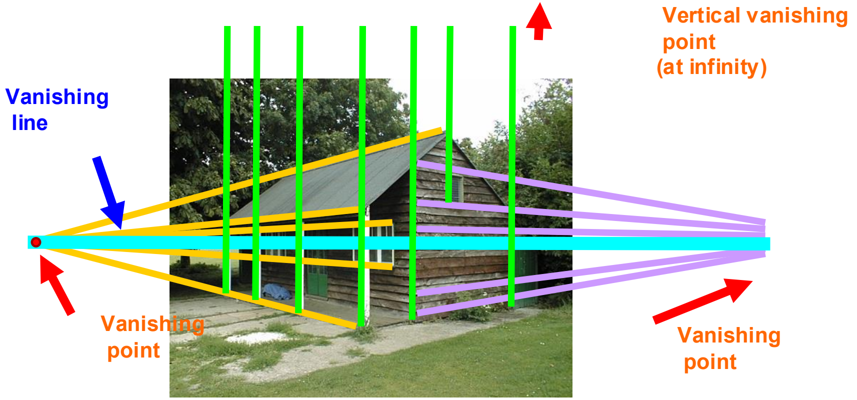
What causes edges?

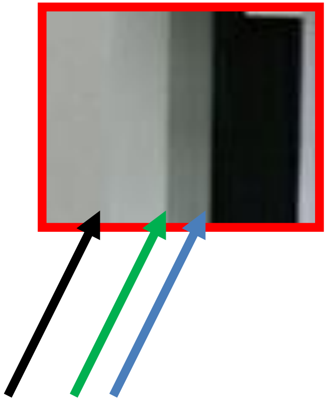


[Source: K. Grauman]

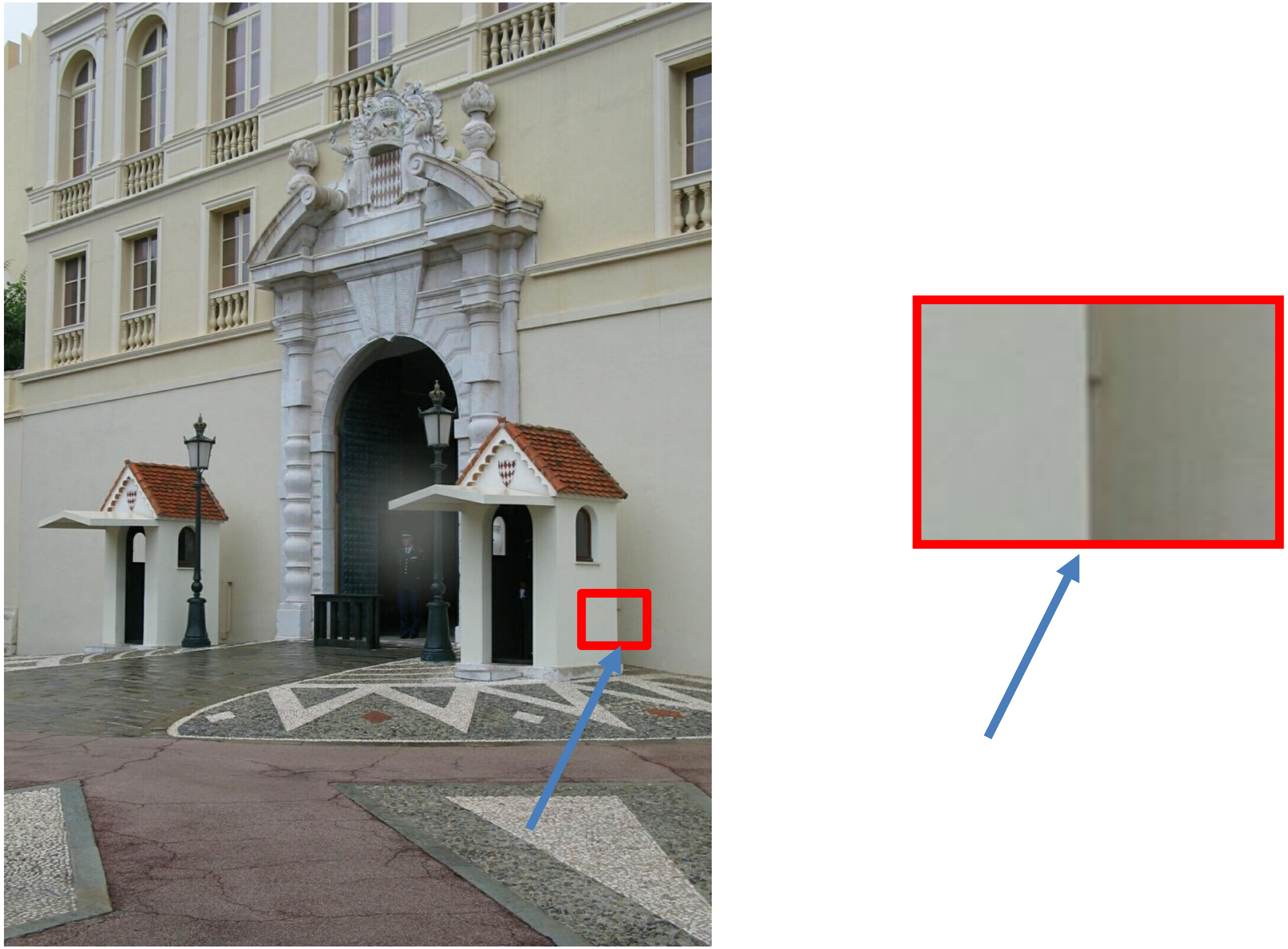
Why edge detection?

- Extract information
- Recognize objects
- Understand scene
- Reconstruct 3D from images
  - Recover viewpoint and geometry





[Source: Steve Seitz]



[Source: Steve Seitz]

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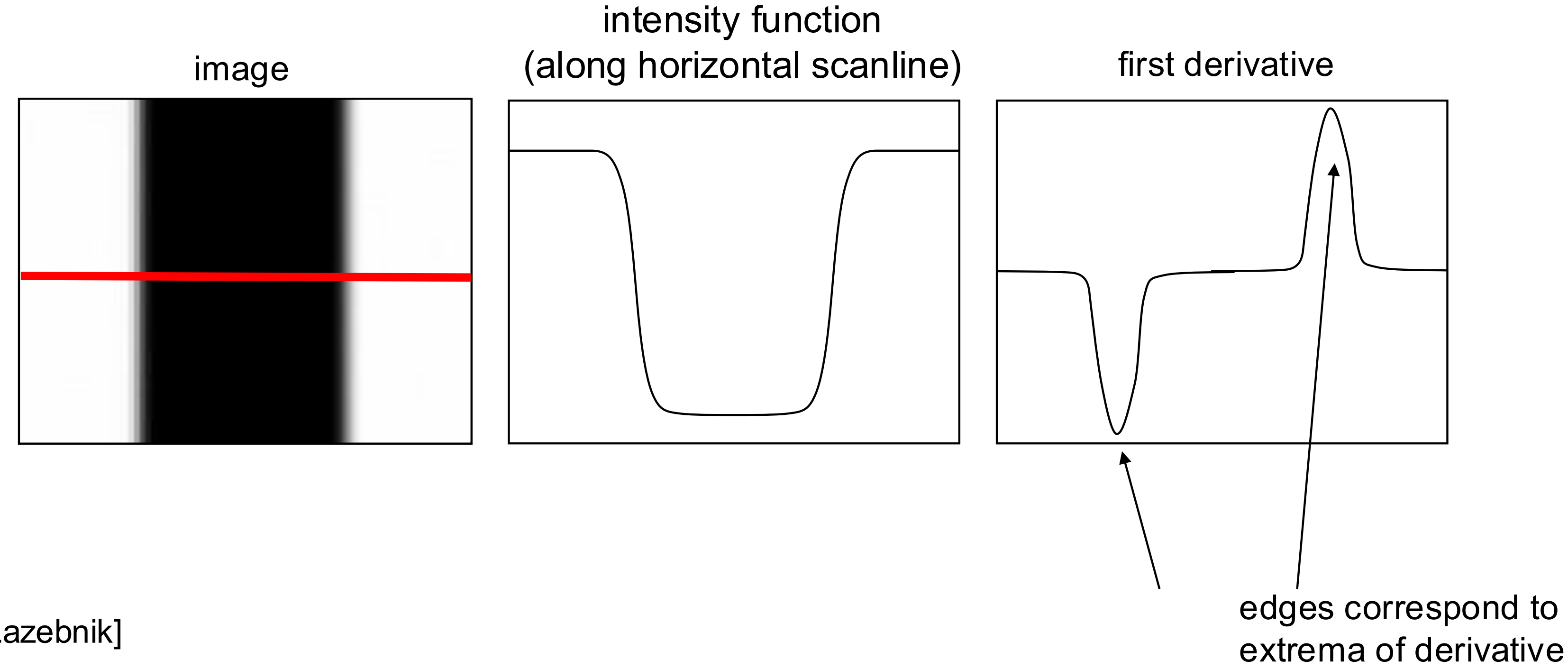
Characterizing Edges

- An edge is a place of rapid change in intensity



[Source: S. Lazebnik]

Summary

- Image gradients
- Finite-difference approximation filters
- Gradient magnitude and direction
- Image noise and gradients
- Edges and their importance