

Image Formation – Lenses

Computational Photography (CSCI 3240U)

Faisal Z. Qureshi

<http://vclab.science.ontariotechu.ca>





computational photography - 04

Acknowledgments

- These slides borrow and adapt materials developed by others, including
 - Michael Brown
 - Kyros Kutulakos
 - David Lindell
 - Gordon Wetzstein
 - Marc Levoy
 - Fredo Durand
 - Paul Debevec
 - Ramesh Raskar

Slide credits

- A lot of inspiration and quite a few examples for these slides were taken directly from:
 - Kayvon Fatahalian (15-769, Fall 2016).
 - Michael Brown (CVPR 2016 Tutorial on understanding the image processing pipeline).
 - Marc Levoy (Stanford CS 178, Spring 2014).

Special thanks to Ioannis Gkioulekas

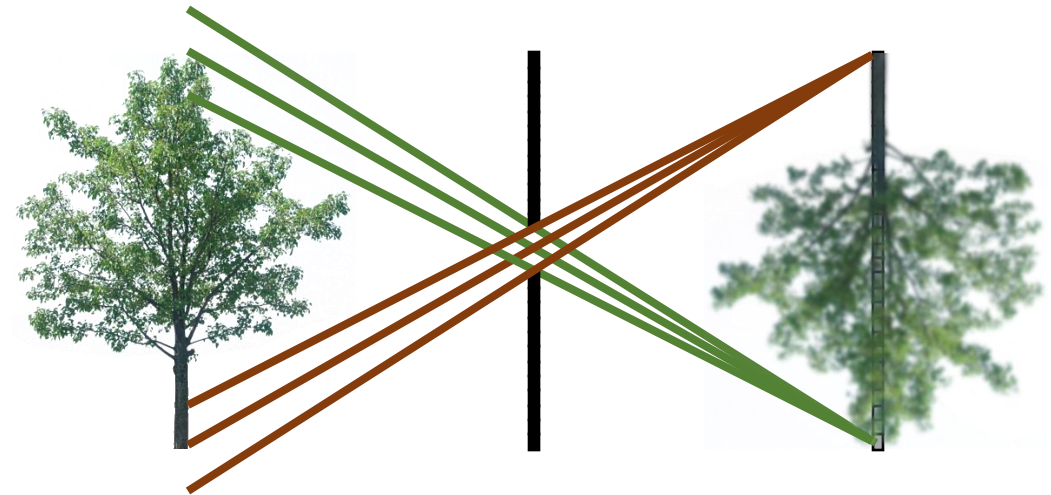
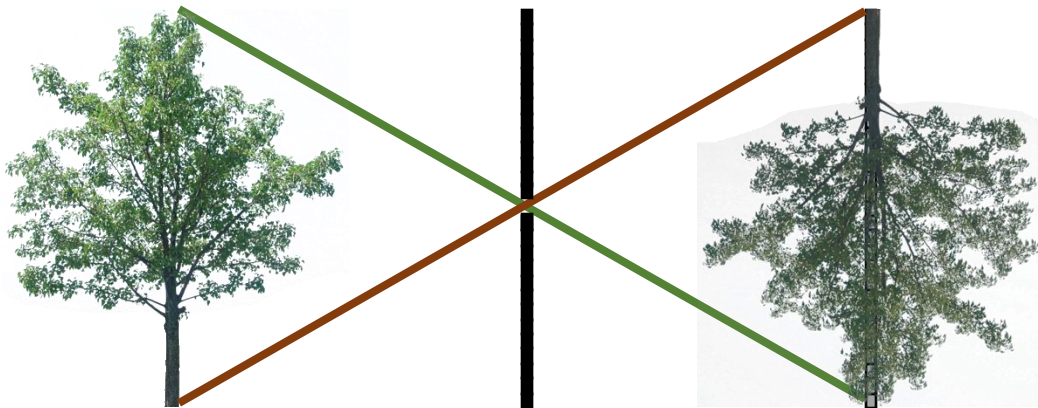
- Many of the slides are taken with his permission from the computational photography course that he has developed at CMU

Pinhole Camera Trade-off

Previously

- Small (ideal) pinhole
 - Sharp image
 - Low signal-to-noise ratio

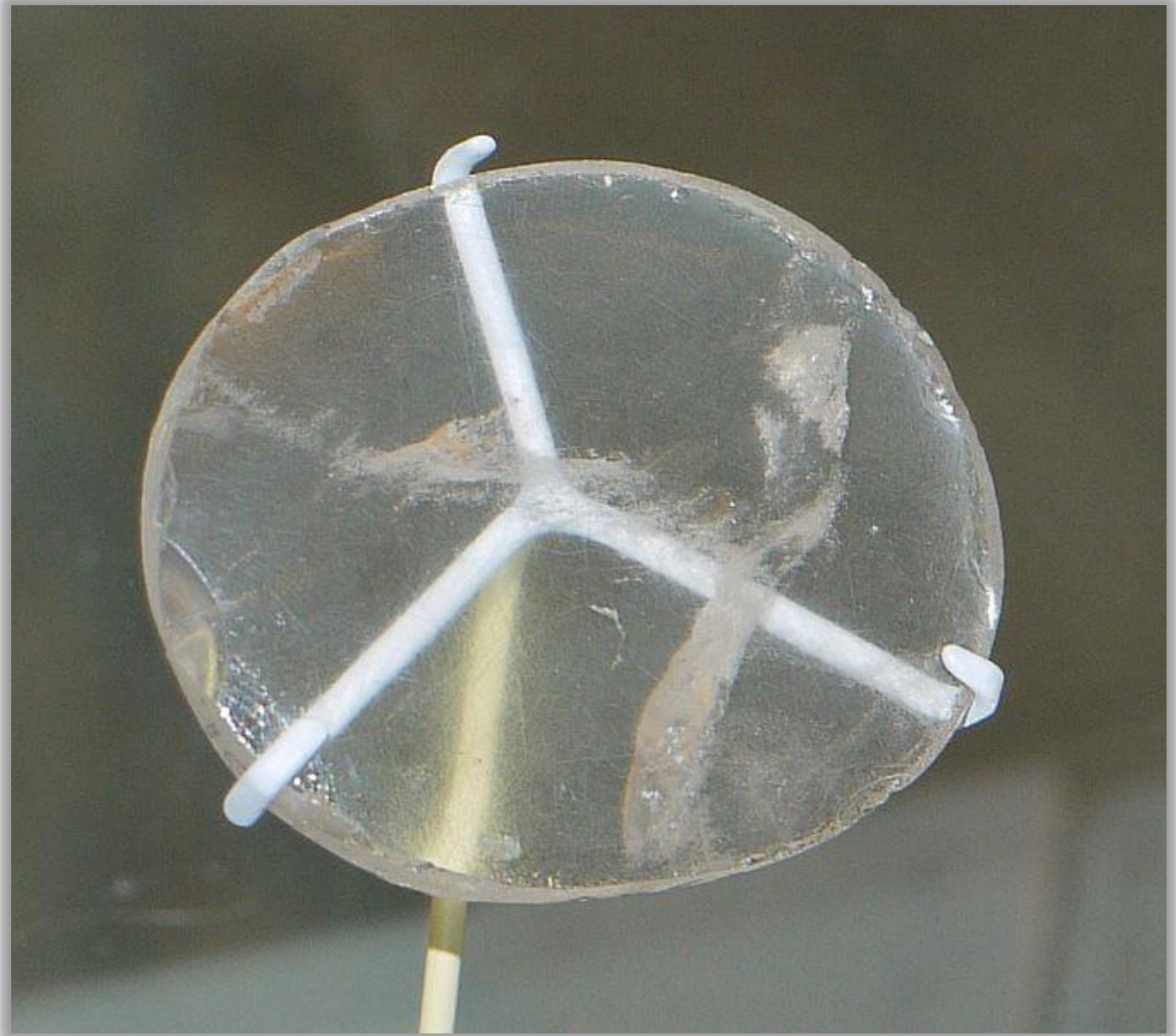
- Large pinhole
 - Blurry image
 - High signal-to-noise ratio



Lenses

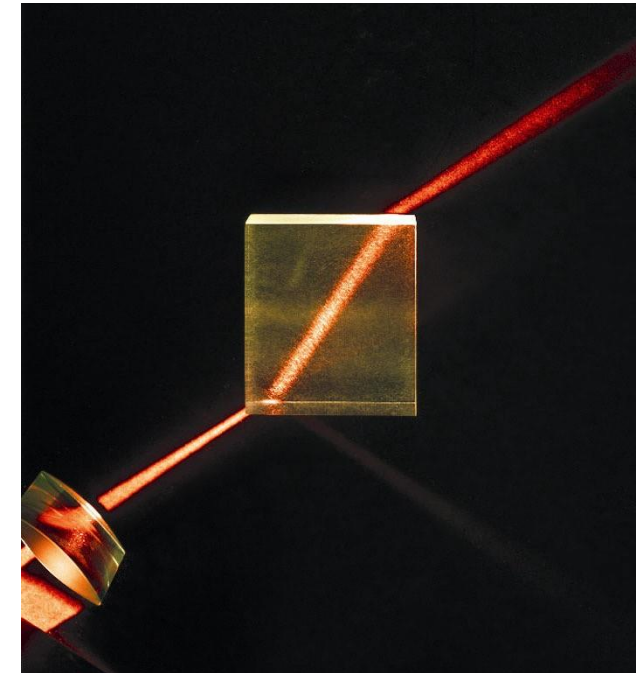
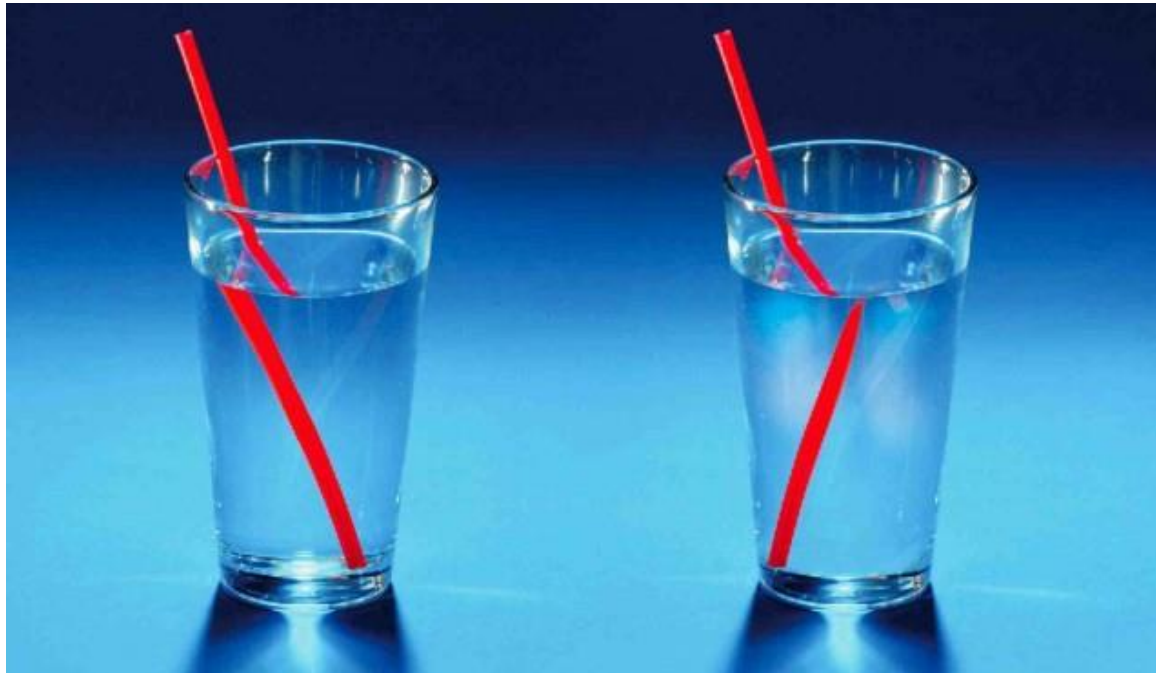
- Focus light
- Magnify objects

Nimrud lens – 2700 years old



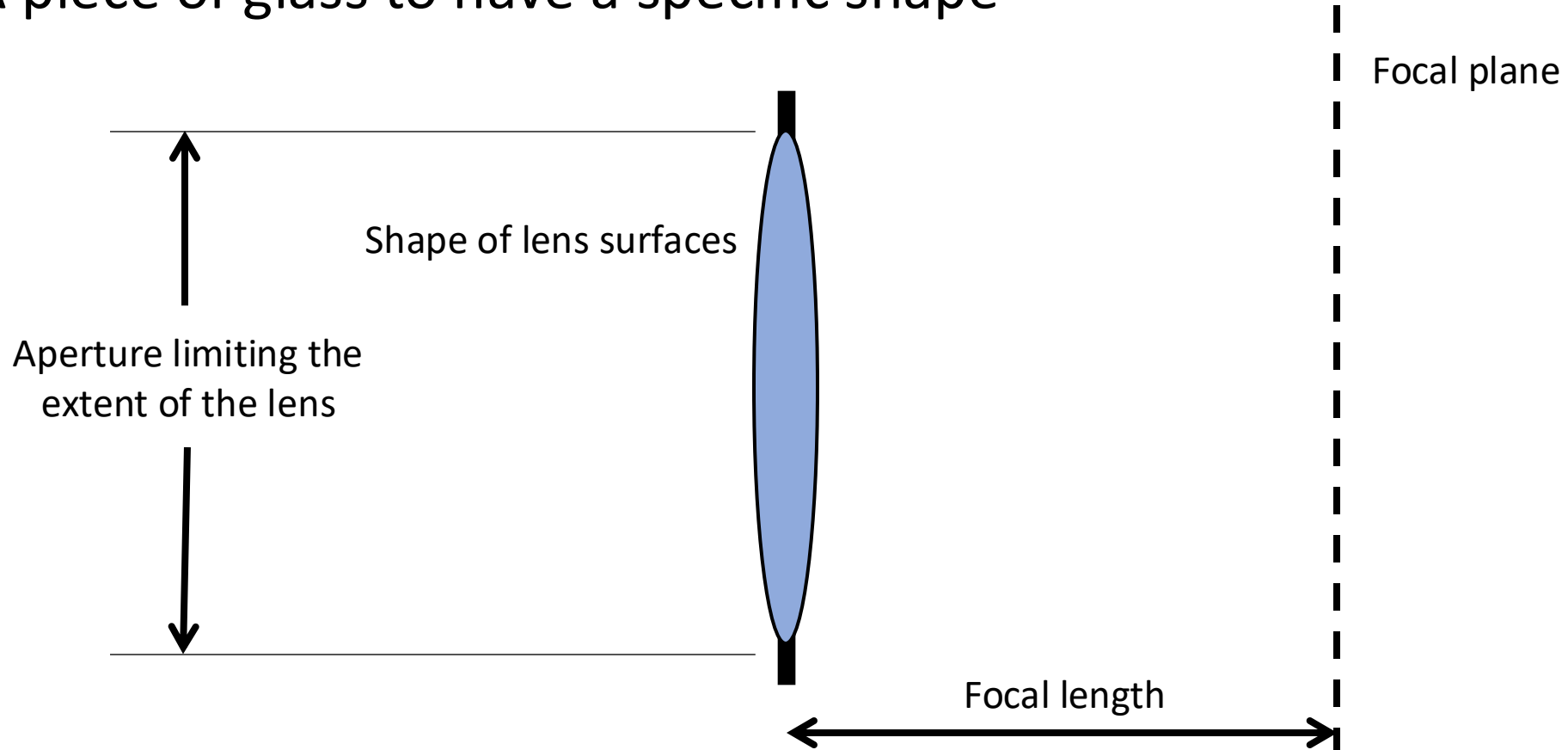
Lenses

- These bend light in a specific way
- The Principle of Refraction: the light changes its direction as it passes from one medium to another



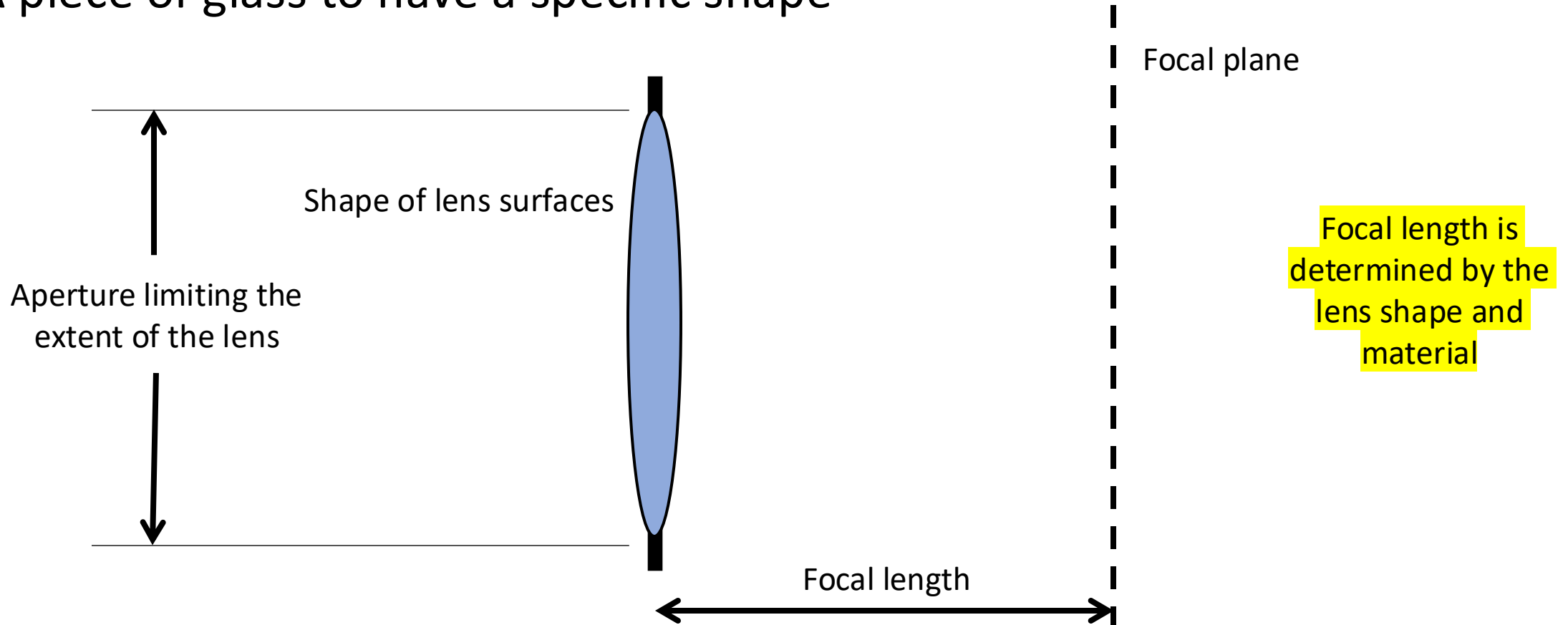
Anatomy of a Lens

- A piece of glass to have a specific shape



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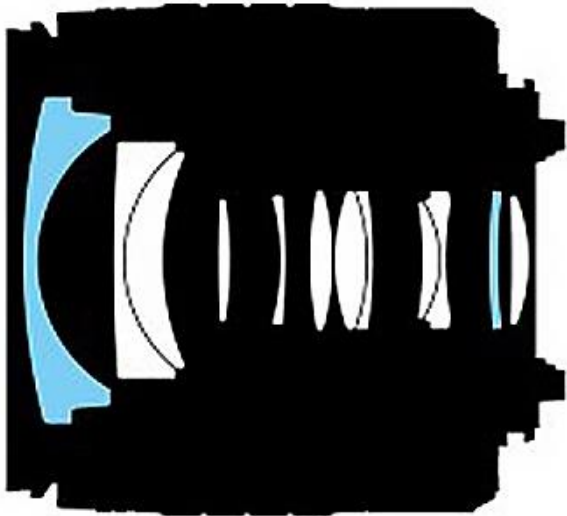


Camera Lenses



Compound Lenses

- Carefully designed glass pieces arranged in a specific pattern to achieve the desired optical qualities

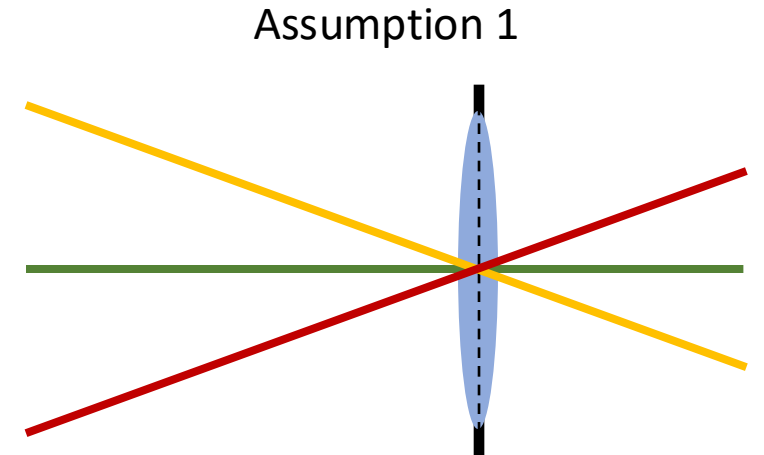


Cross-section of Nikon 18-55 mm lens

The effective aperture size and focal length are determined by the lenses' shape, material, and relative placement.

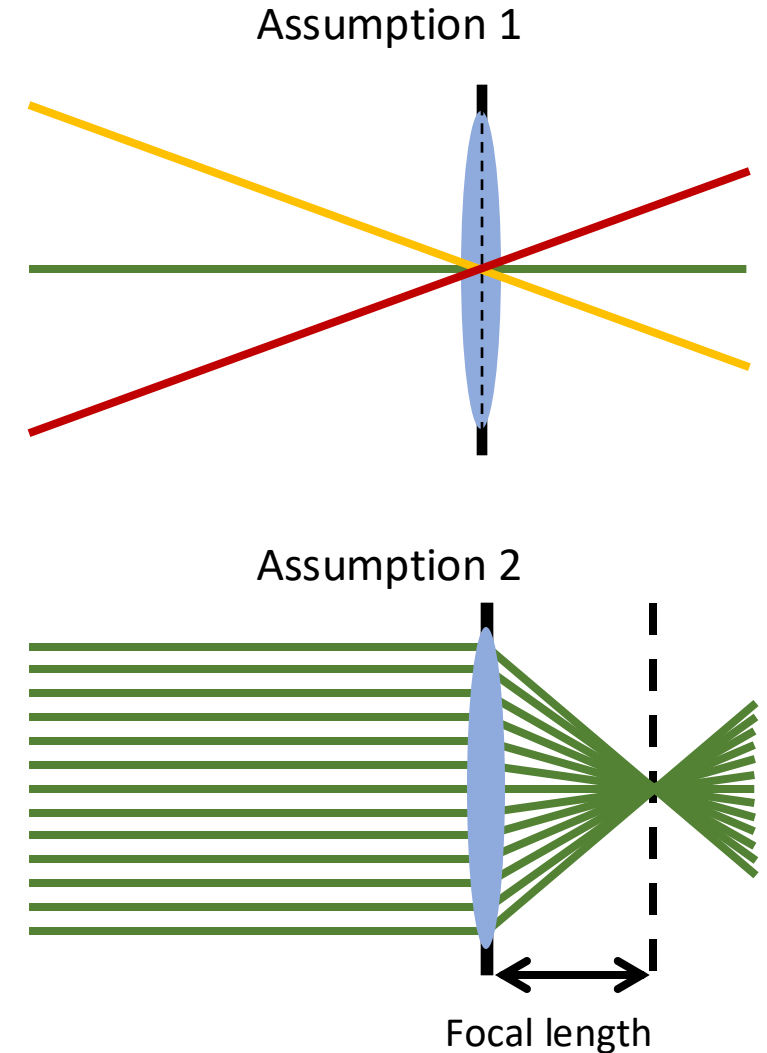
Thin Lens Model

- Simplified geometric optics for **well-designed** lenses
- Assumptions
 - Rays passing through the center of the lens remain unaffected



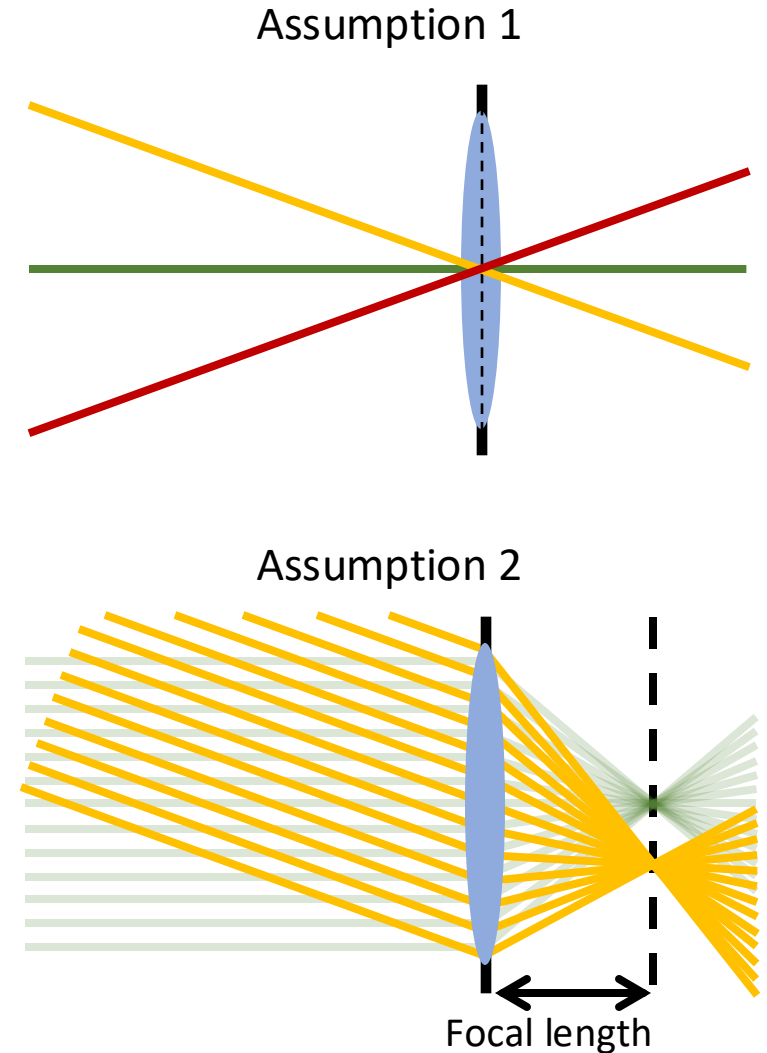
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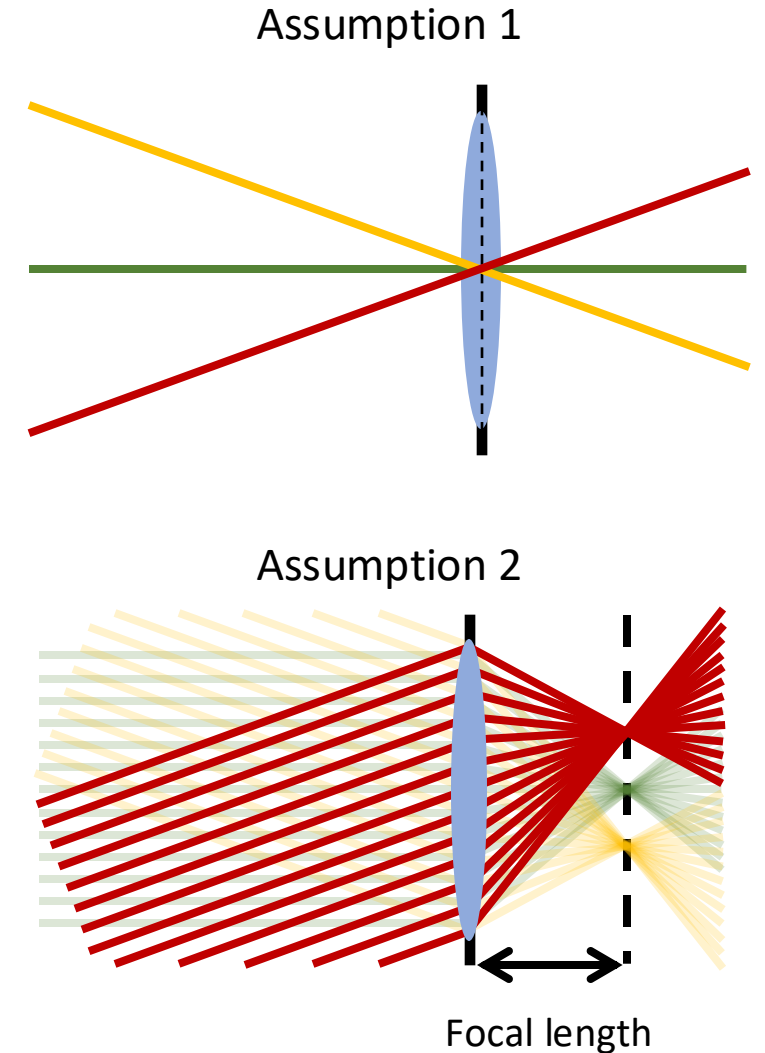
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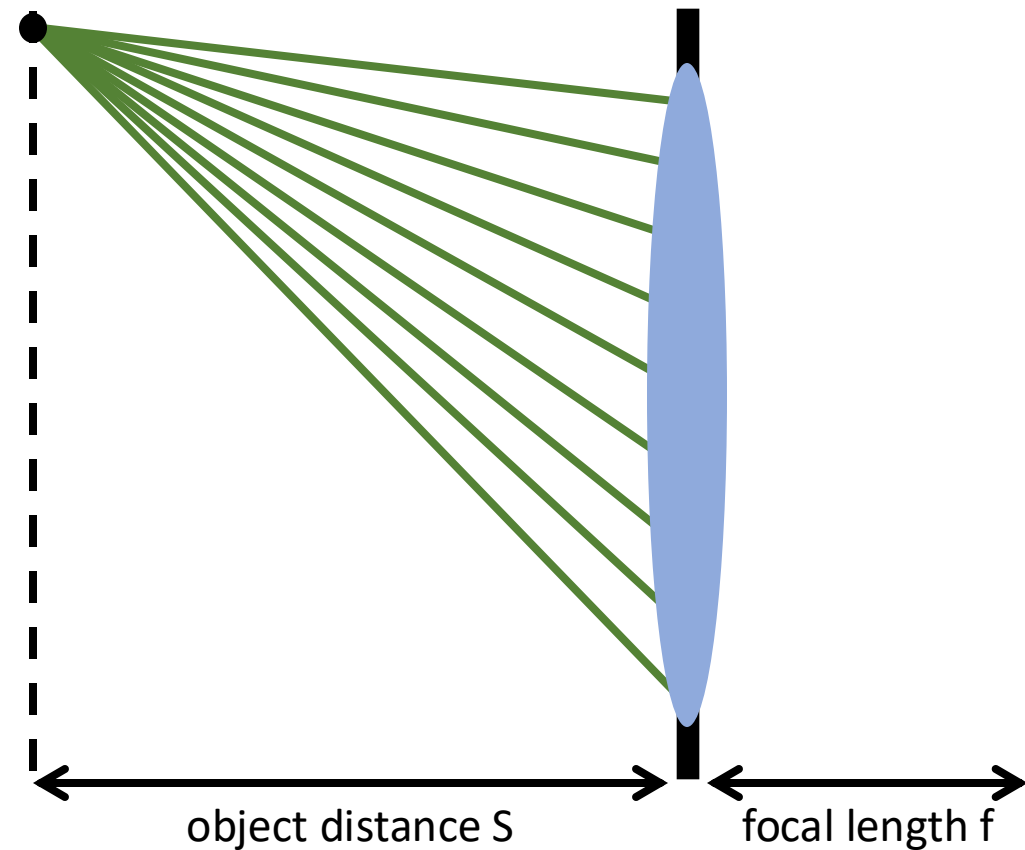
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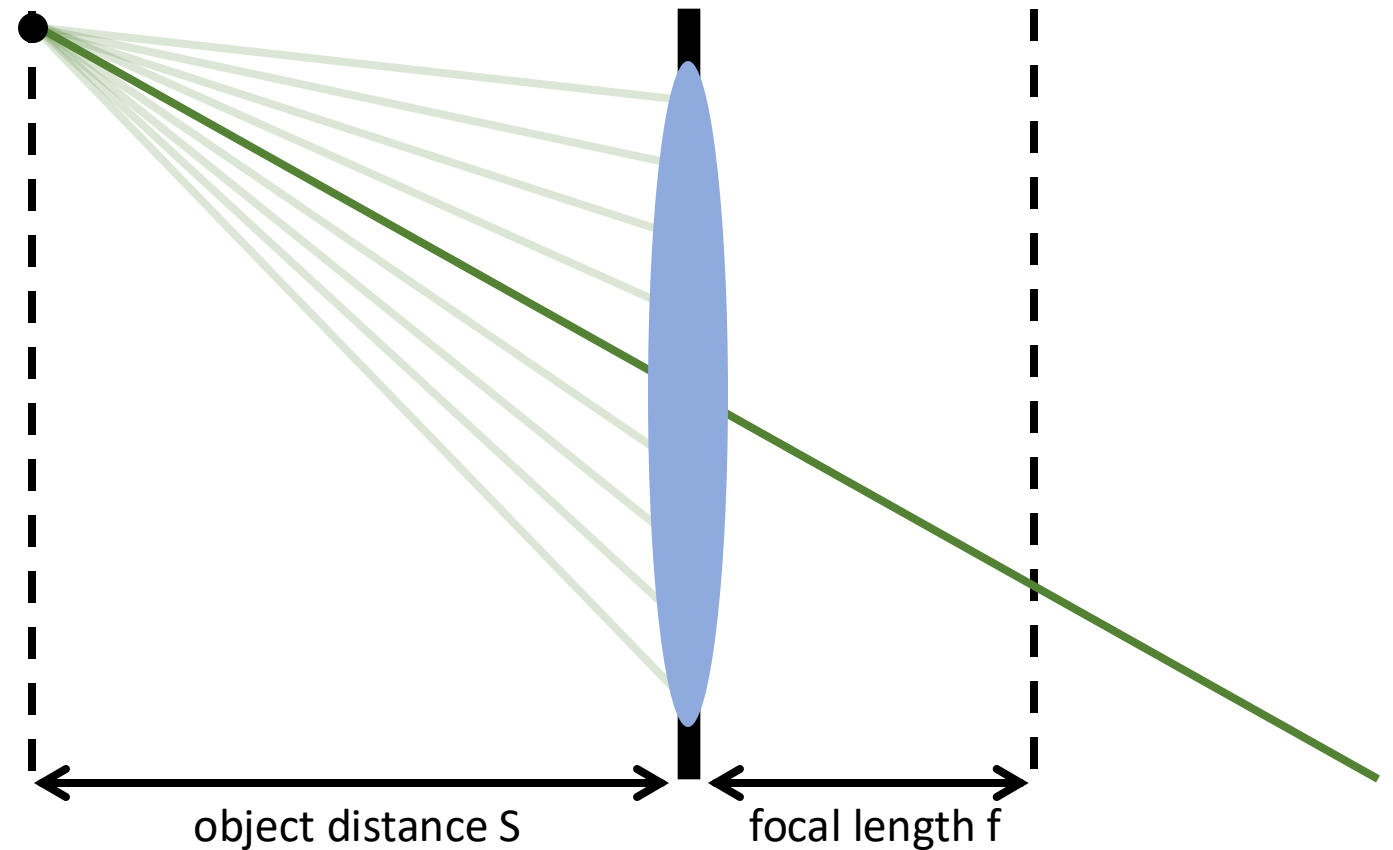
Tracing Rays Through a Thin Lens

1. Trace rays through lens center.
2. For all other rays:
 - a. Trace their parallel through lens center.
 - b. Connect on focal plane.



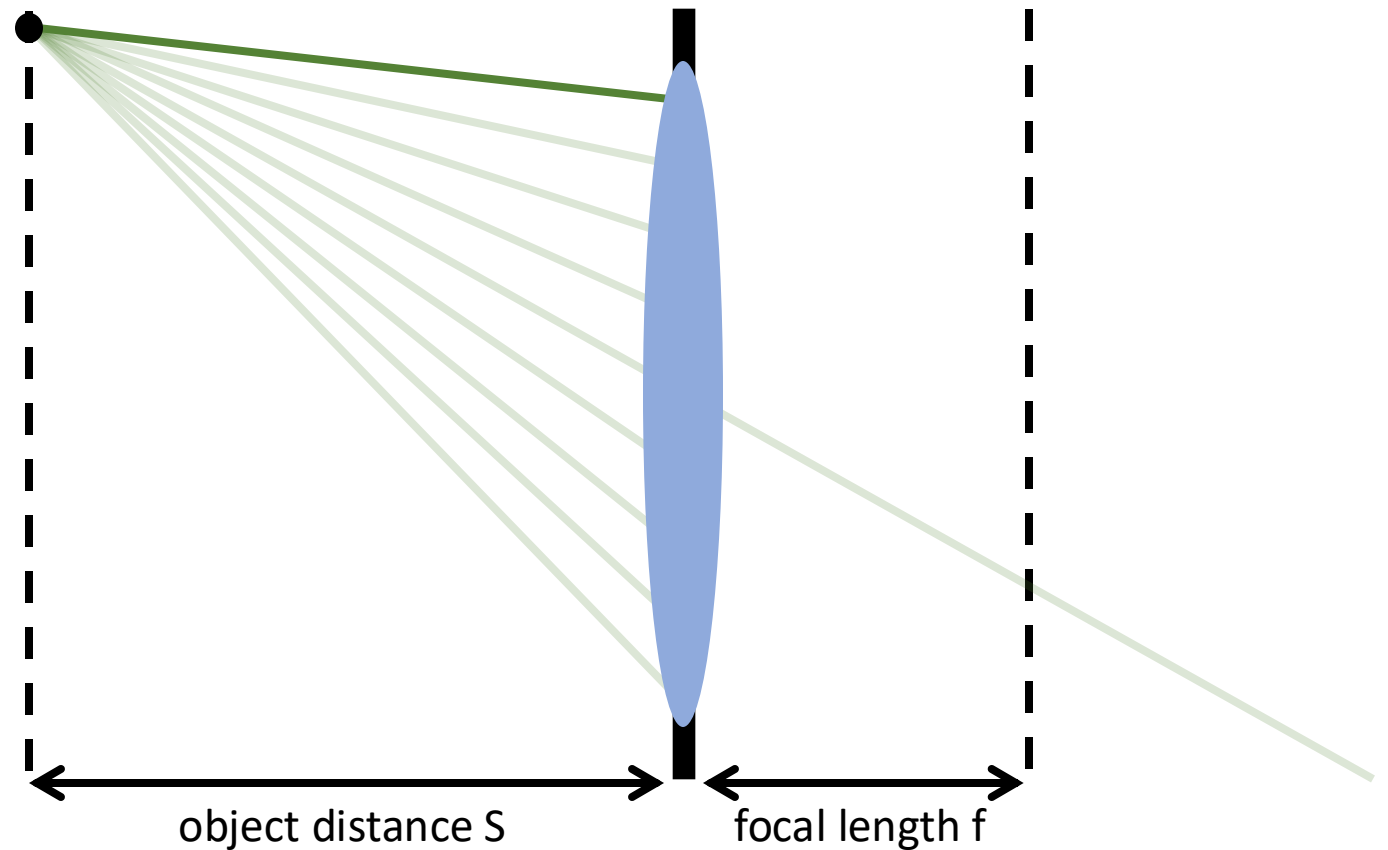
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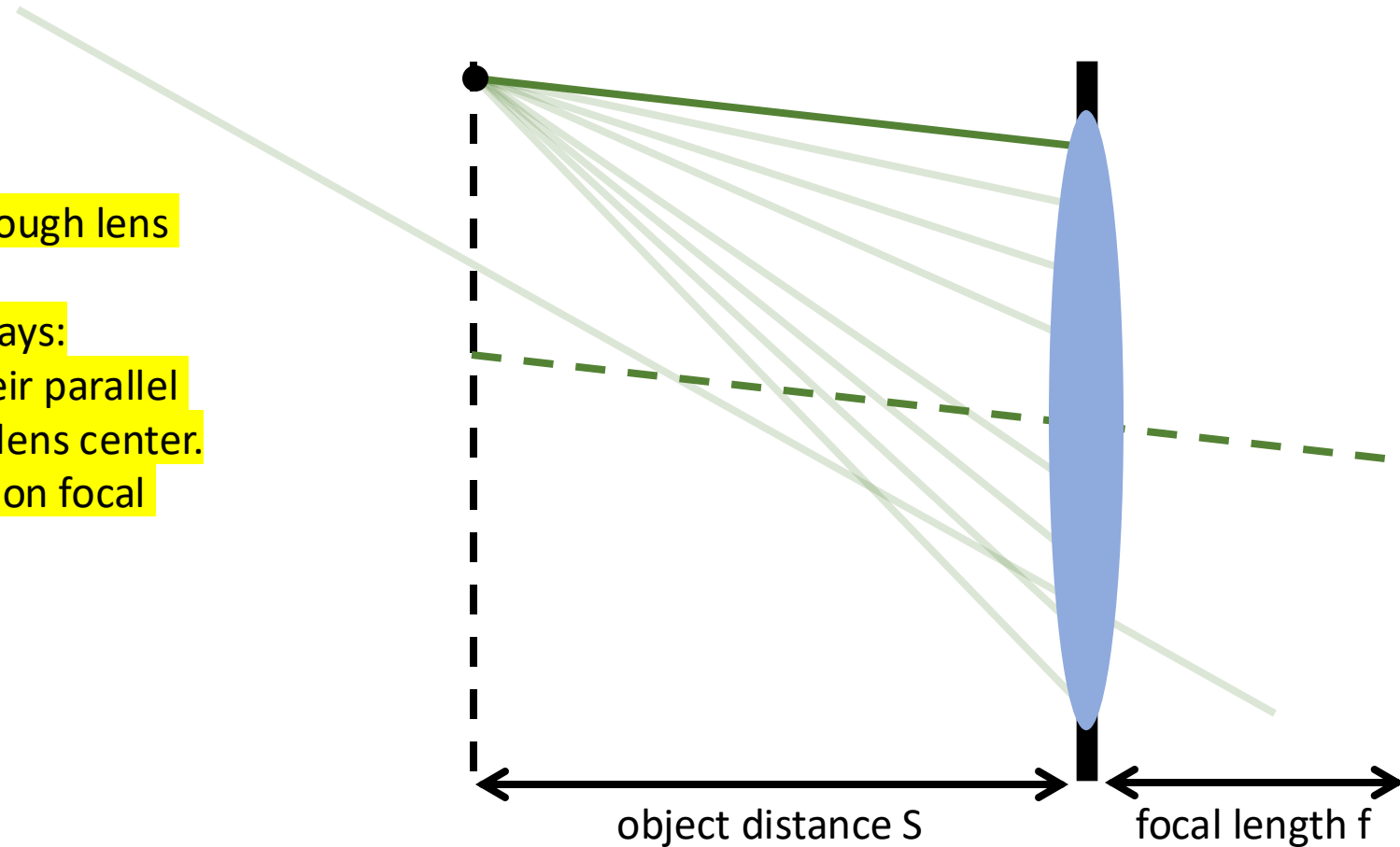
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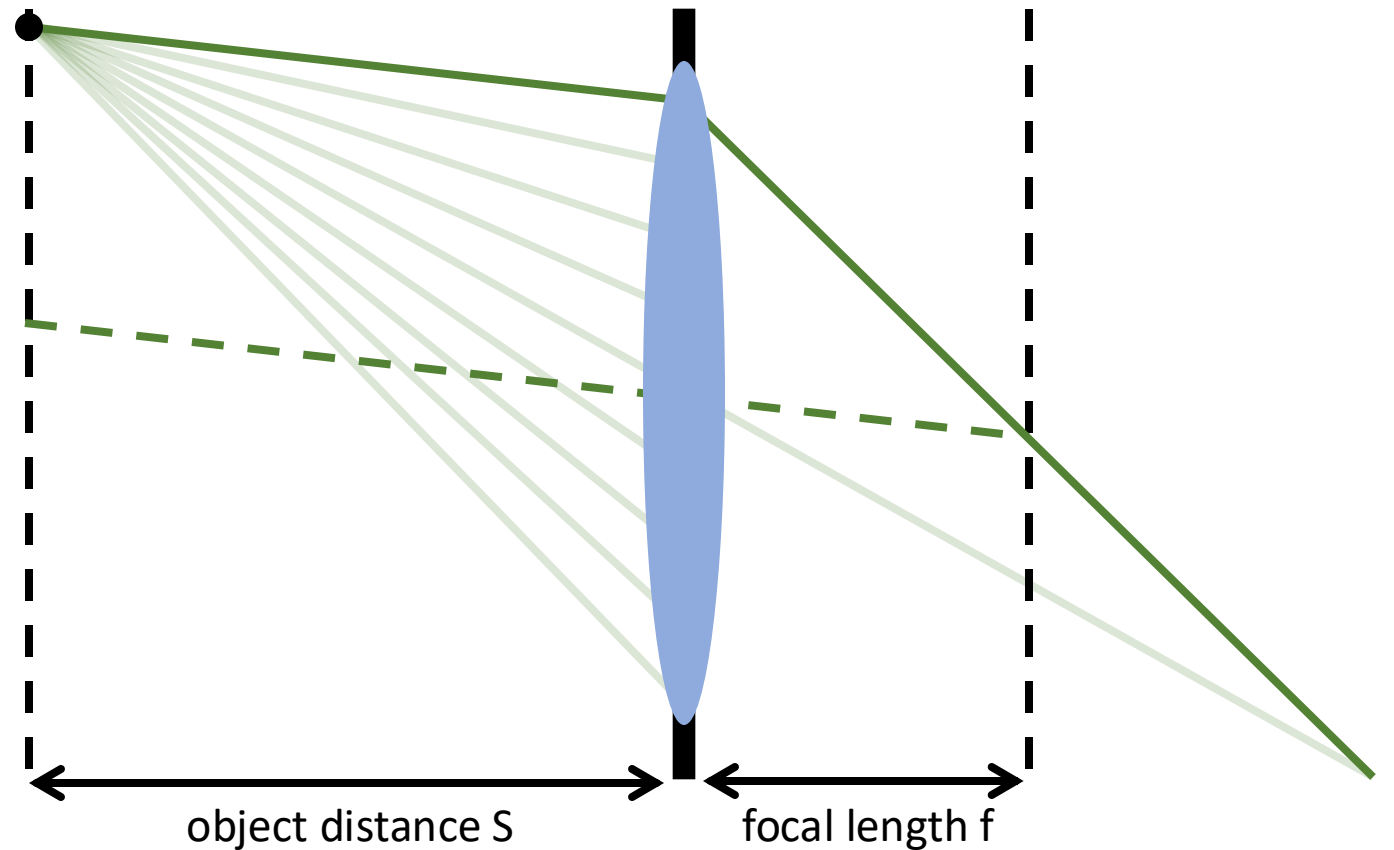
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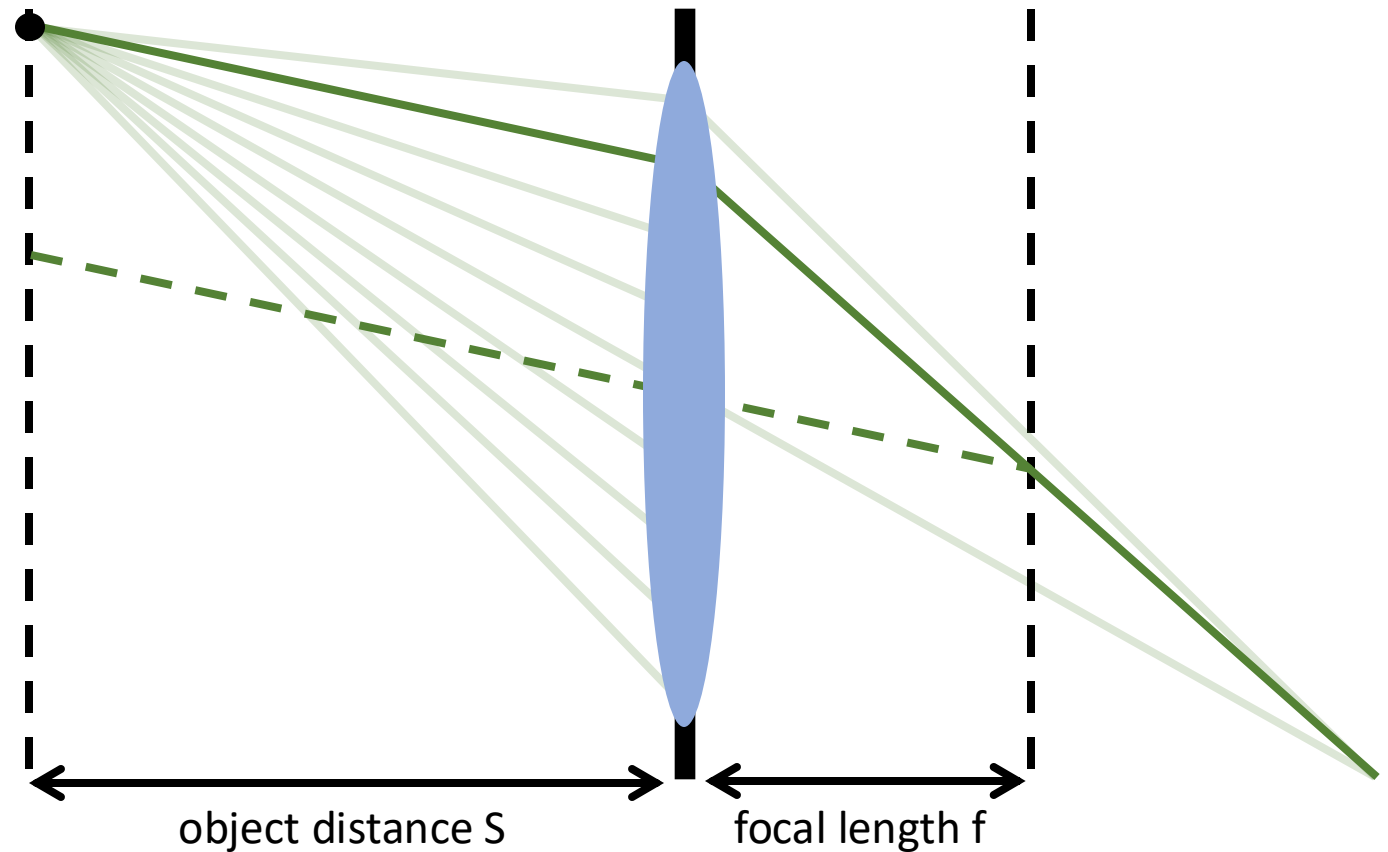
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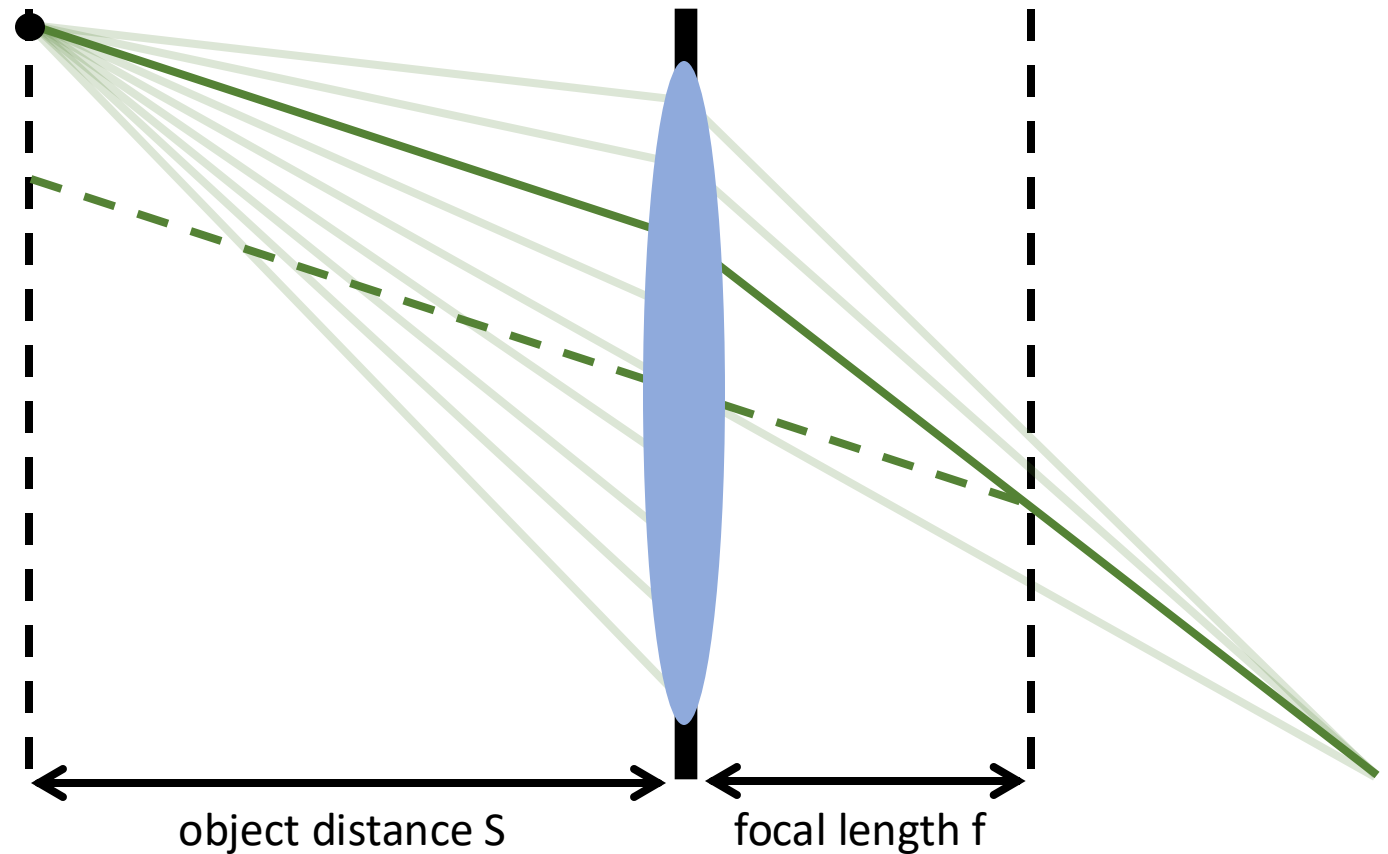
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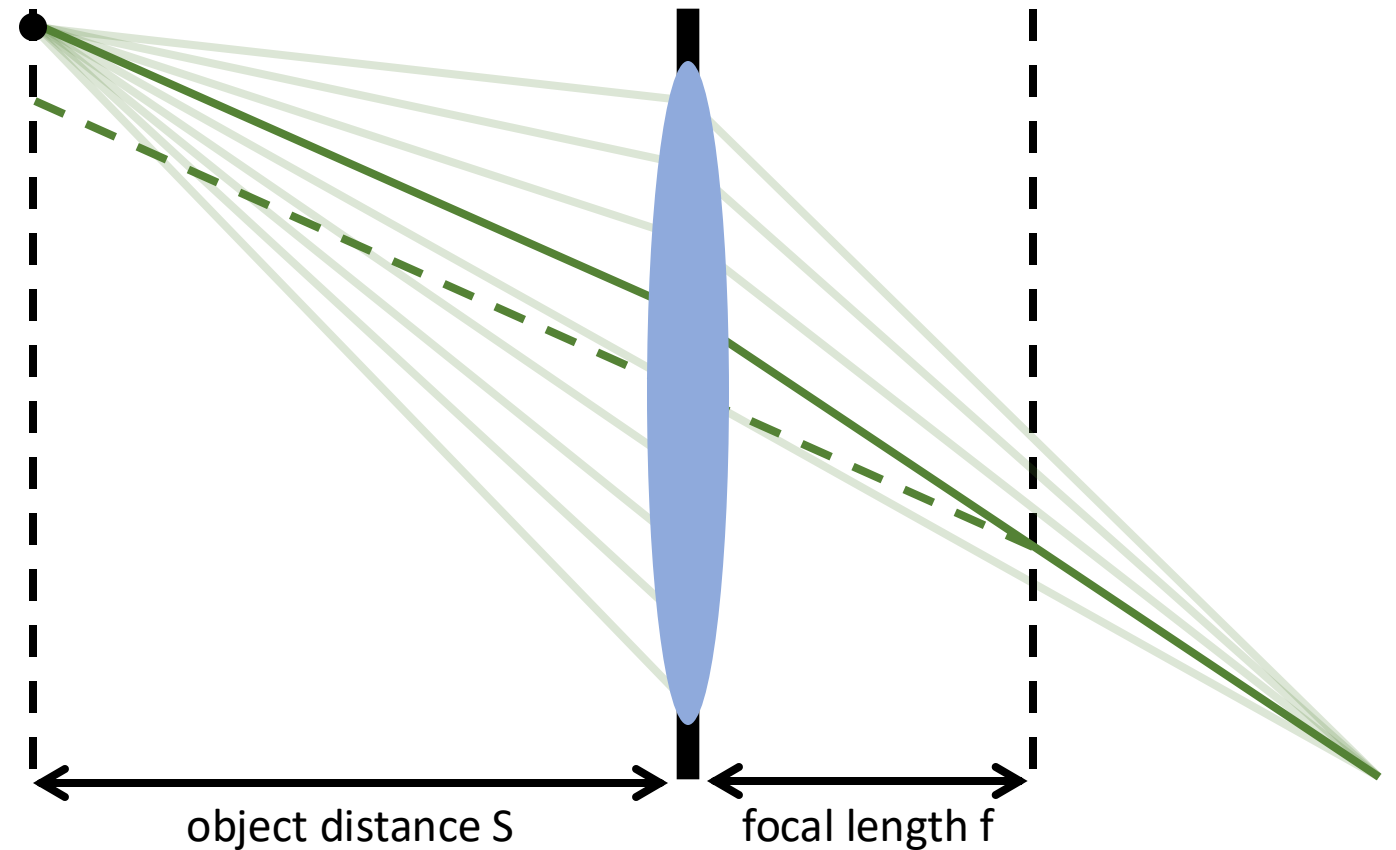
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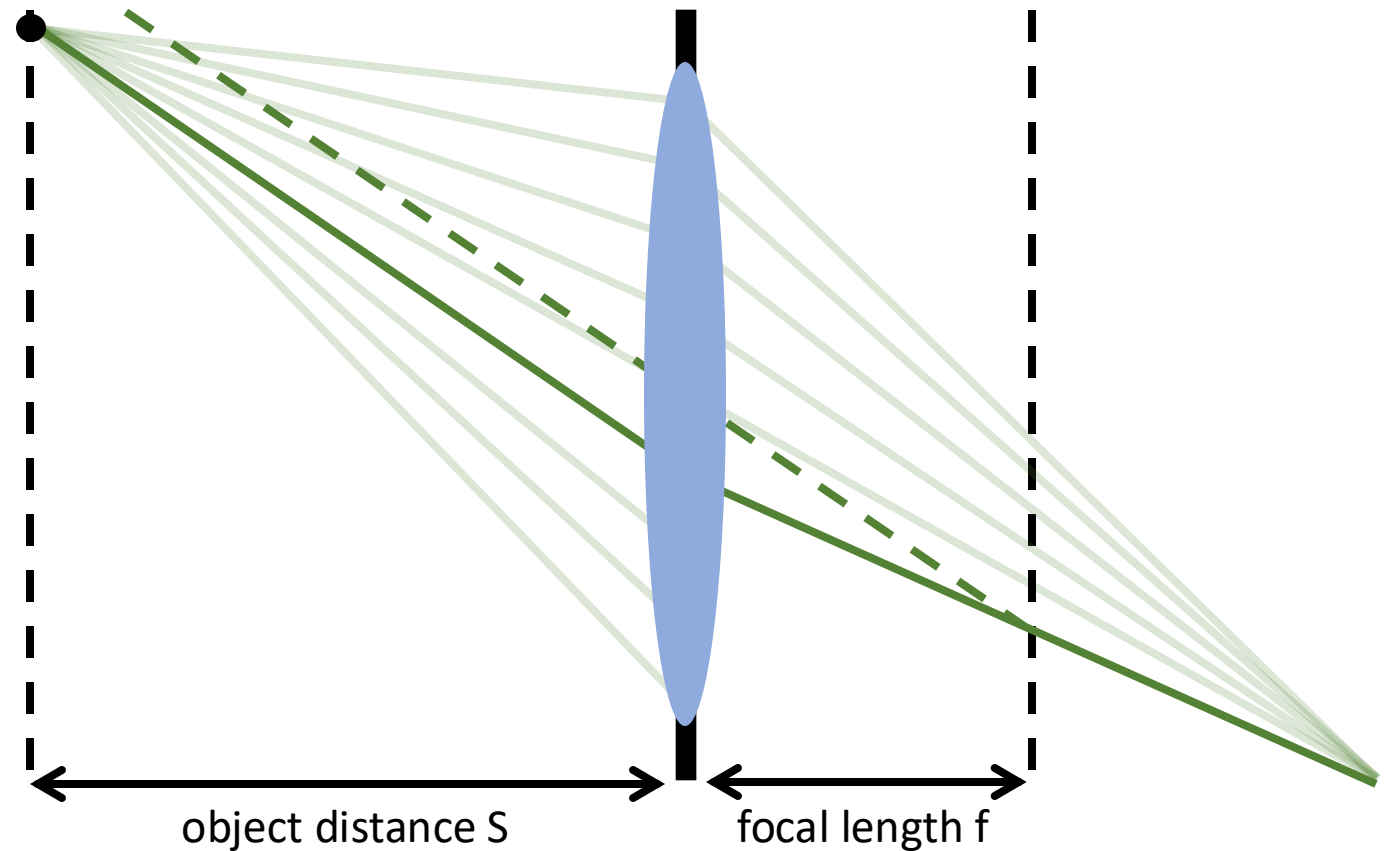
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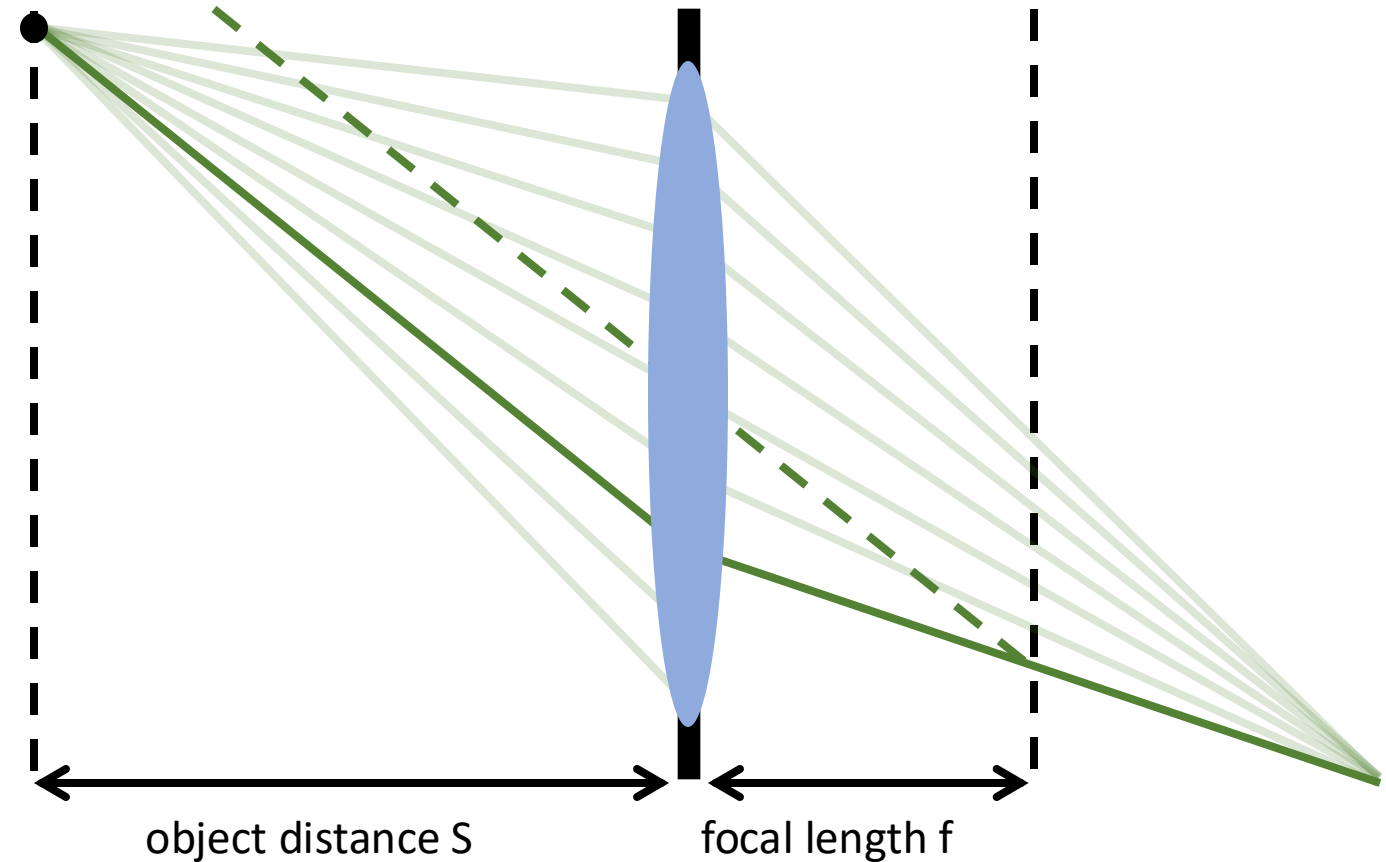
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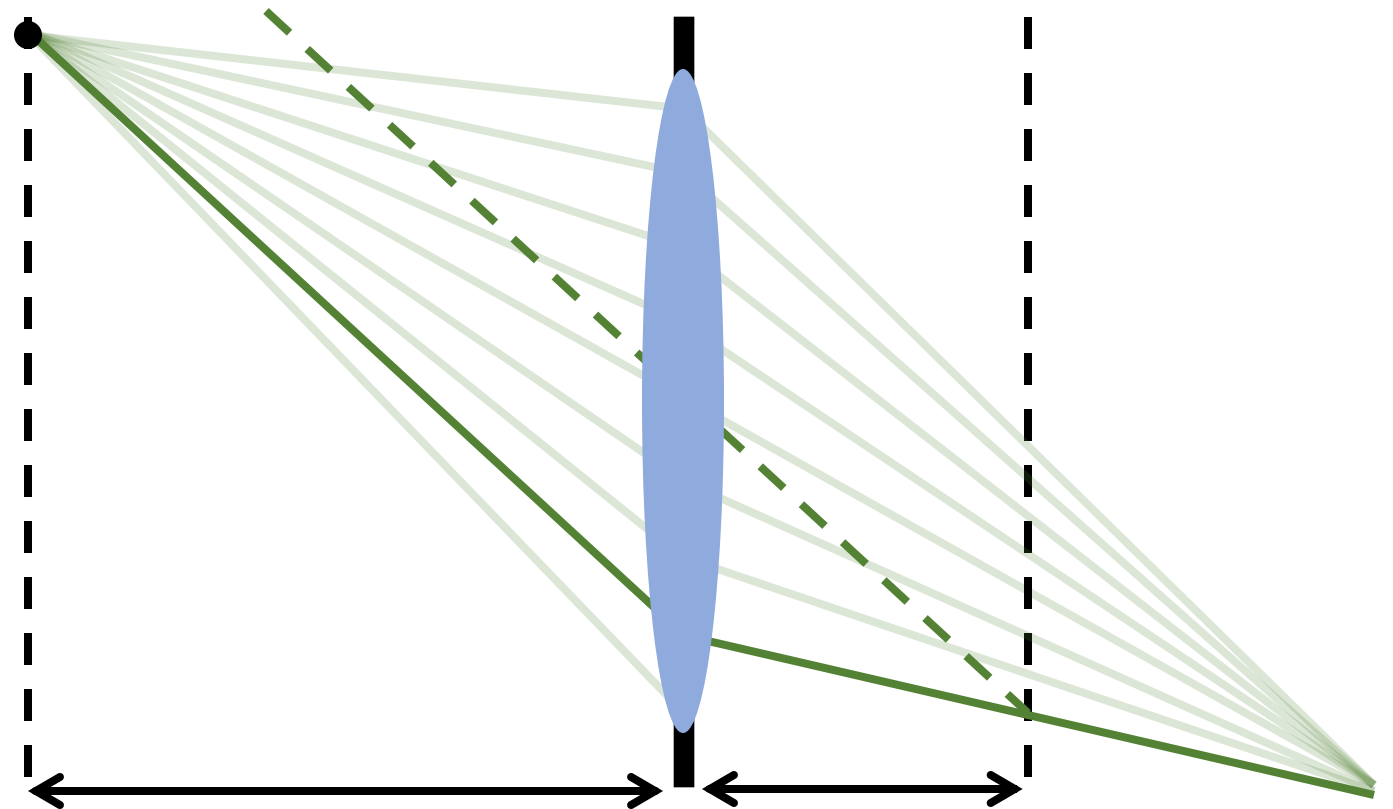
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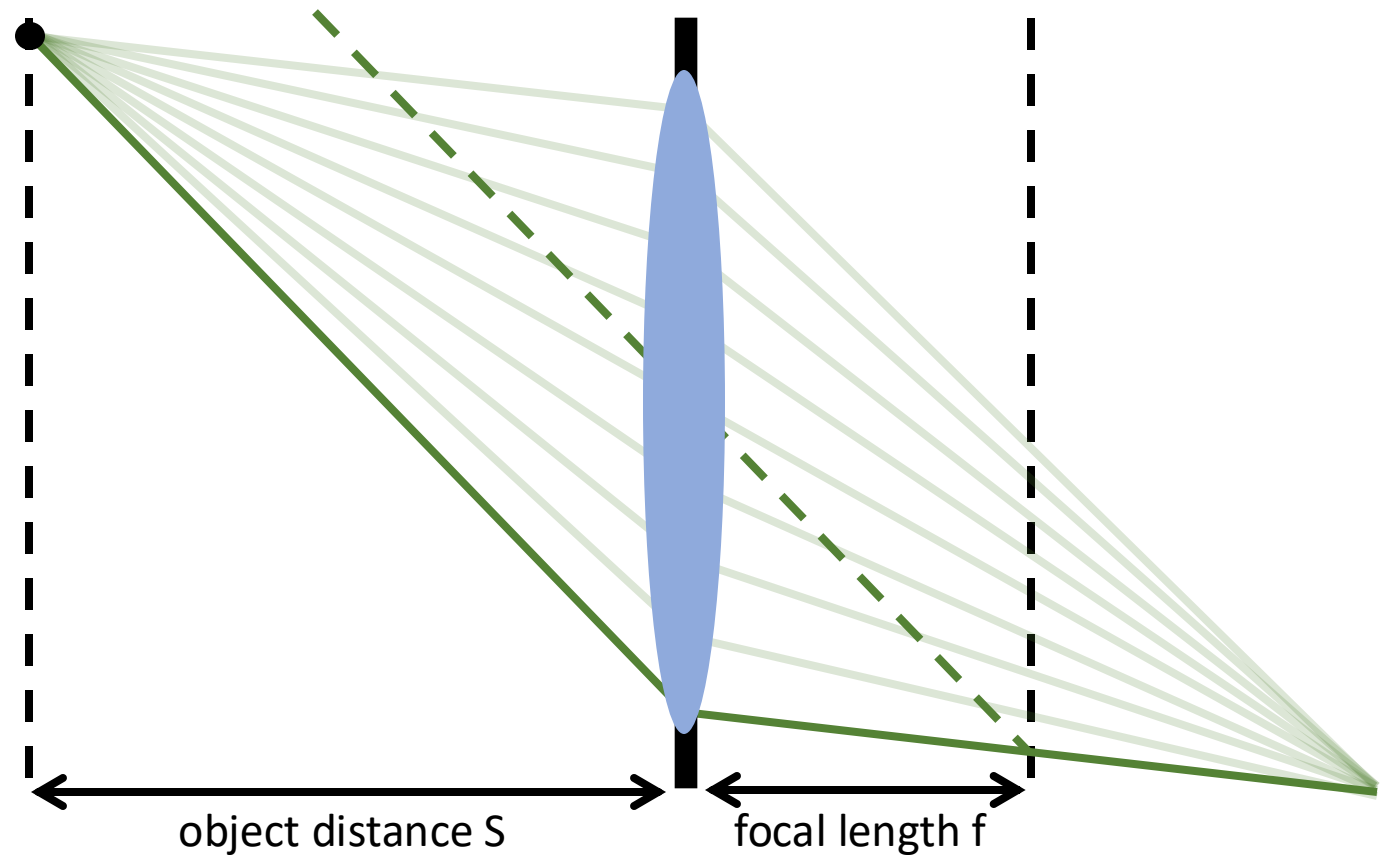
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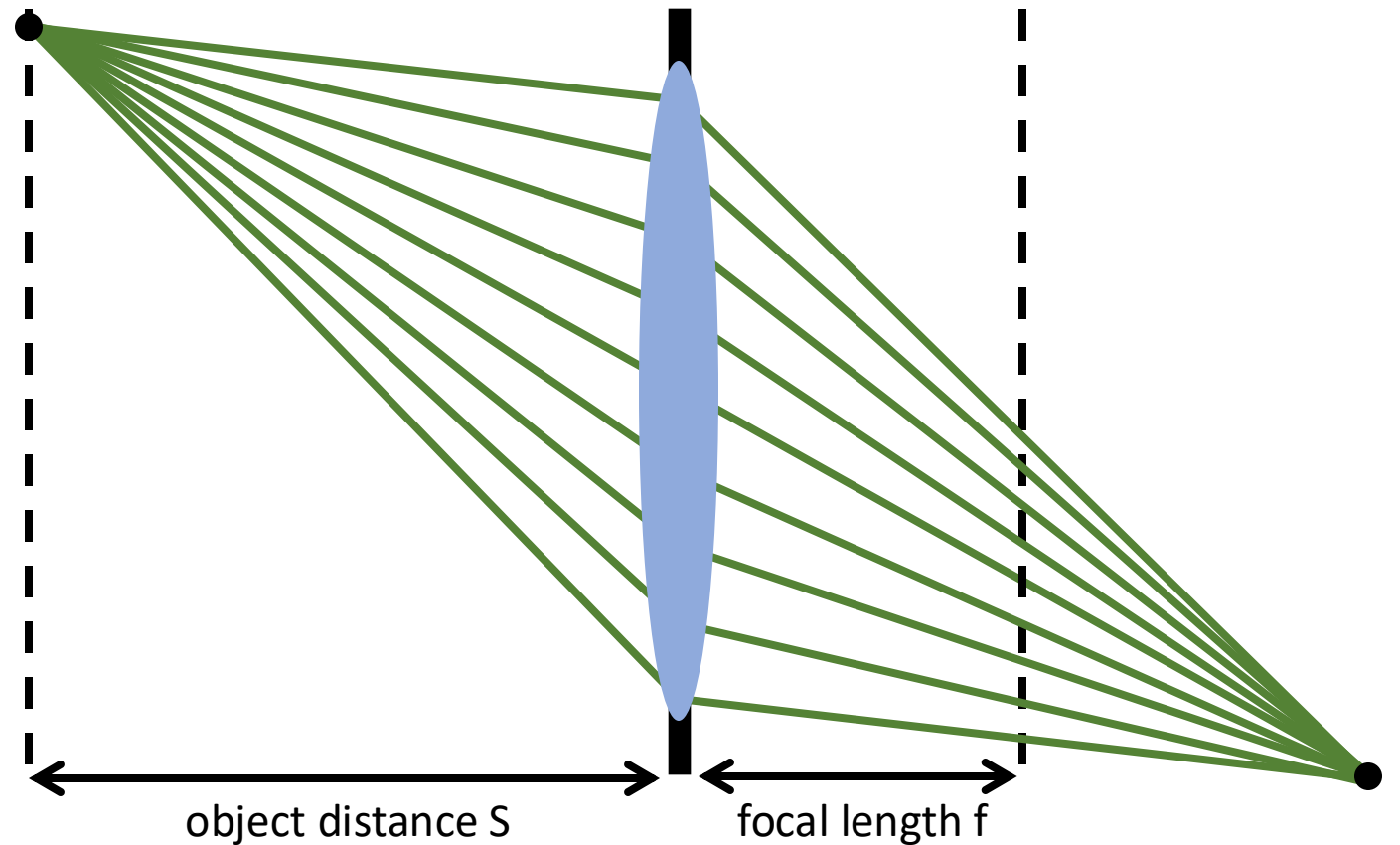
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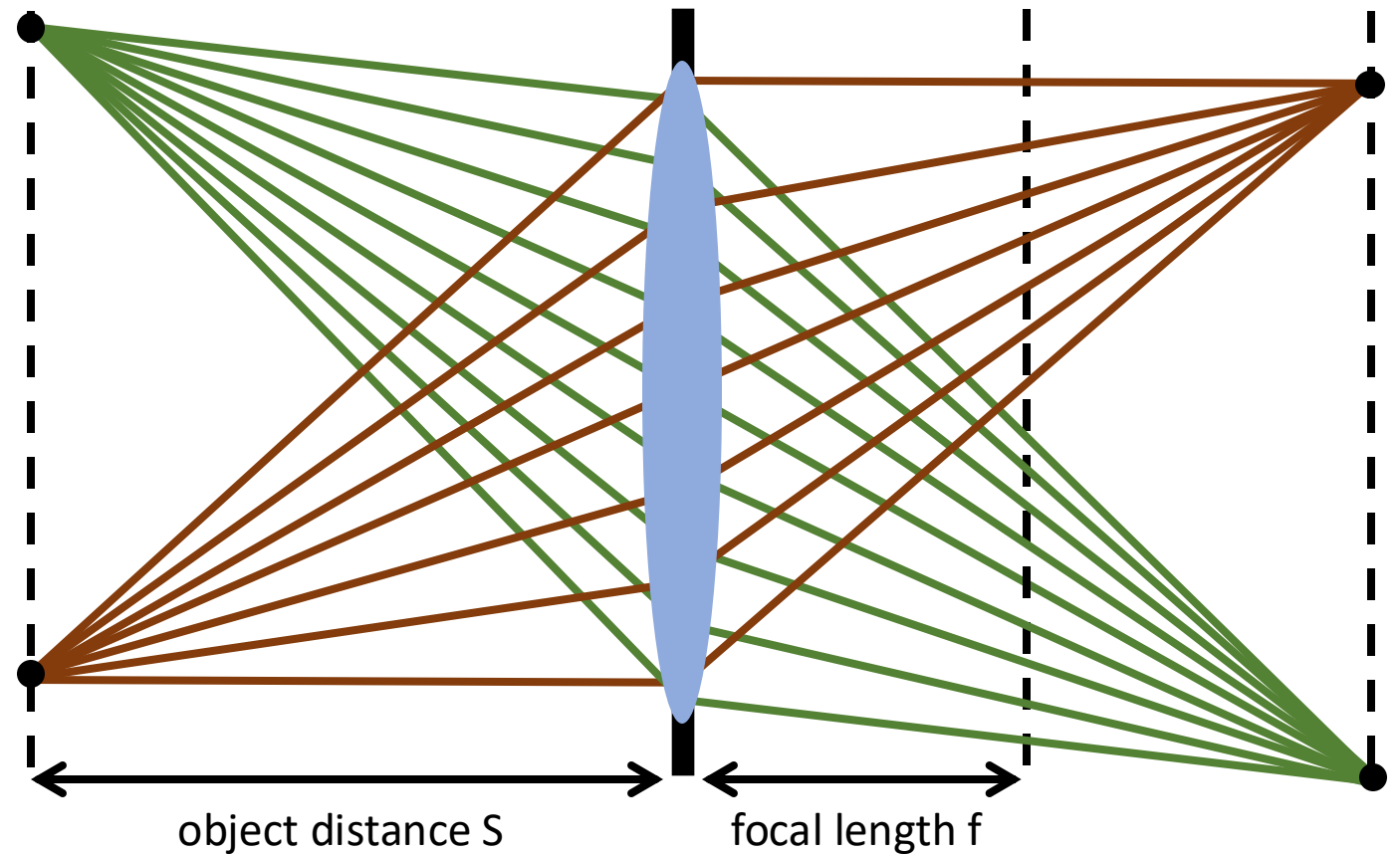
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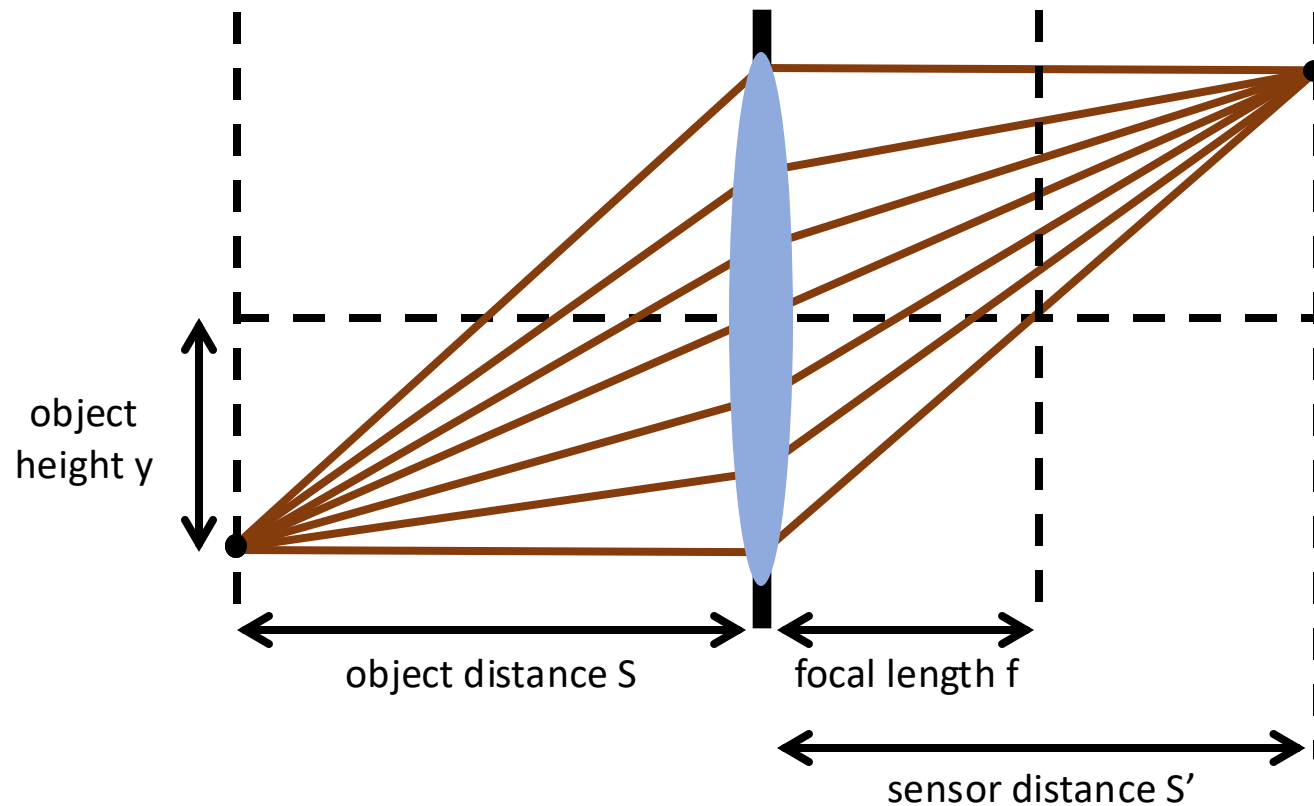
Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.



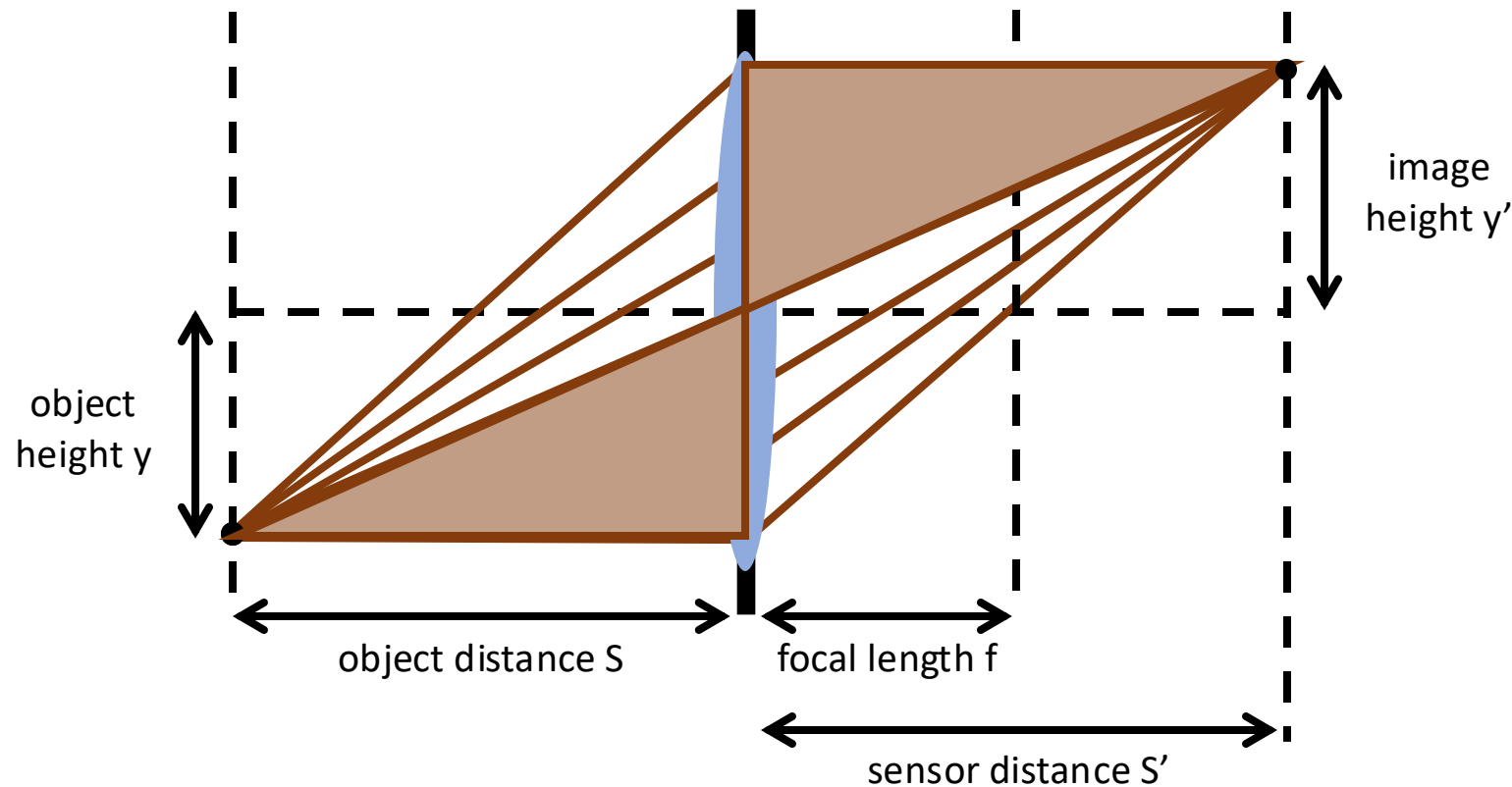
Gaussian Lens Formula

- Relationship between scene-space (S, y) and image-space (S', y') quantities



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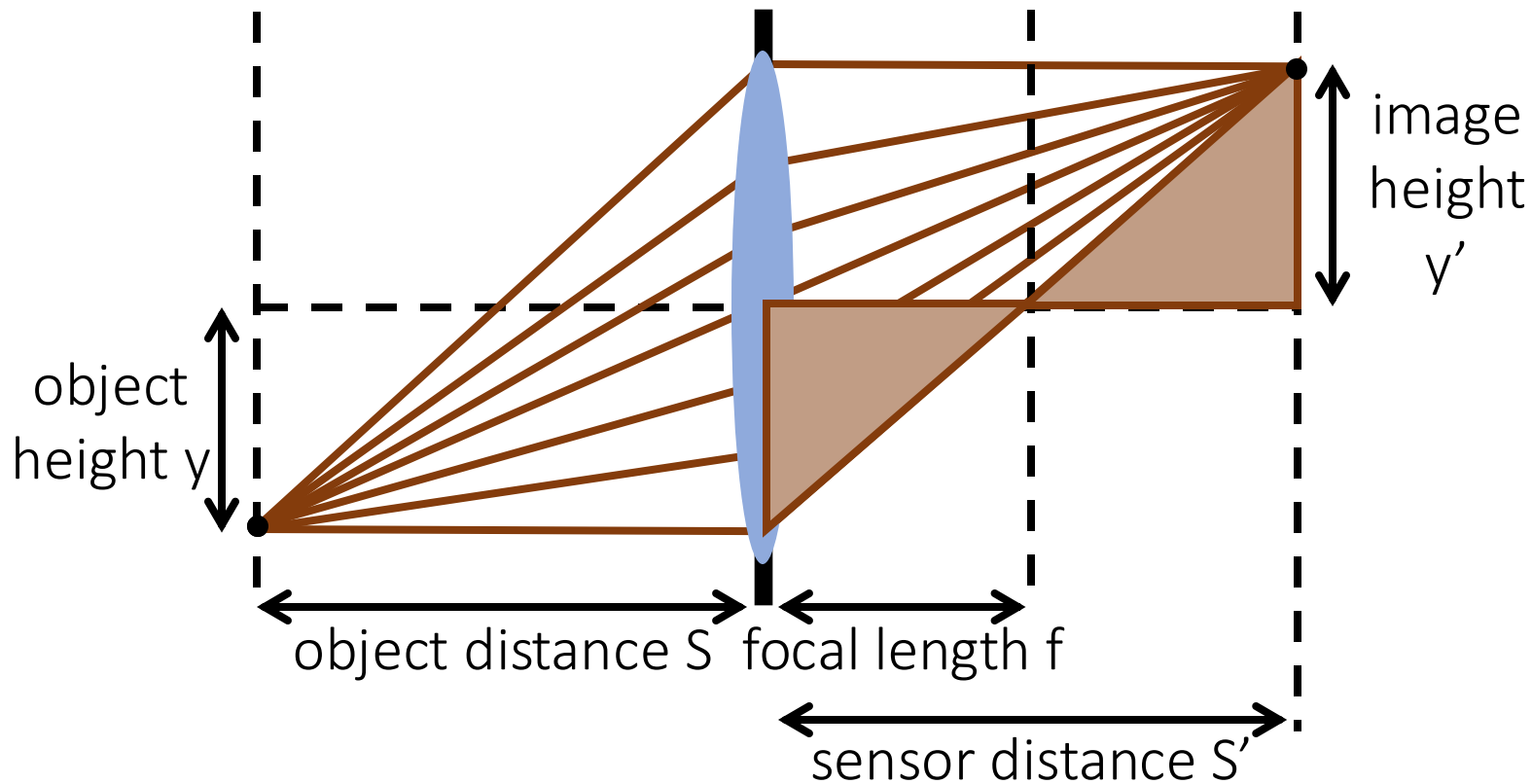


Similar triangles

$$\frac{y'}{y} = \frac{S'}{S}$$

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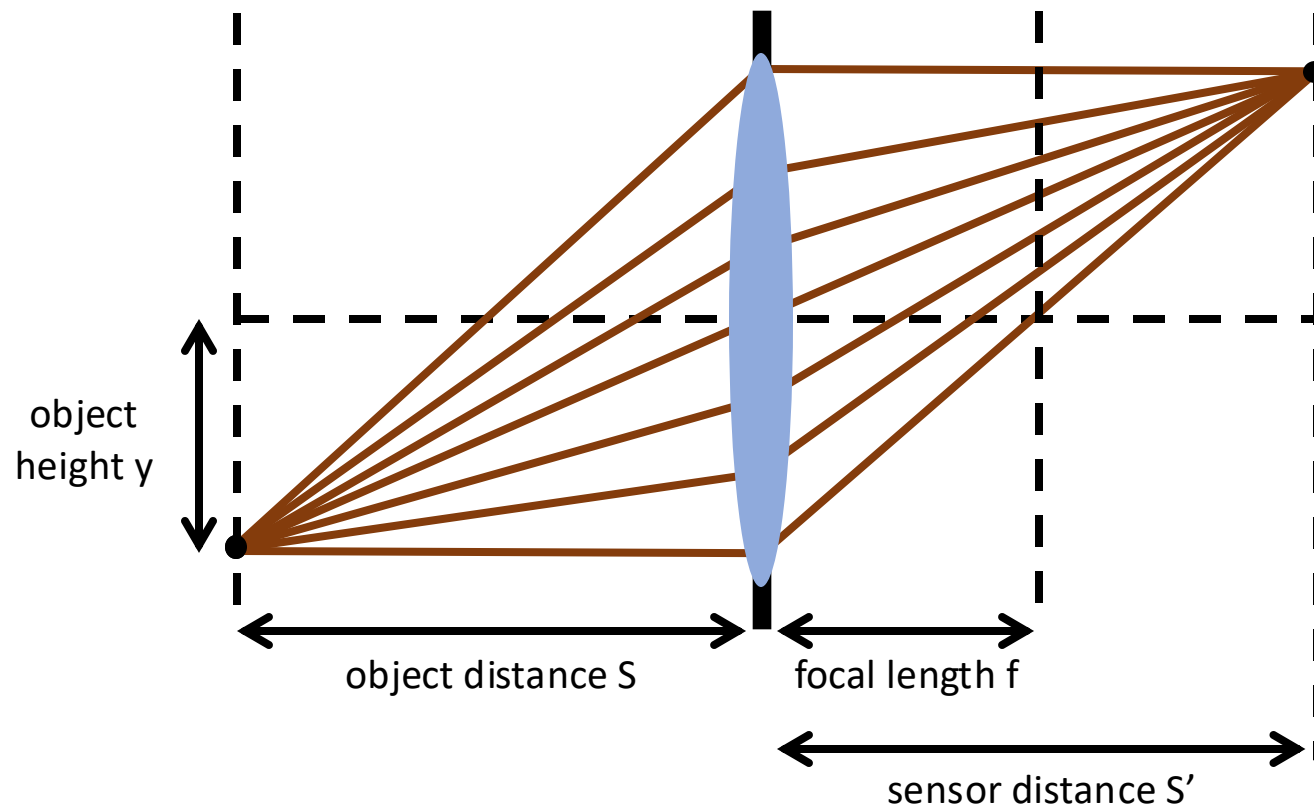


Similar triangles

$$\frac{y'}{y} = \frac{S'}{S}$$
$$\frac{y'}{y} = \frac{S' - f}{f}$$

Gaussian Lens Formula

- Relationship between scene-space (S, y) and image-space (S', y') quantities



$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{S' - f}{f}$$
$$m = \frac{f}{S - f}$$
$$m = \frac{y'}{y}$$

magnification

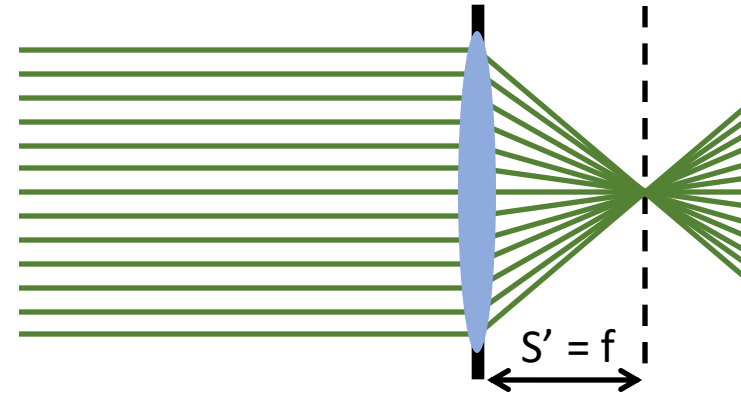
Special Focus Distances

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{f}{S - f}$$

$$S' = f, S = \infty$$

Special Focus Distances

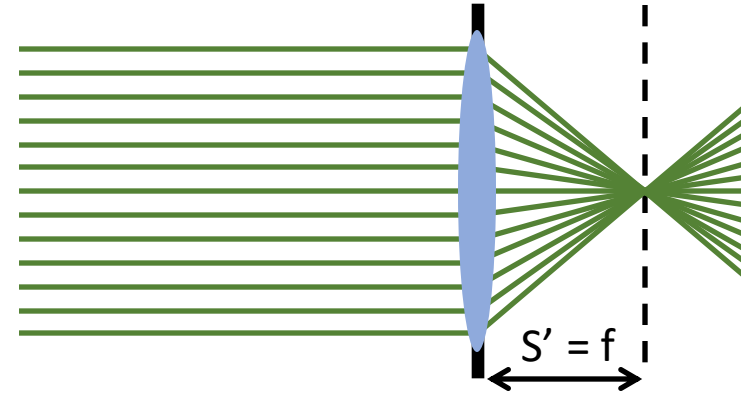
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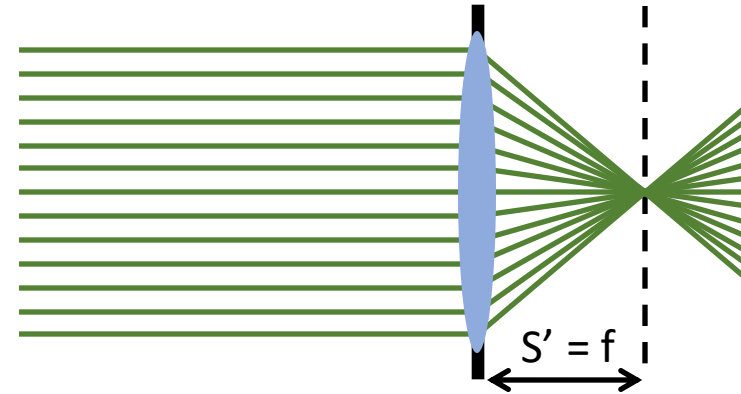


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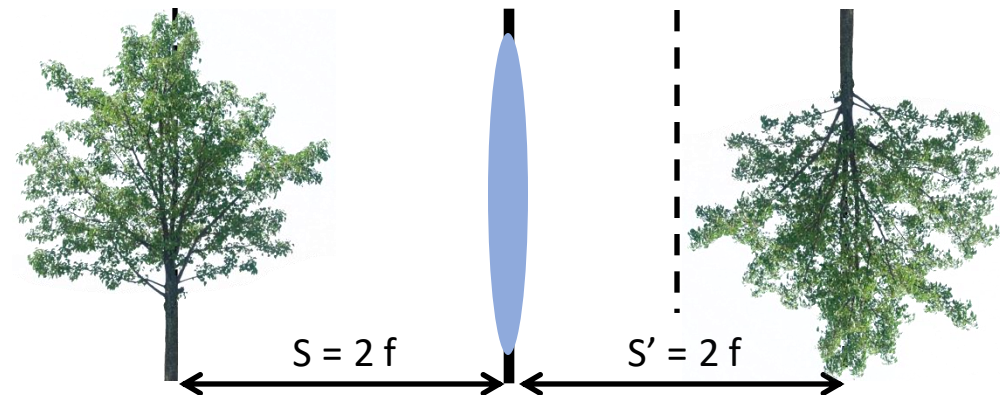
$$S' = S = 2f$$

Special Focus Distances

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$$m = \frac{f}{S - f}$$



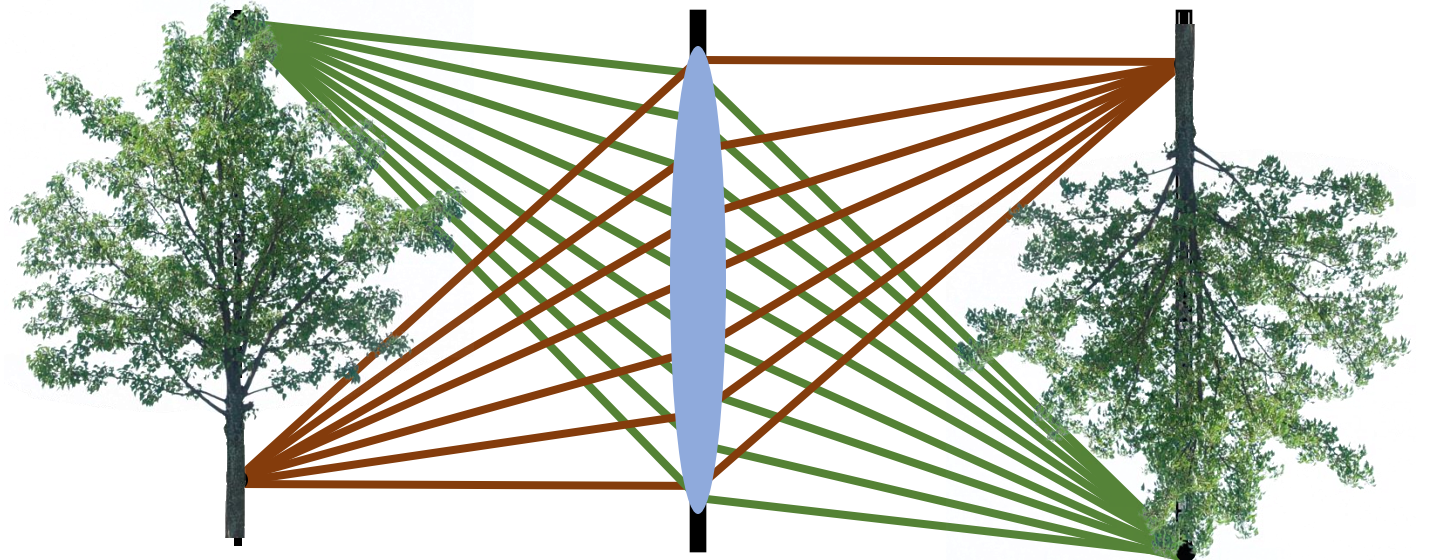
$S' = f, S = \infty, m = 0 \rightarrow$ infinity focus (parallel rays)



$S' = S = 2f, m = 1 \rightarrow$ object is reproduced in real-life size

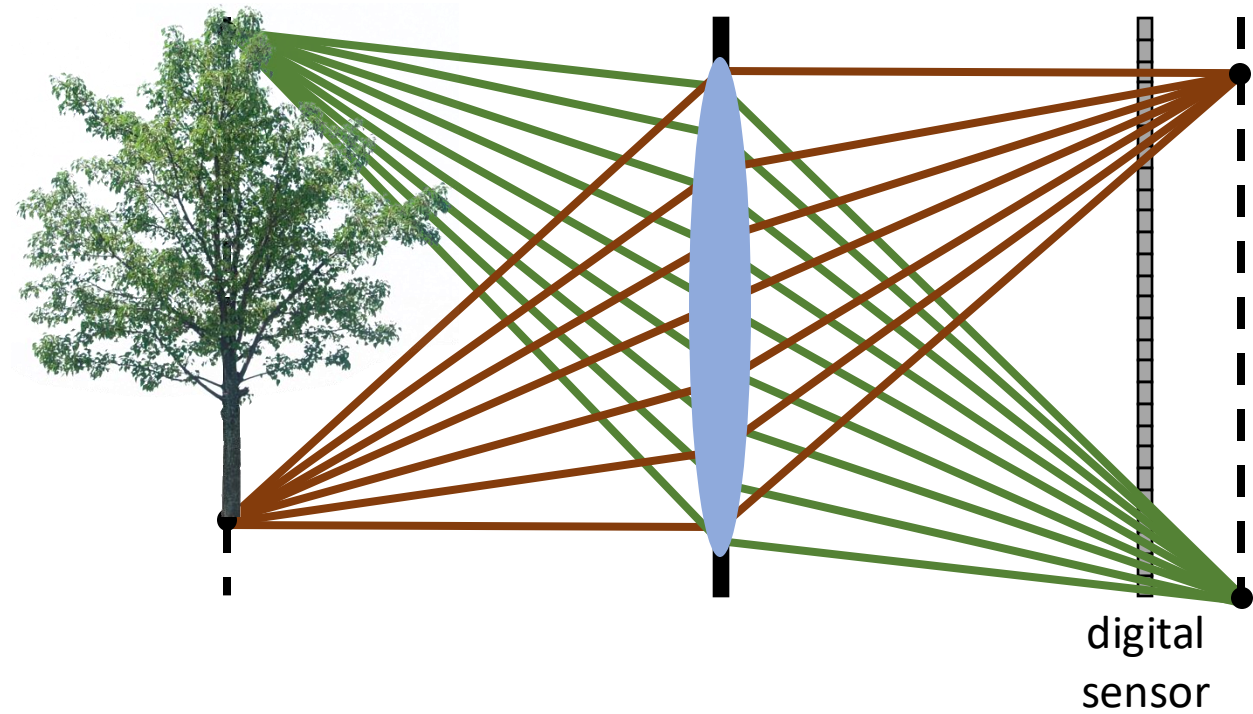
Free Lunch?

- By using a lens, we simultaneously achieve:
 - Sharp image; and
 - High signal-to-noise ratio
- Is there any downside?



Defocus

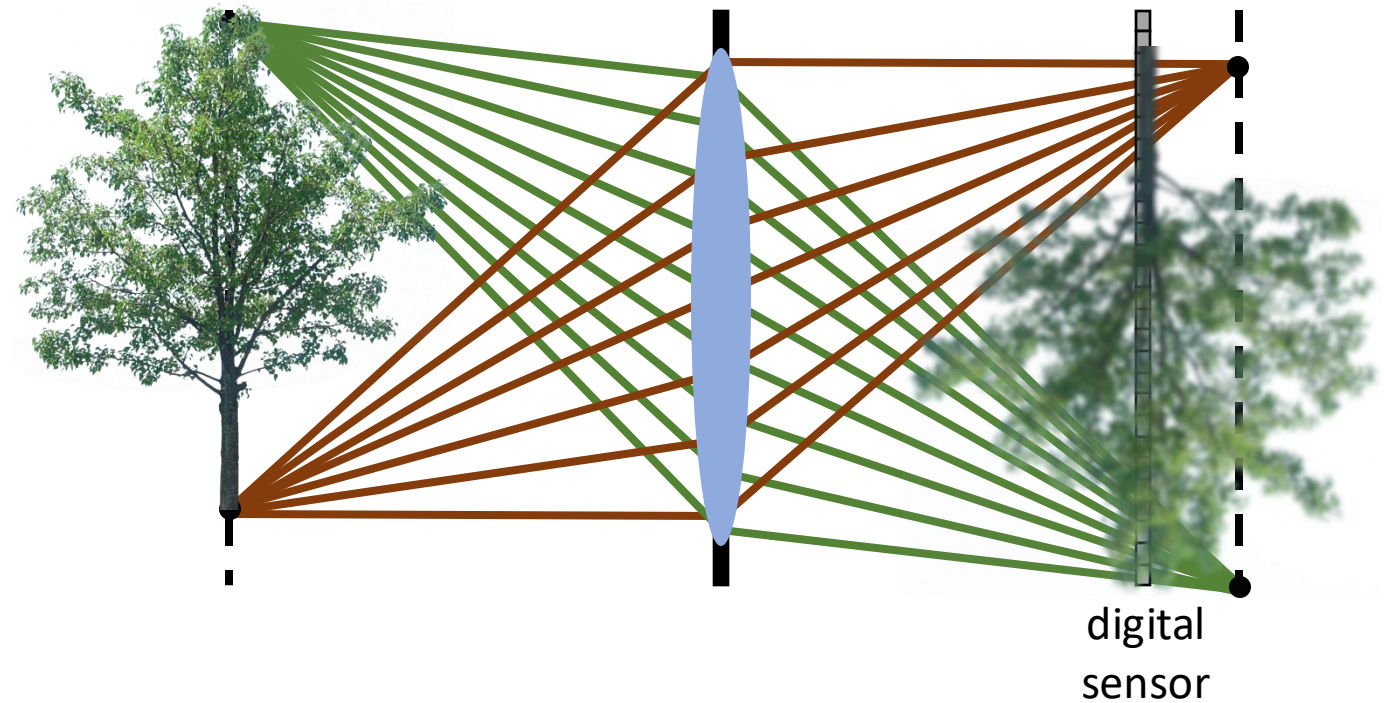
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Defocus

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We get a blurry image.

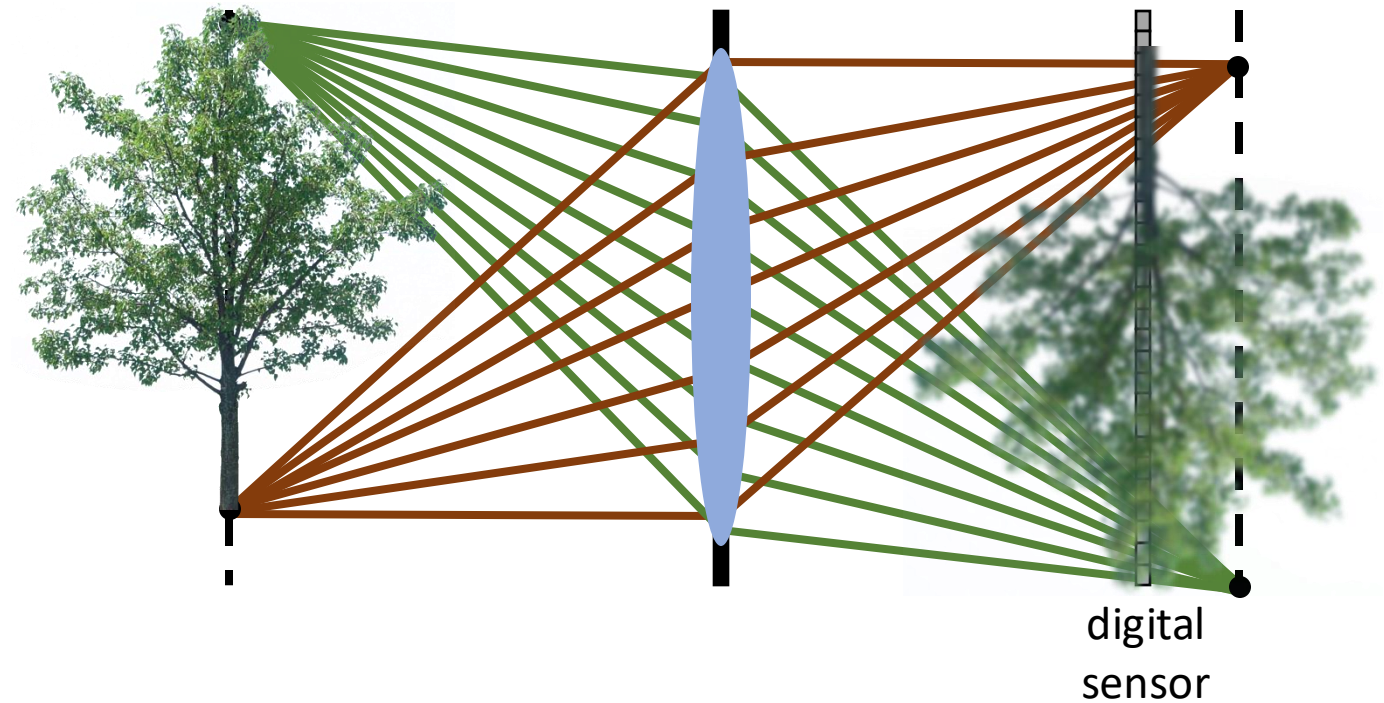


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This is called **defocus**, which never happens in an ideal pinhole camera.



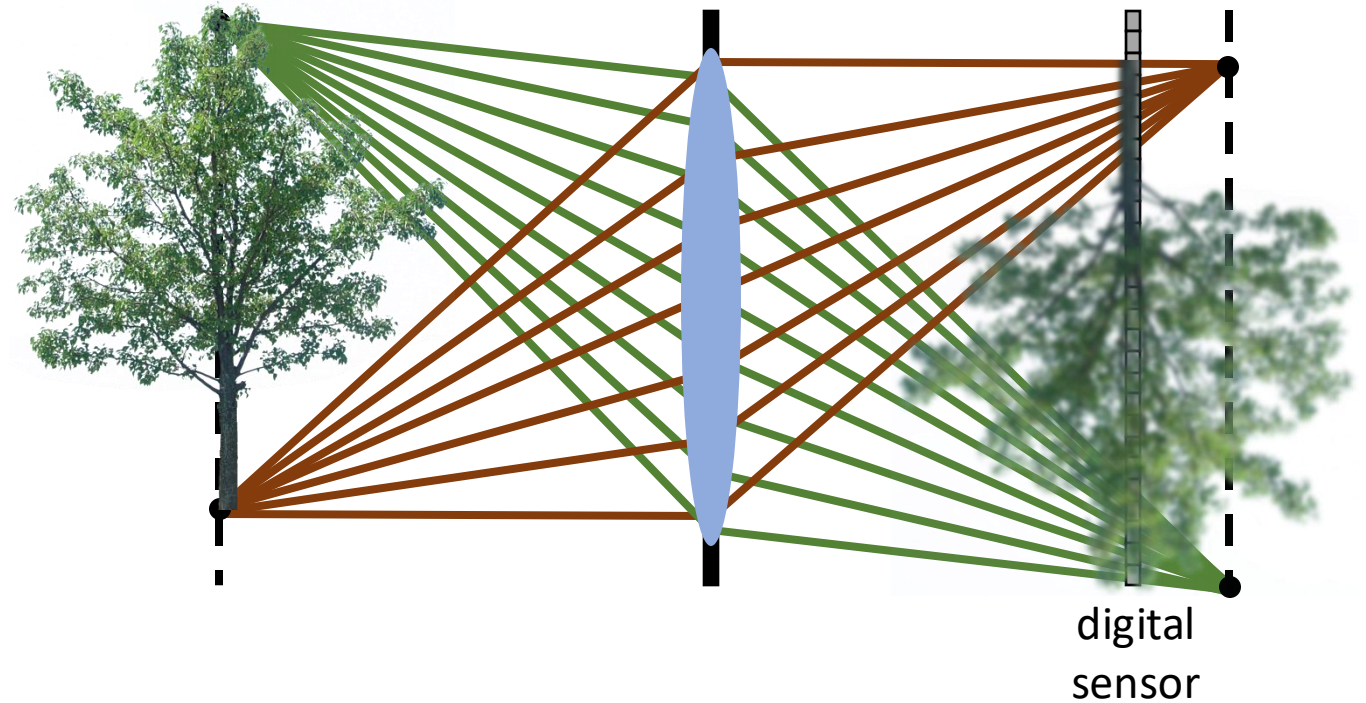
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How do we fix it? Can't we just move the sensor to the correct distance?



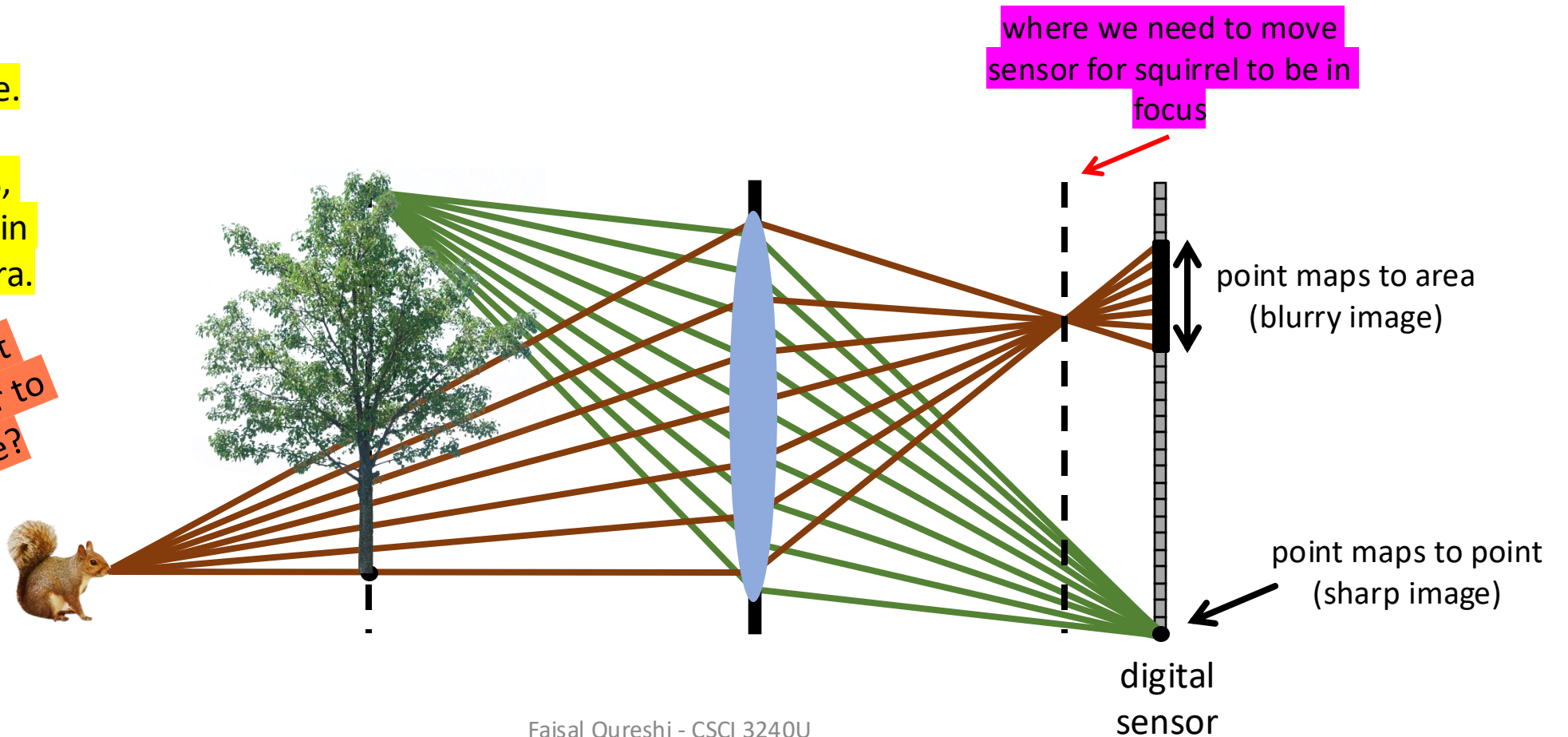
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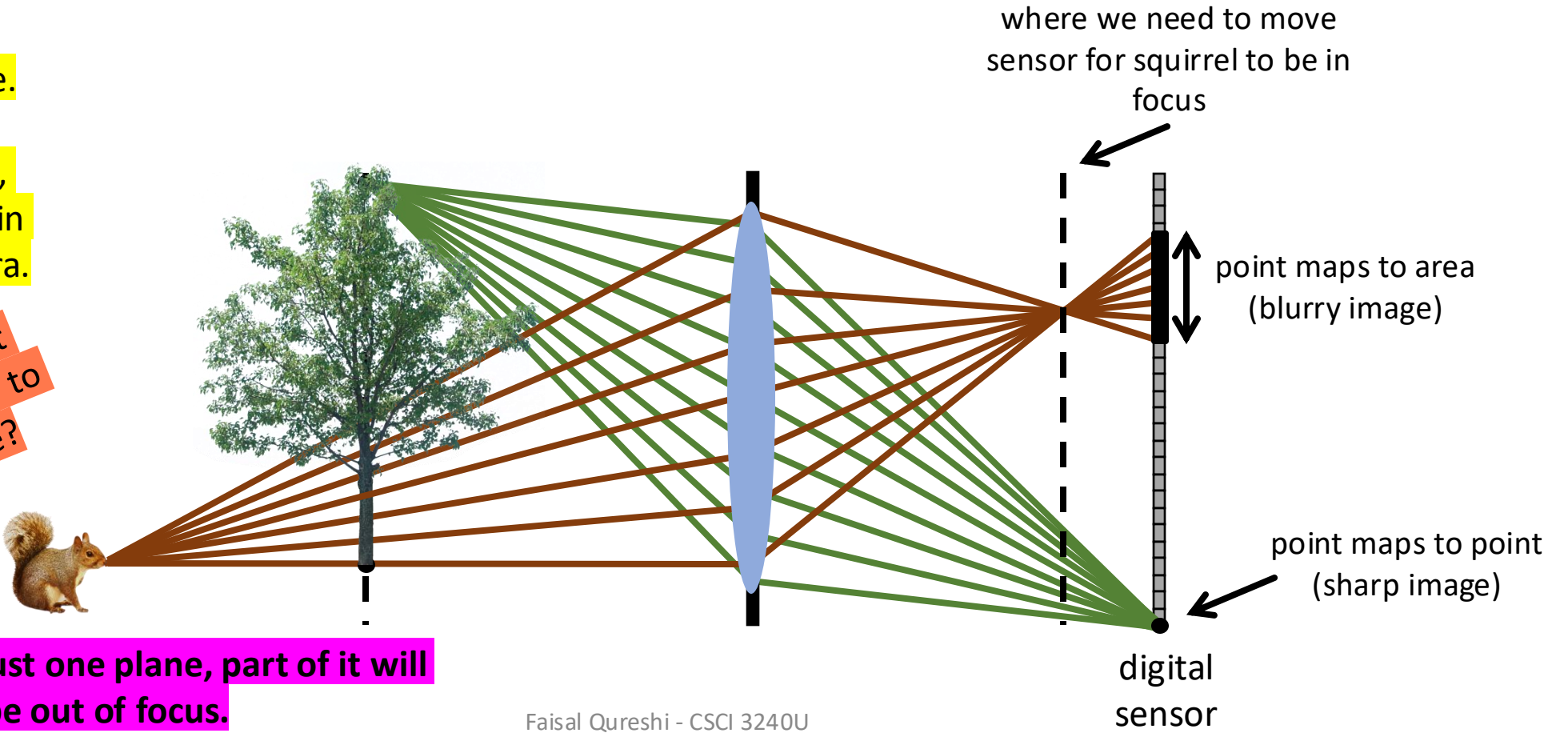
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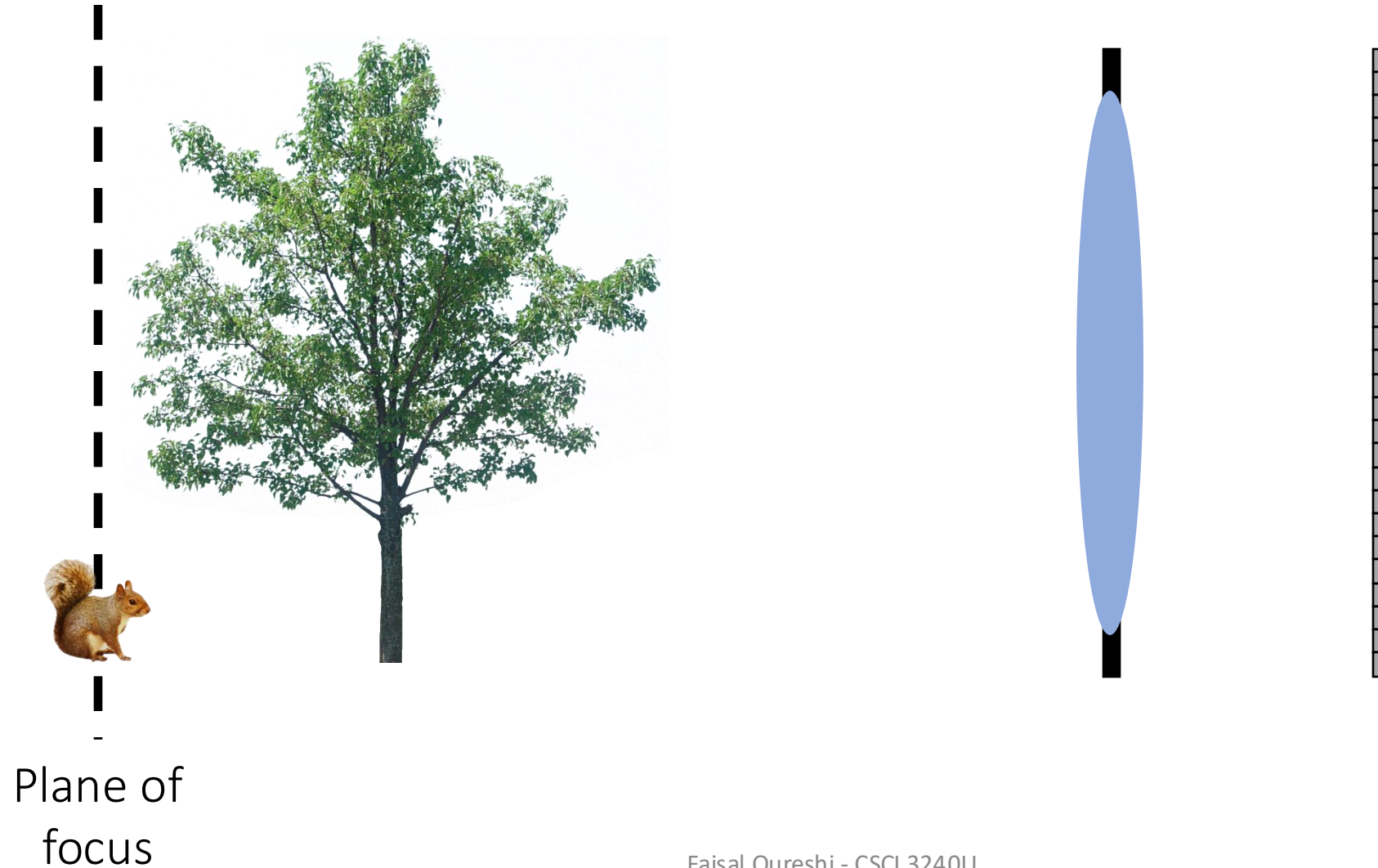
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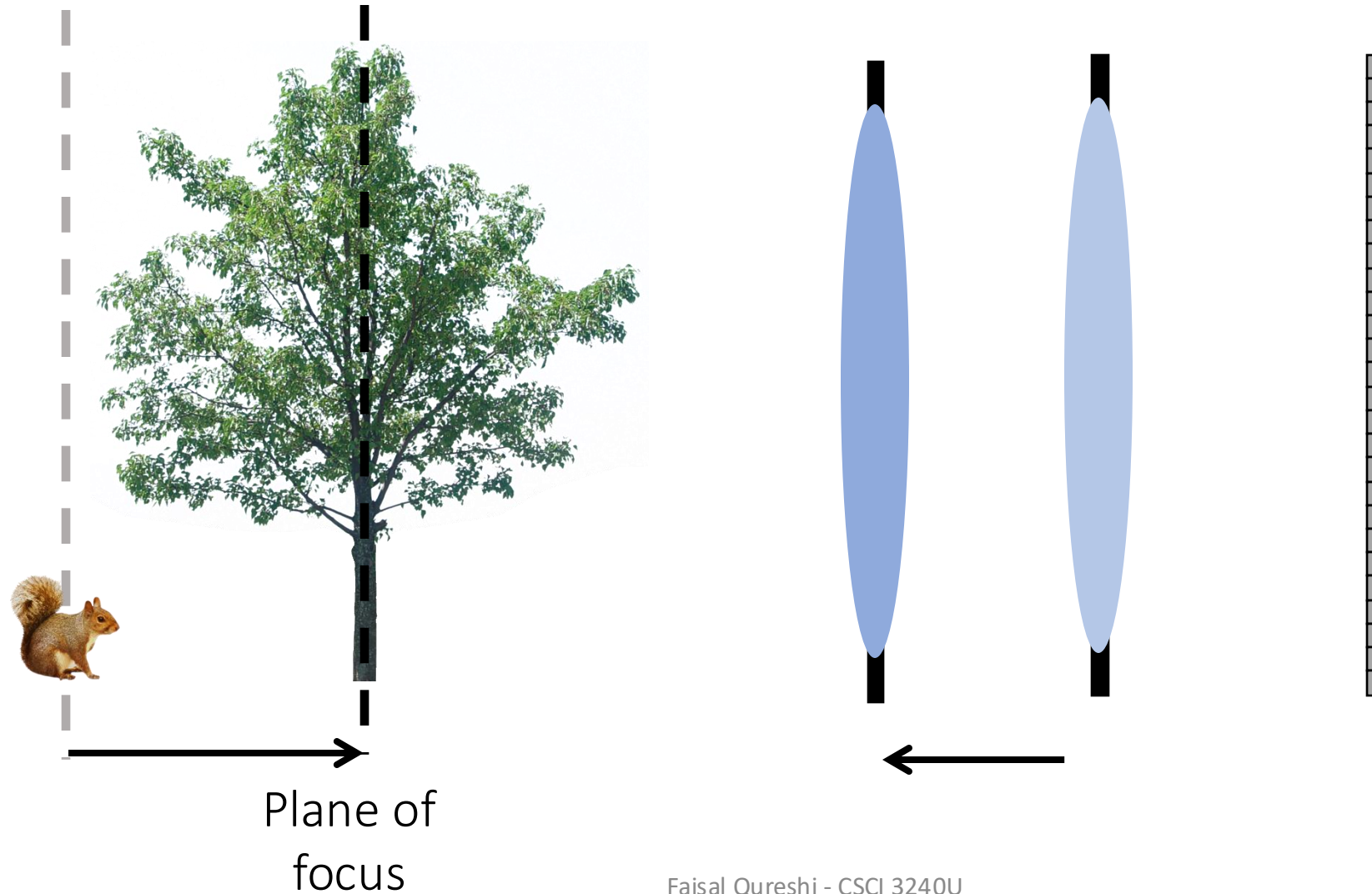


Unless our scene is just one plane, part of it will always be out of focus.

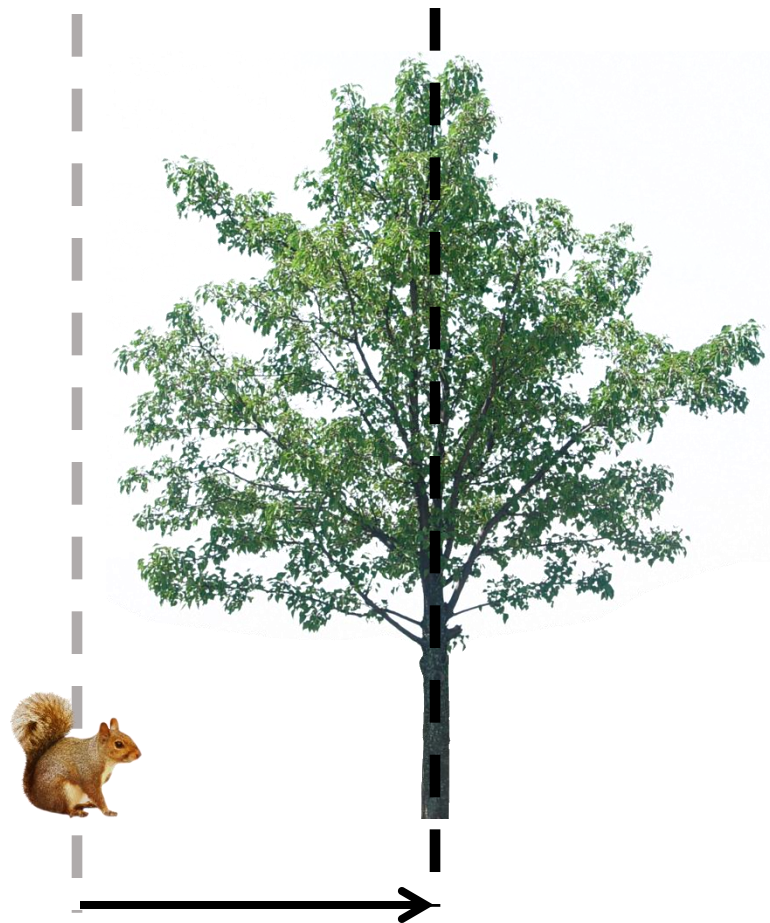
How Do We Control What is in Focus?



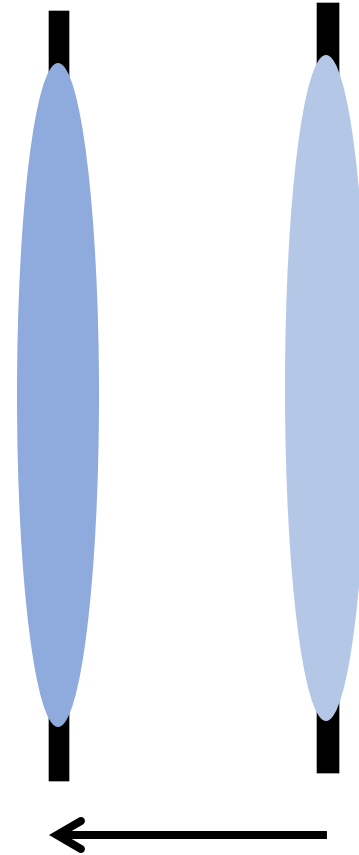
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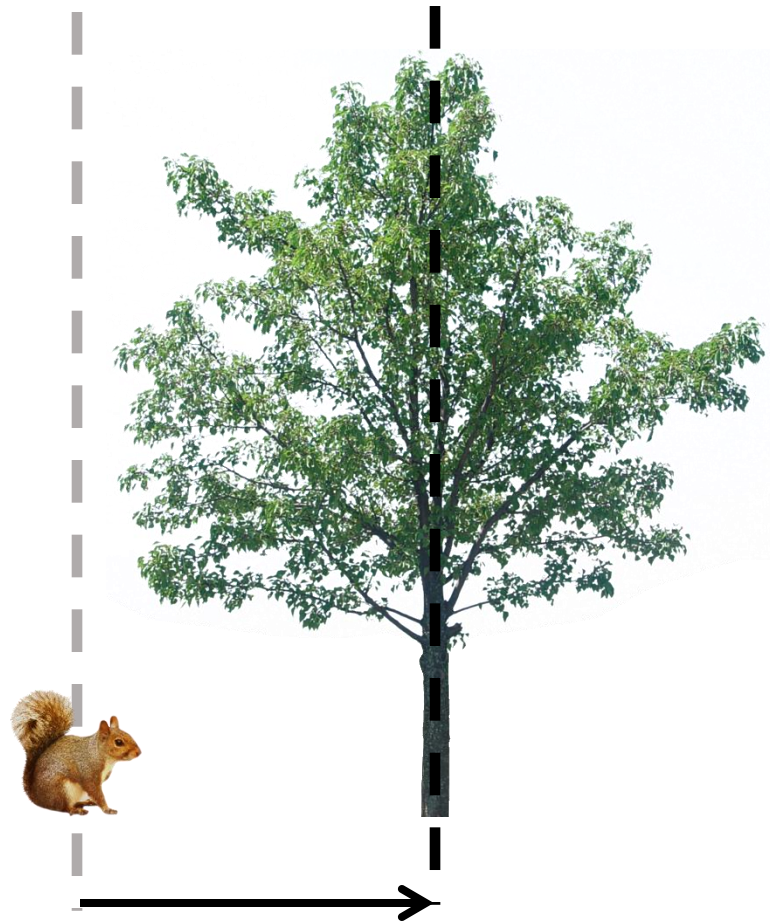


Plane of
focus

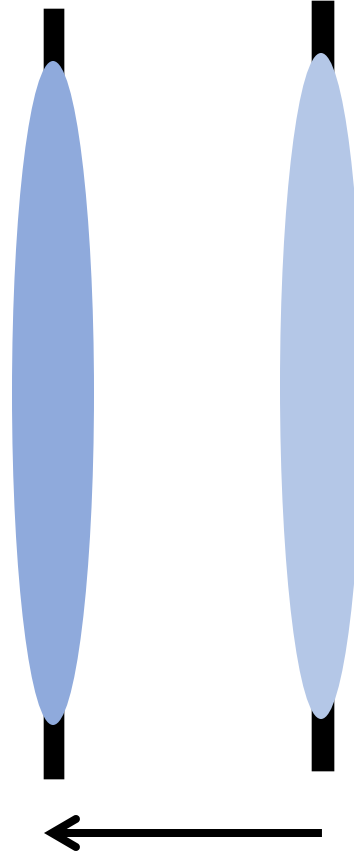


As lens moves away
from the sensor the
plane of focus moves
closer

How Do We Control What is in Focus?



Plane of
focus



We change the distance
between the lens and
the sensor



As lens moves away
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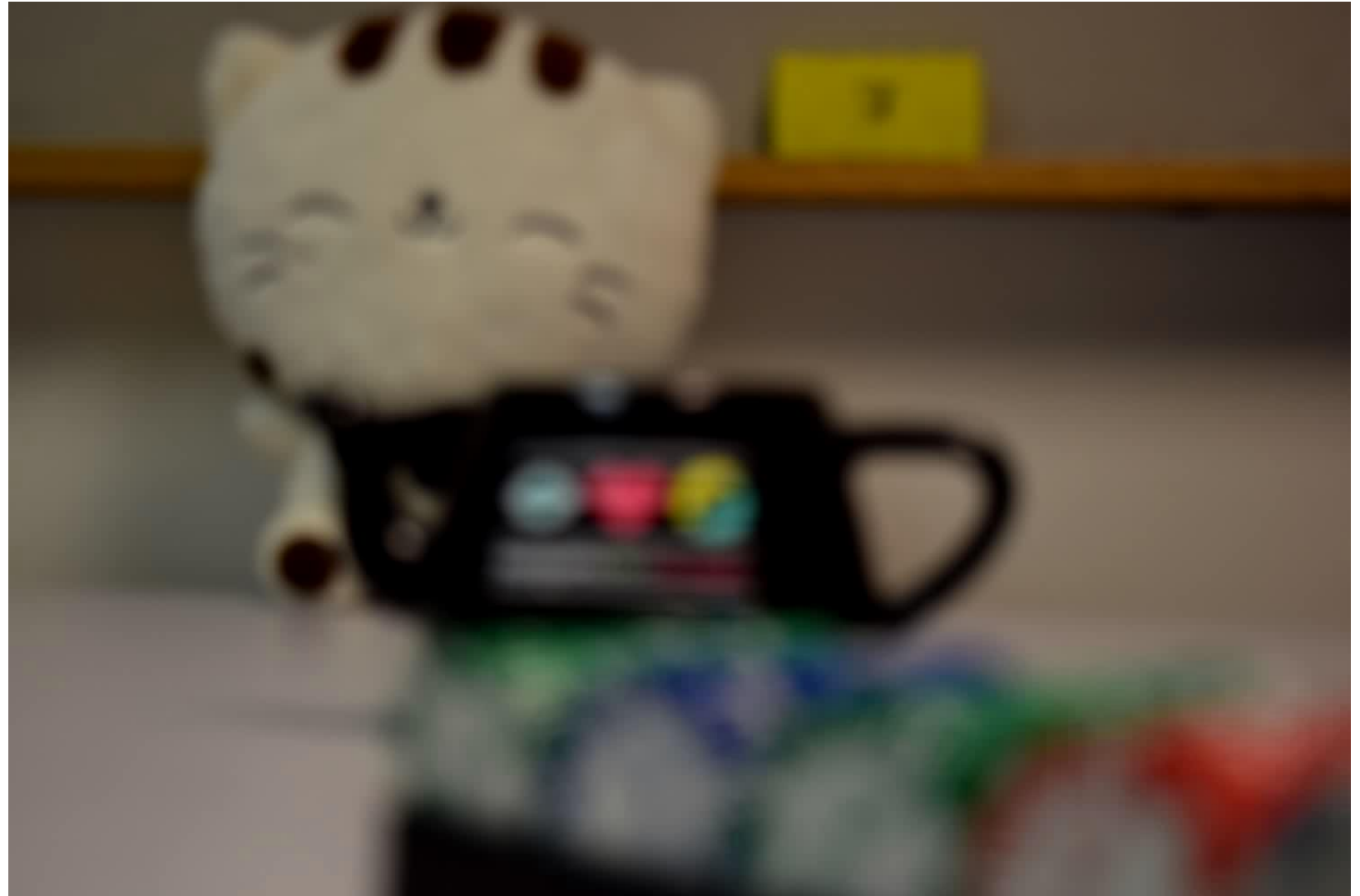
How Do We Control What is in Focus?

Focus ring: controls distance of lens from sensor



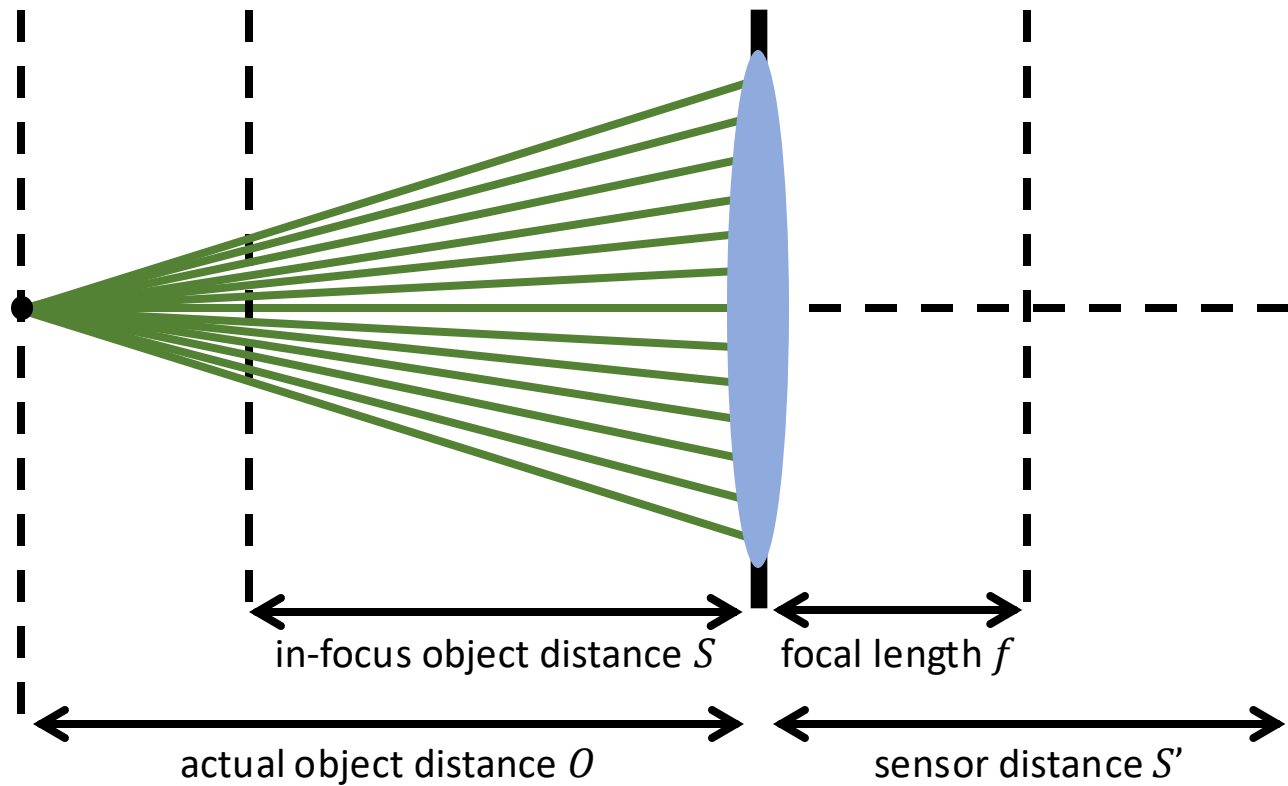
Changing Focus

How do we deal
with non-planar
scenes?



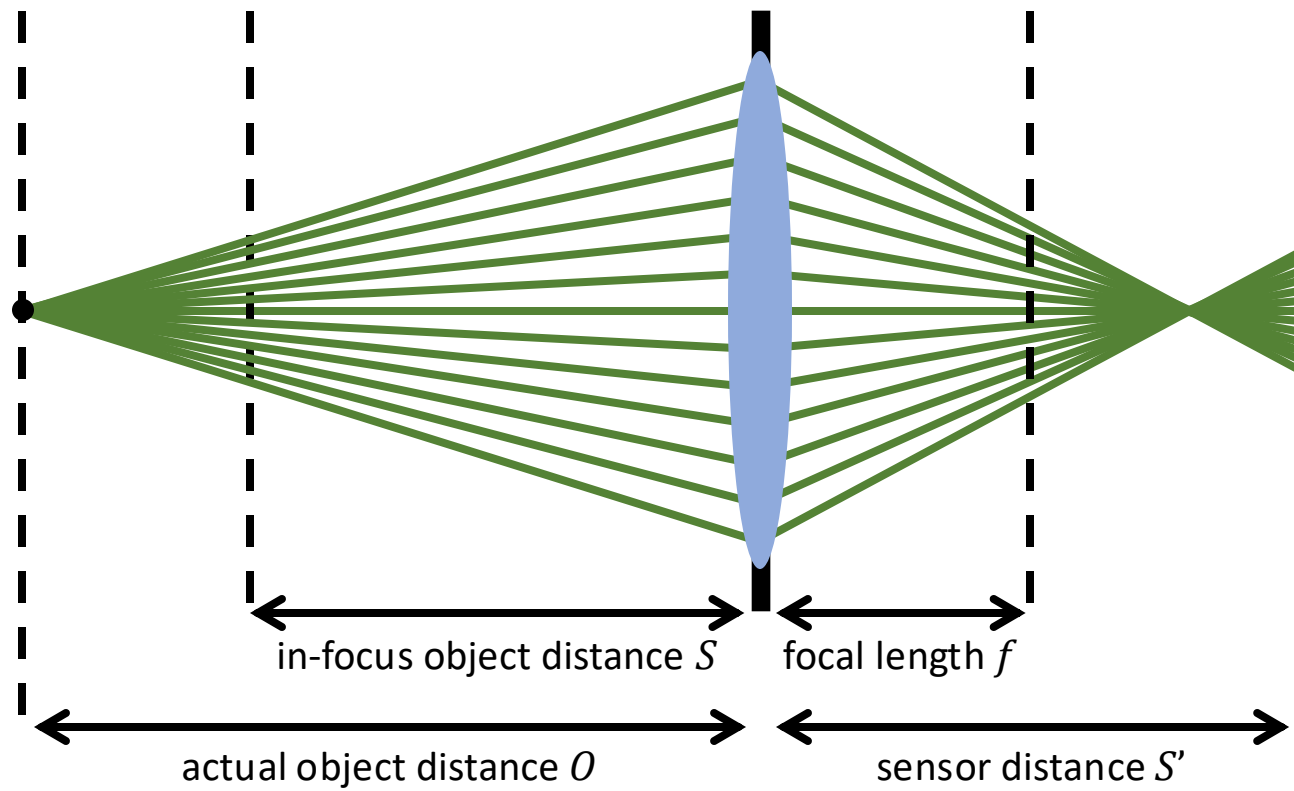
Circle of Confusion

- Will the points focus at a distance smaller or larger than S' ?



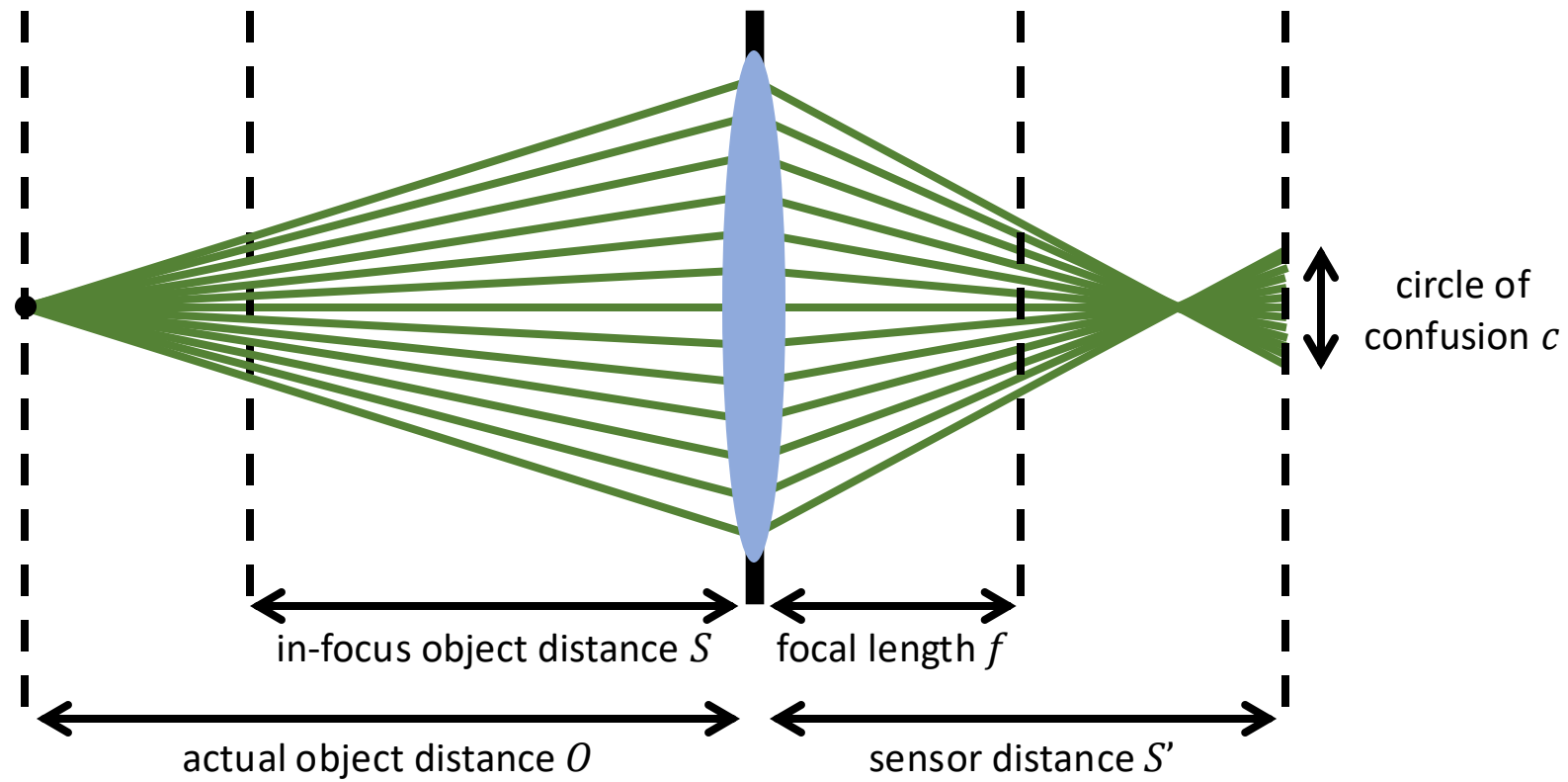
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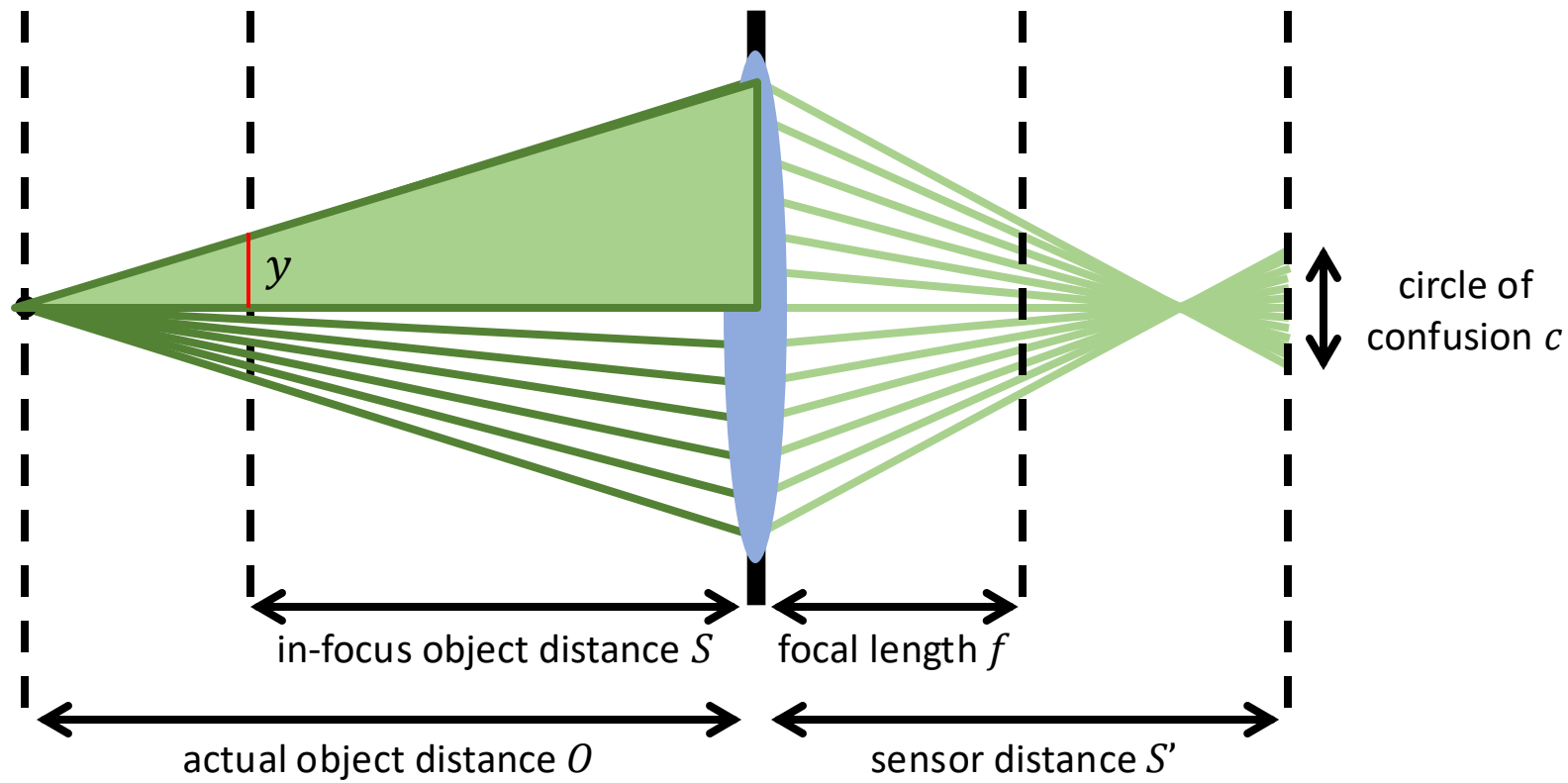
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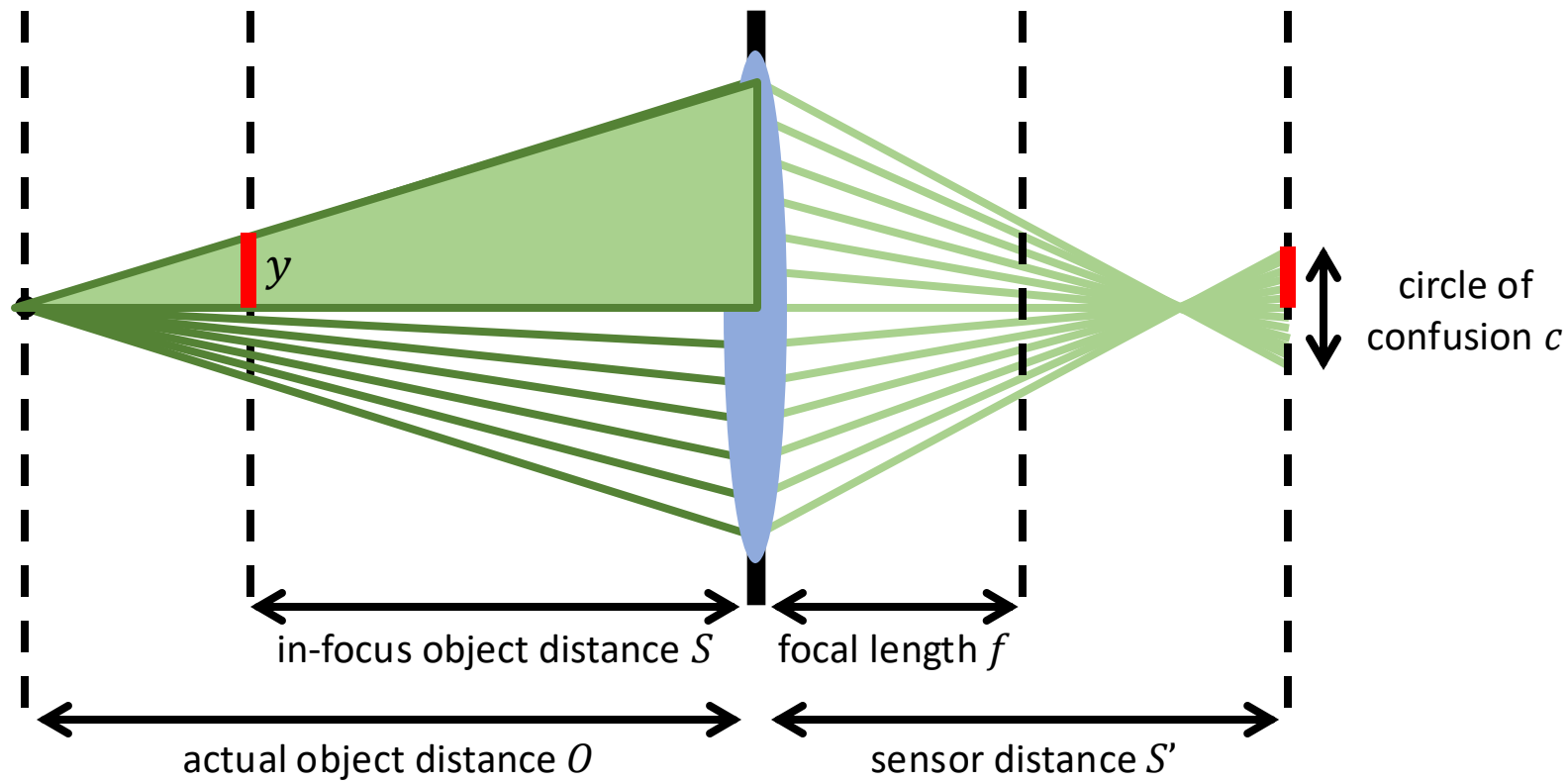


Similar triangles

$$\frac{y}{(D/2)} = \frac{|O - S|}{O}$$

Circle of Confusion

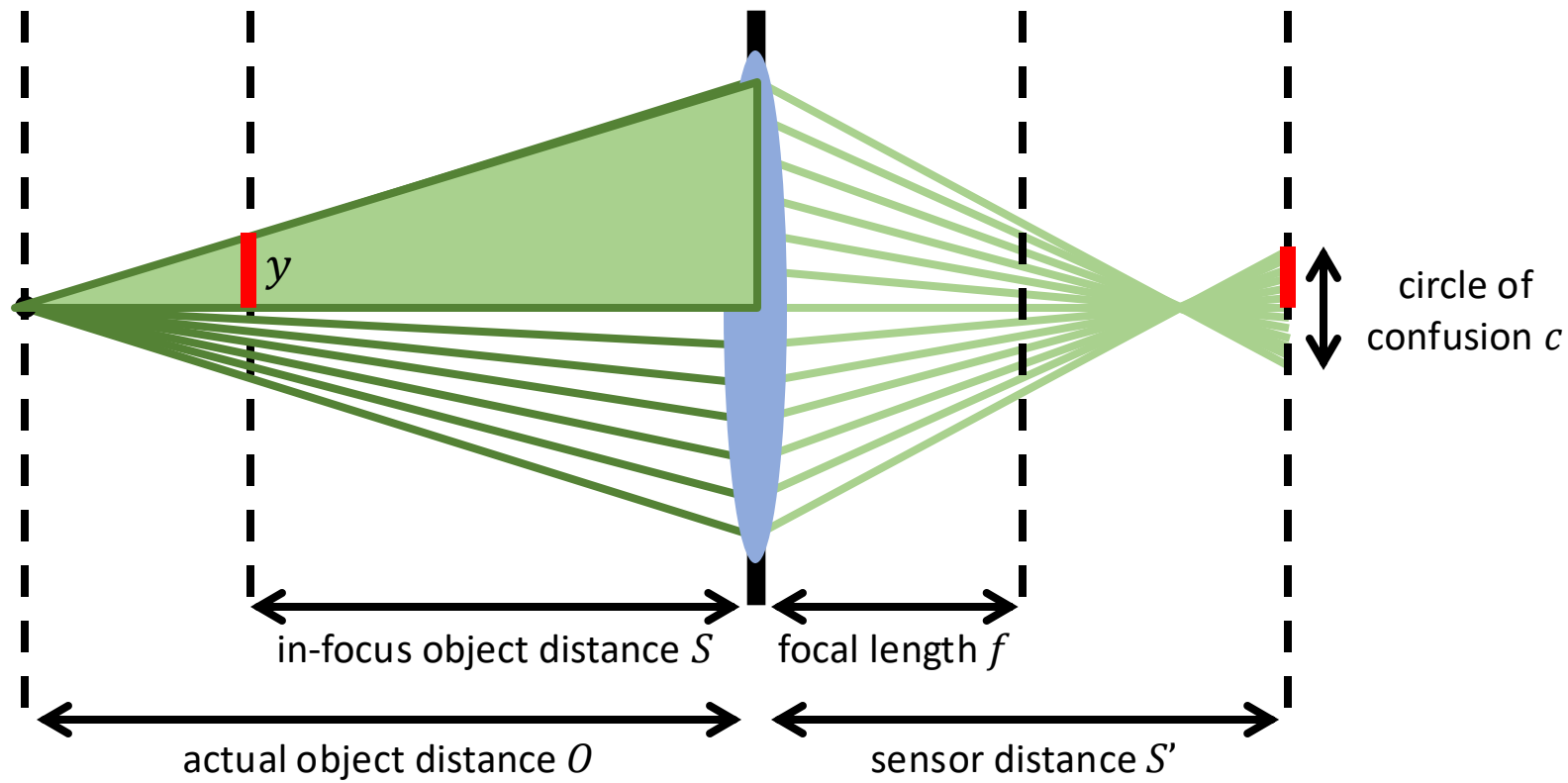
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$$\frac{y}{(D/2)} = \frac{|O - S|}{O}$$
$$\frac{y}{(c/2)} = m$$

Circle of Confusion

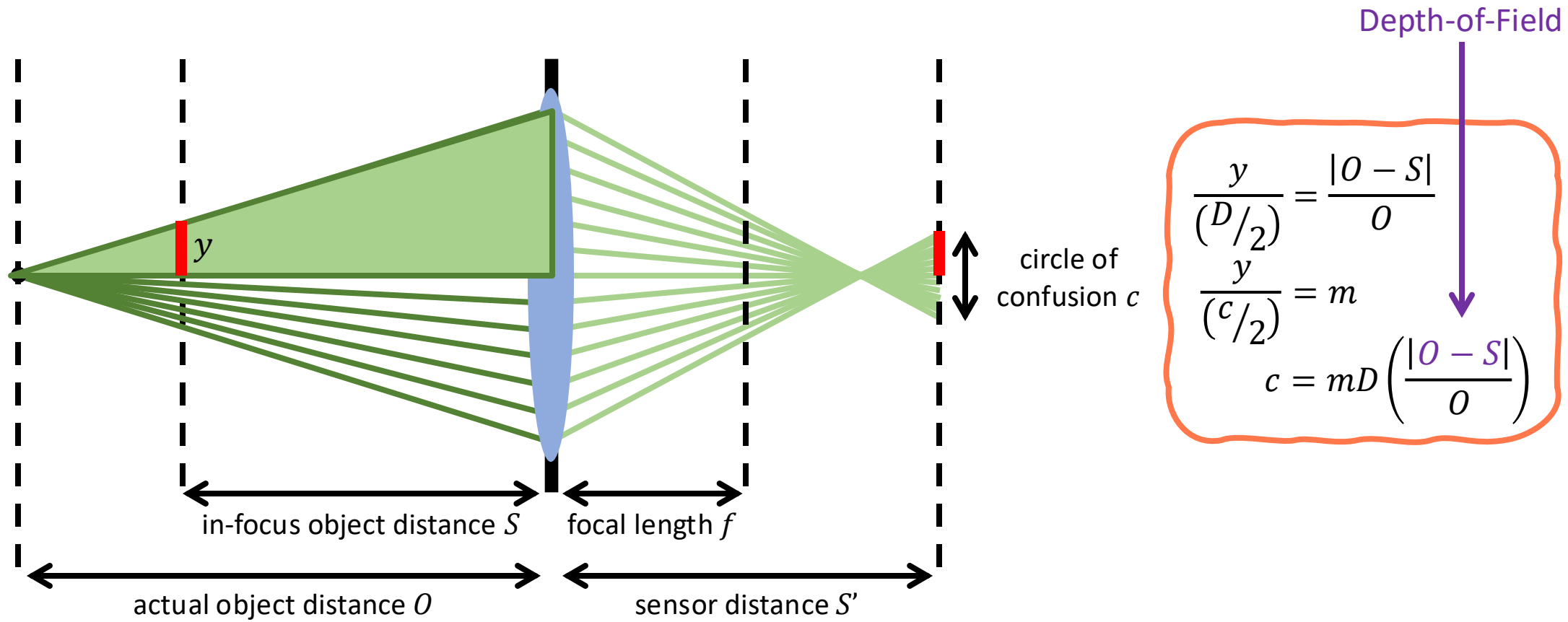
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$$\frac{y}{(D/2)} = \frac{|O - S|}{O}$$
$$\frac{y}{(c/2)} = m$$
$$c = mD \left(\frac{|O - S|}{O} \right)$$

Circle of Confusion

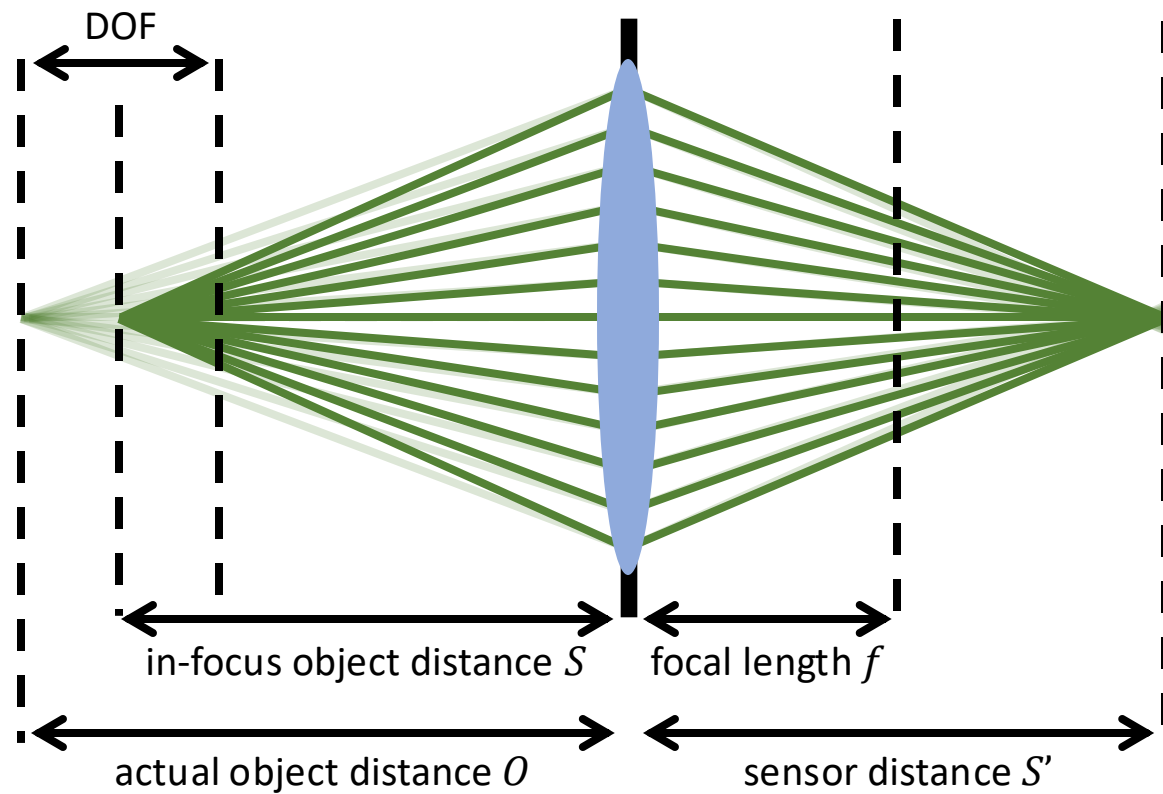
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Circle of Confusion

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Note: in reality, DOF is slightly asymmetrical!

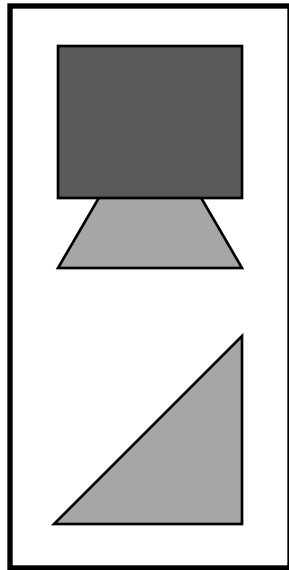


circle of confusion
 $c < \epsilon$

e.g., equal to 4-5 pixels
↓
 $c < \epsilon \Rightarrow$
 $DOF = \frac{2\epsilon O}{mD}$

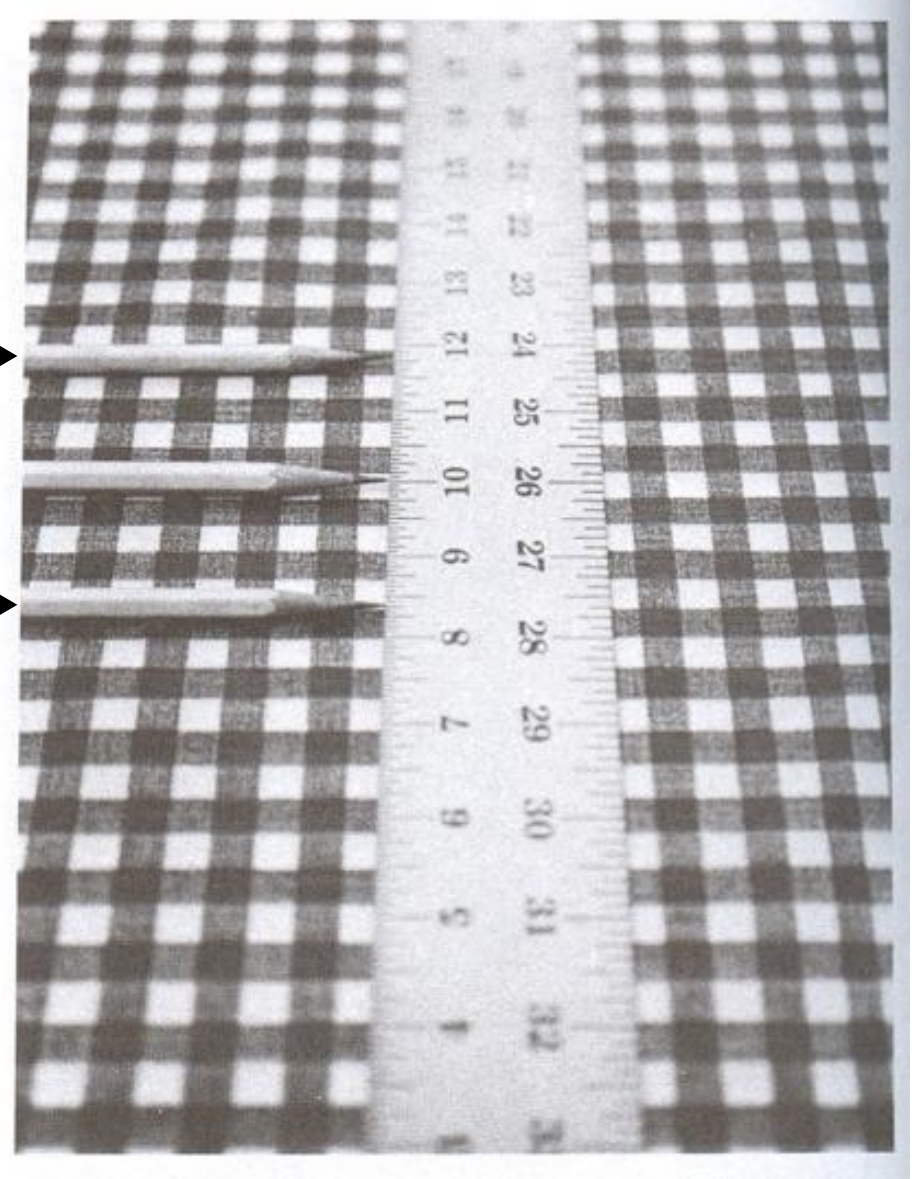
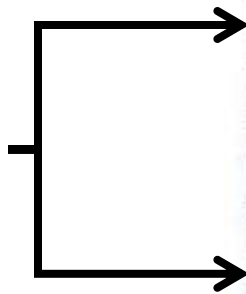
Distance around the in-focus plane where objects are imaged within an acceptable level of blurriness

Depth of Field (DOF)

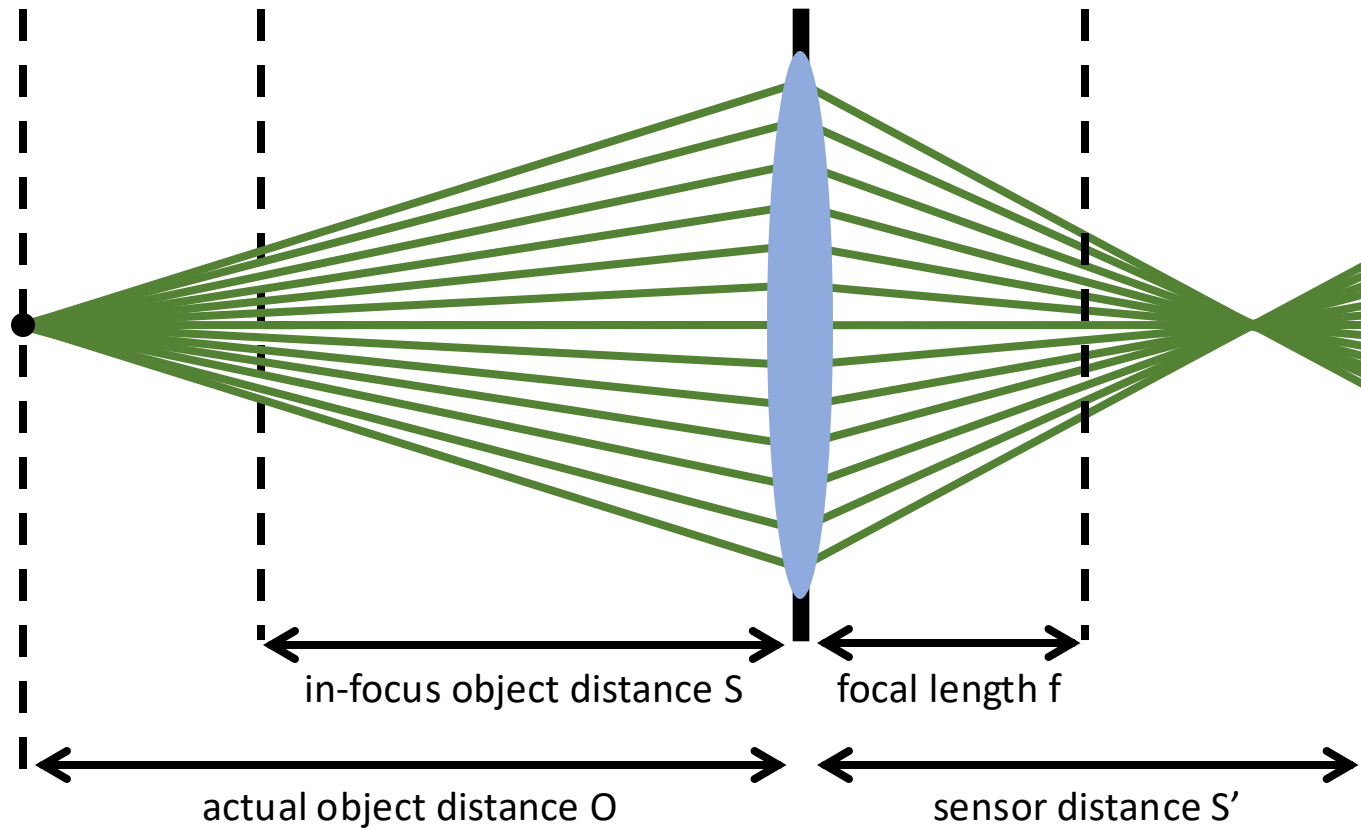


scene

depth of field

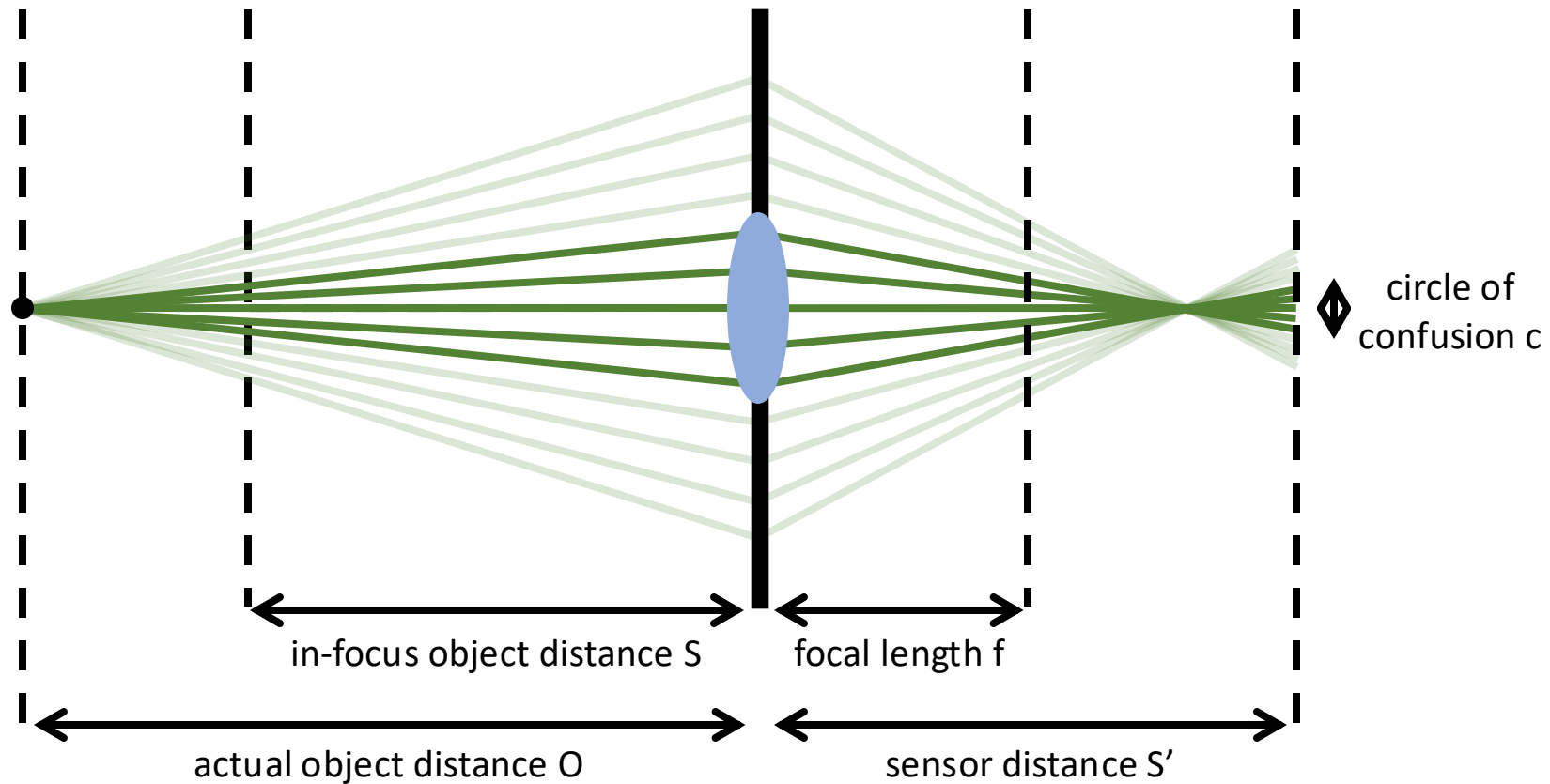


Defocus Depends Upon Aperture Diameter



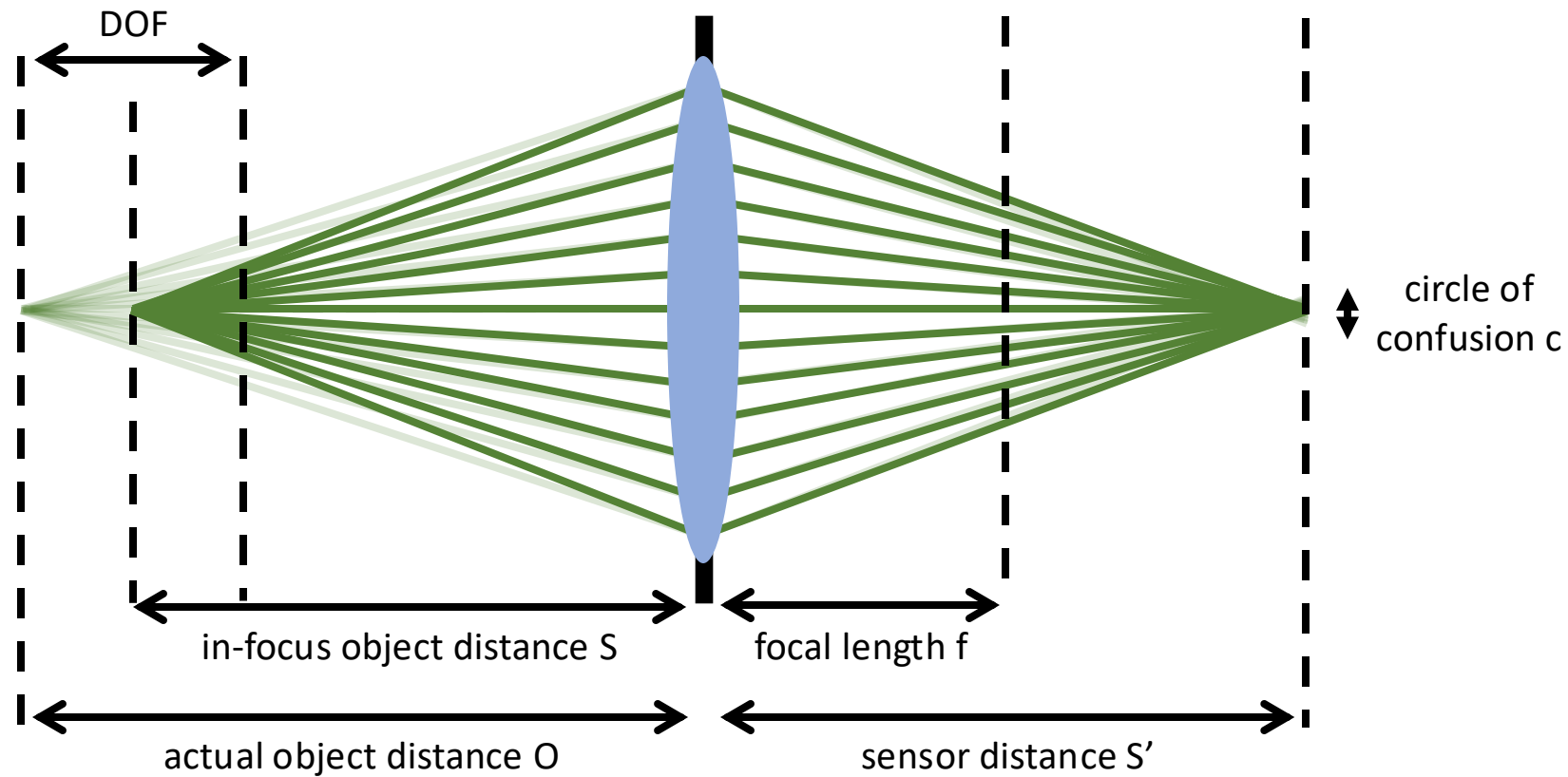
$$c = mD \frac{|O - S|}{O}$$
$$\text{DOF} = \frac{2\varepsilon O}{mD}$$

Defocus Depends Upon Aperture Diameter



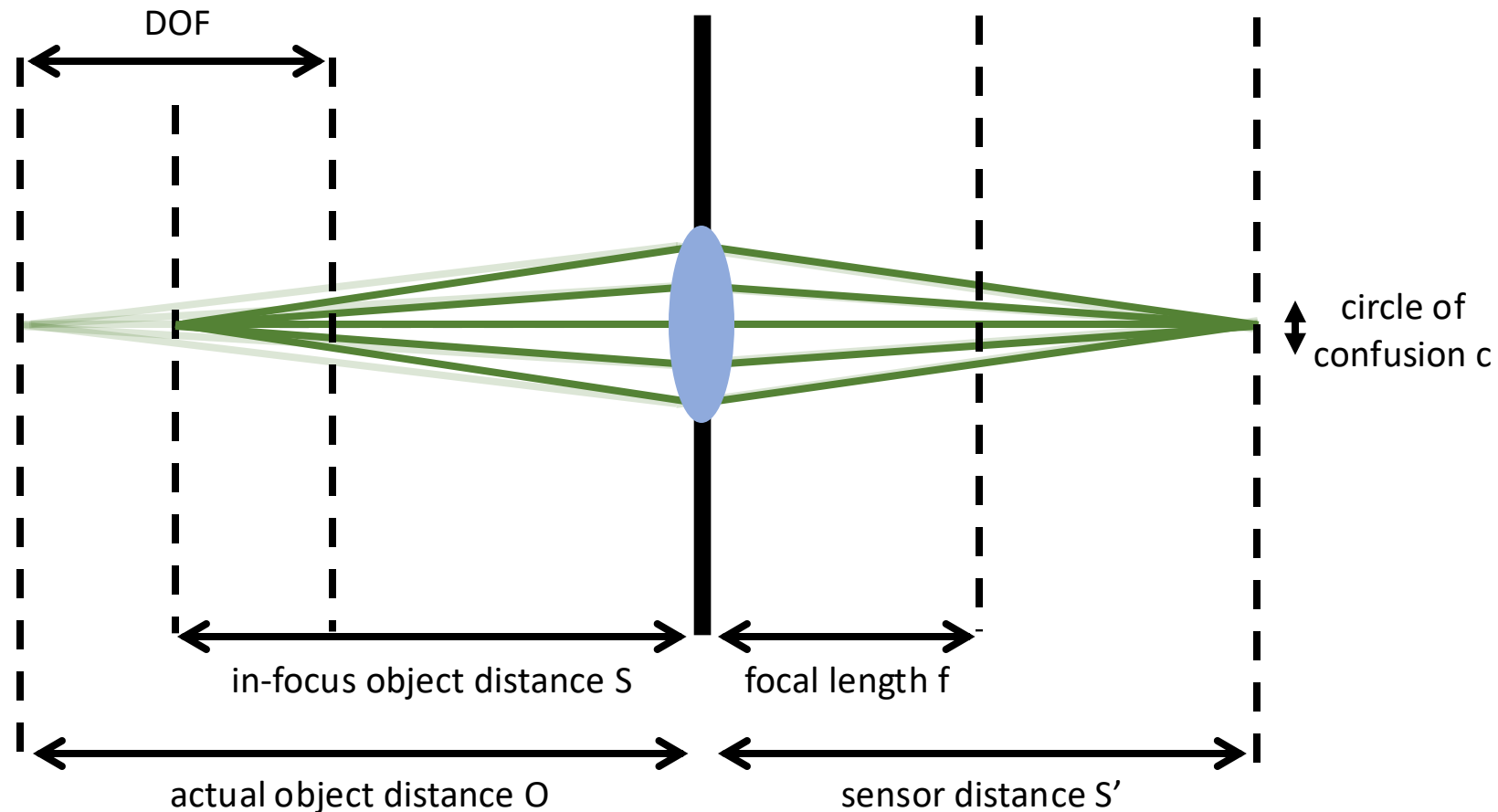
$$c = mD \frac{|O - S|}{O}$$
$$\text{DOF} = \frac{2\varepsilon O}{mD}$$

Defocus Depends Upon Aperture Diameter



$$c = mD \frac{|O - S|}{O}$$
$$\text{DOF} = \frac{2\varepsilon O}{mD}$$

Defocus Depends Upon Aperture Diameter



$$c = mD \frac{|O - S|}{O}$$
$$\text{DOF} = \frac{2\varepsilon O}{mD}$$

- Most lenses have apertures of variable size.
- The size of the aperture is expressed as the “f-number”: The bigger this number, the smaller the aperture.



f / 1.4



f / 2.8



f / 4



f / 8



f / 16

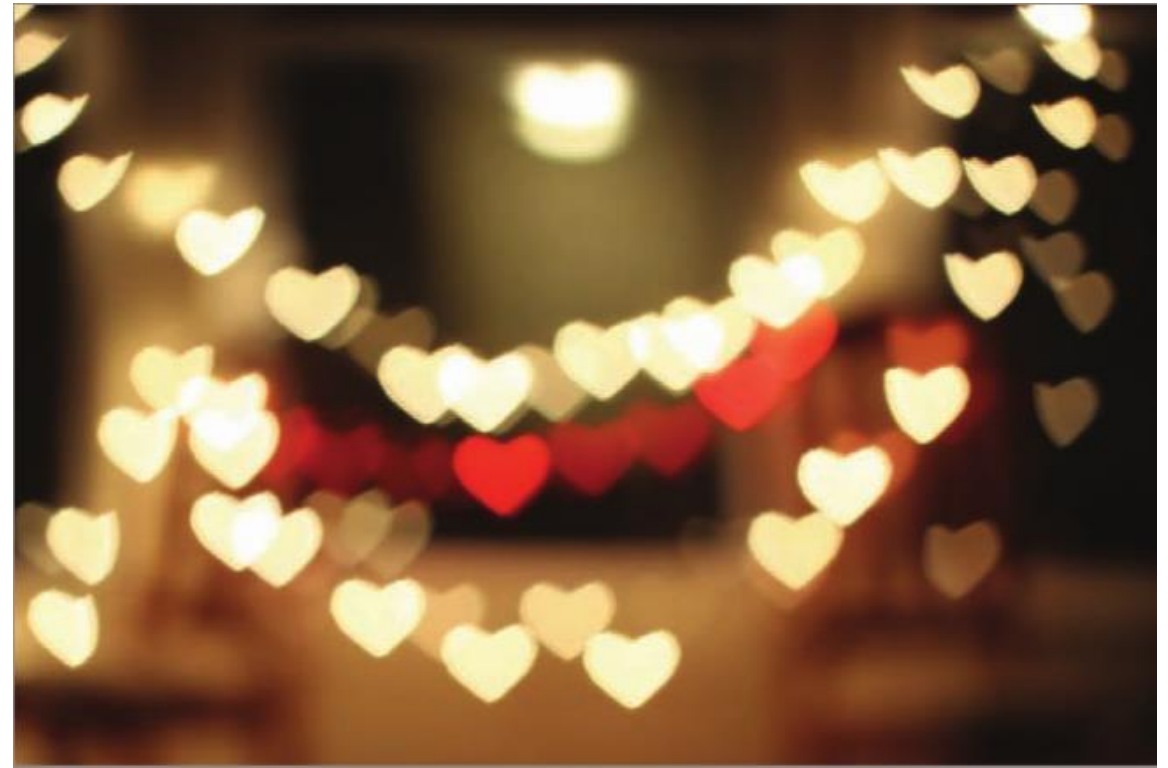
Aperture Size

- Aperture sizes are denoted as f-stops, i.e., f/N , where f represent the focal length and N is a number.
- Represents the ratio of the lens focal length to the diameter of the aperture
- Aperture sizes shift at steps of one-half or one-third f-stops:
 $\frac{f}{1}$ ' $\frac{f}{1.4}$ ' $\frac{f}{2}$ ' $\frac{f}{2.8}$ ' $\frac{f}{4}$ ' $\frac{f}{5.6}$ ' $\frac{f}{8}$ ' $\frac{f}{11}$ ' $\frac{f}{16}$ ' $\frac{f}{22}$
- Smaller f-stops means larger aperture sizes
- Larger f-stops means smaller aperture sizes
- Each step lets in half as much light as the previous step, i.e., $\frac{f}{2.8}$ lets in twice as much light as $\frac{f}{4}$



Depth of Field

- Form of defocus is determined by the shape of the aperture



Defocus Depends upon the Aperture Diameter

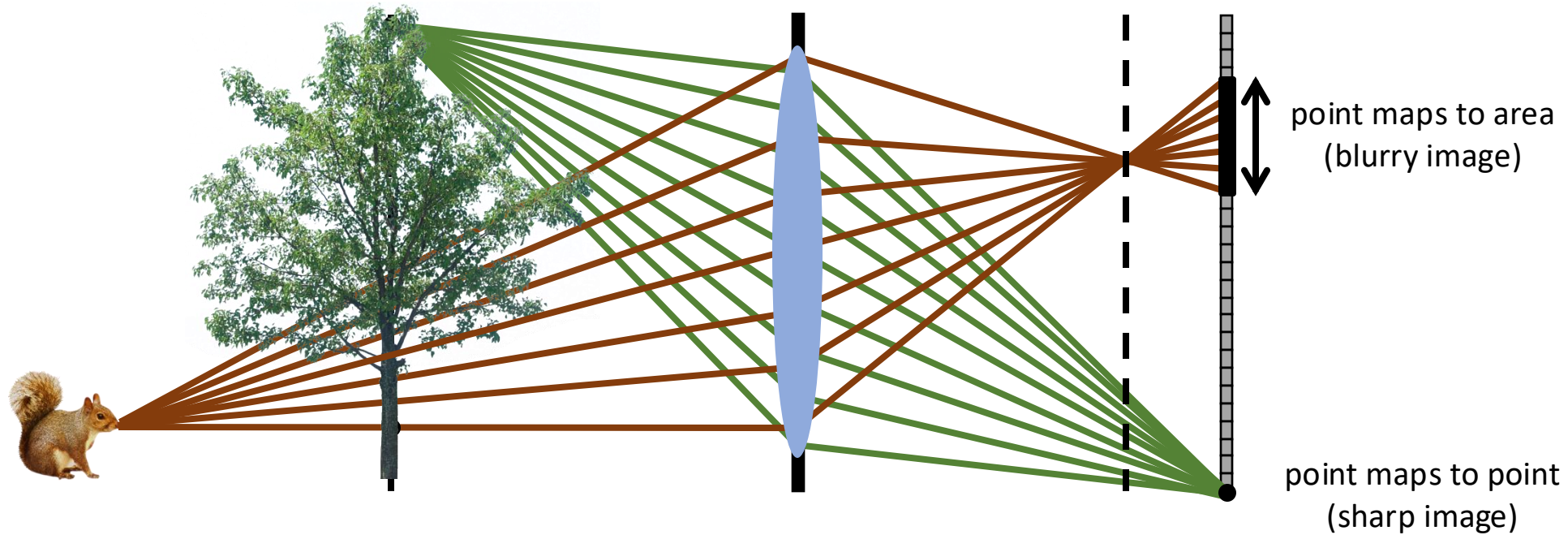
- Small aperture diameter reduce defocus blur. Should we always use smaller apertures?

Defocus Depends upon the Aperture Diameter

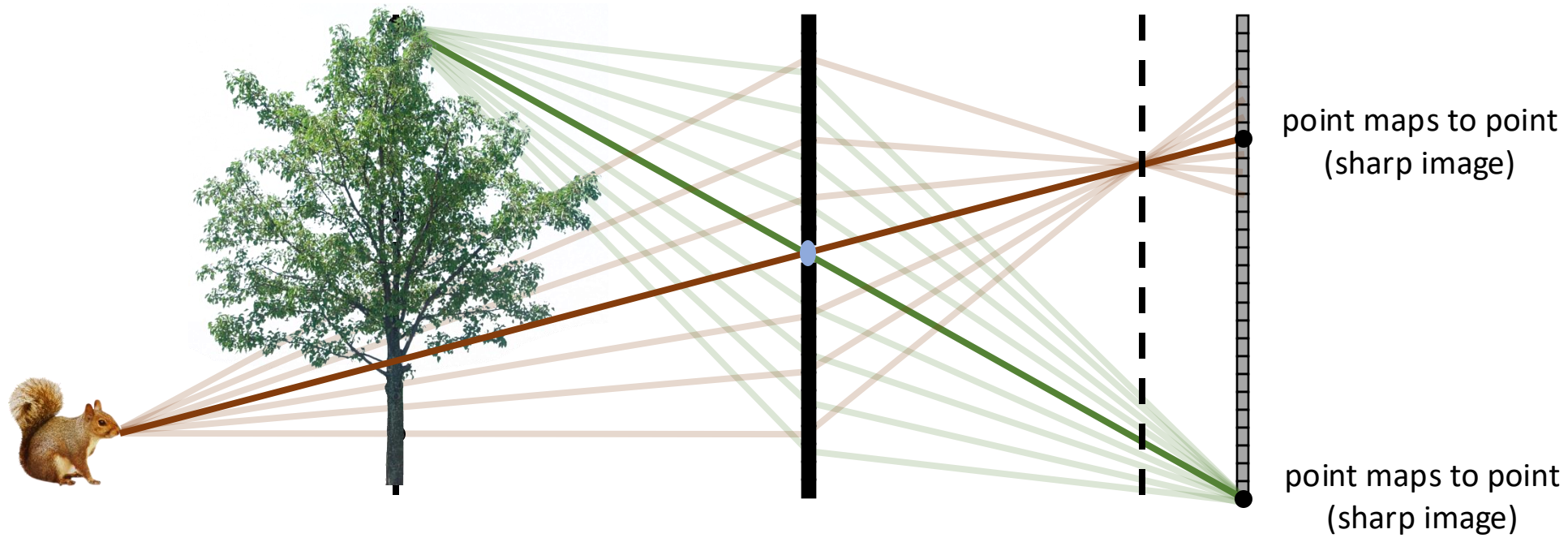
- Small aperture diameter reduce defocus blur. Should we always use smaller apertures?
 - No. Sharp depth of field, also known as bokeh, is often desirable and an artistic choice



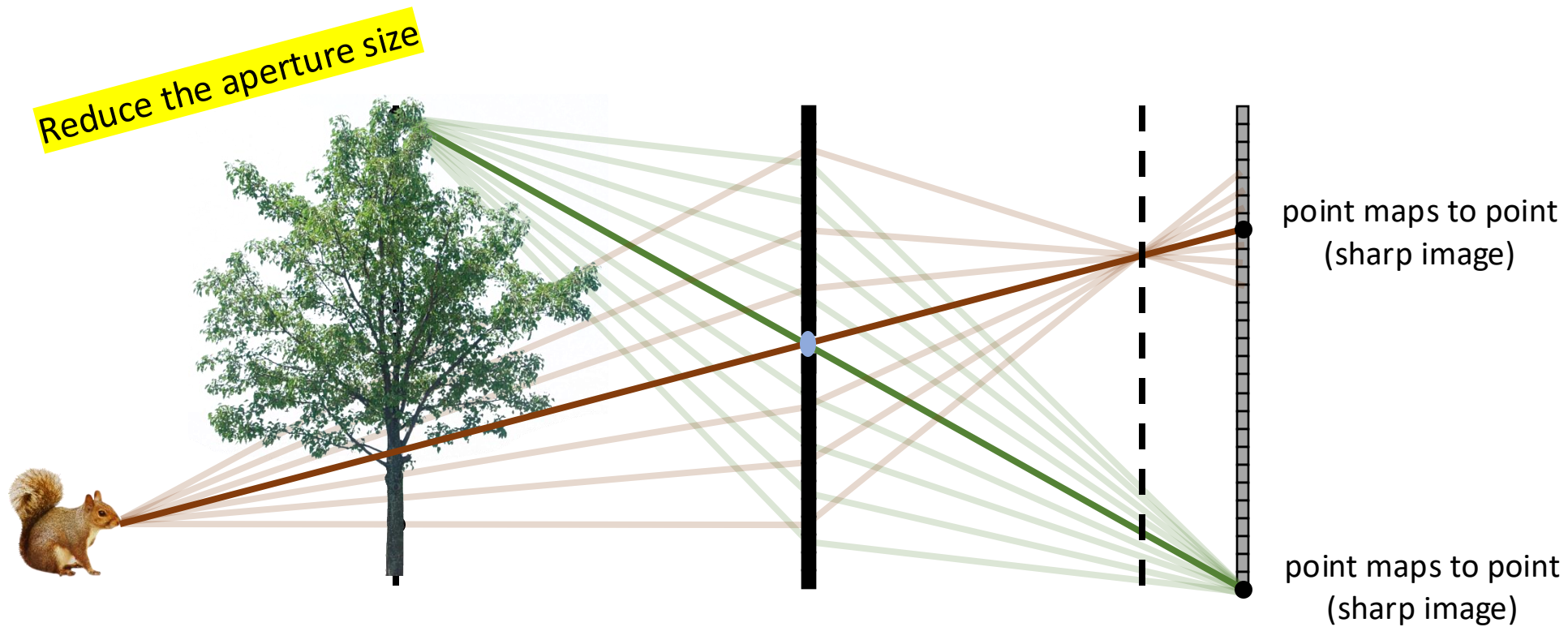
How to Ensure that “Everything” is in Focus?



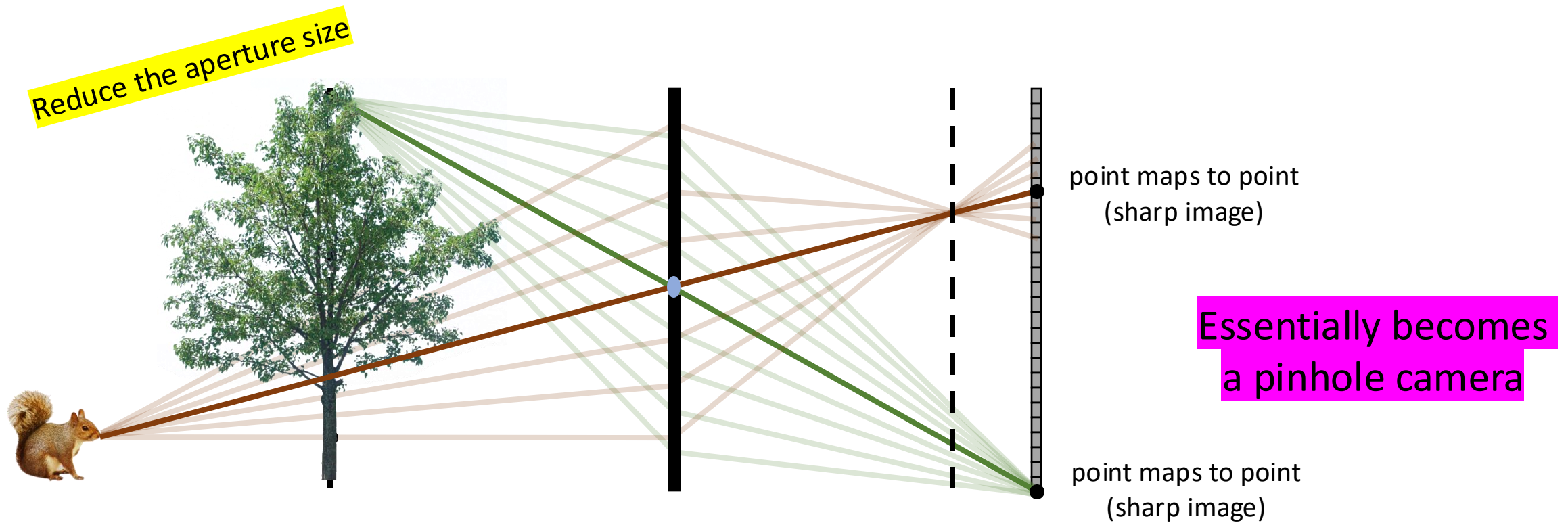
How to Increase the Depth of Field?



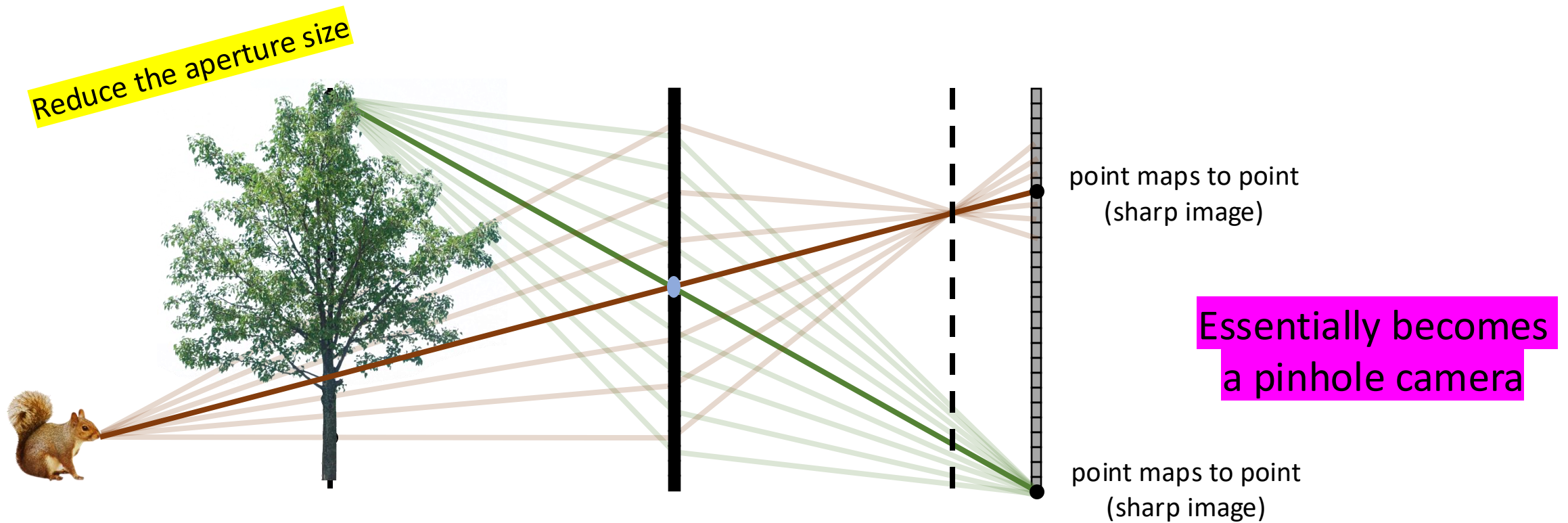
How to Increase the Depth of Field?



How to Increase the Depth of Field?



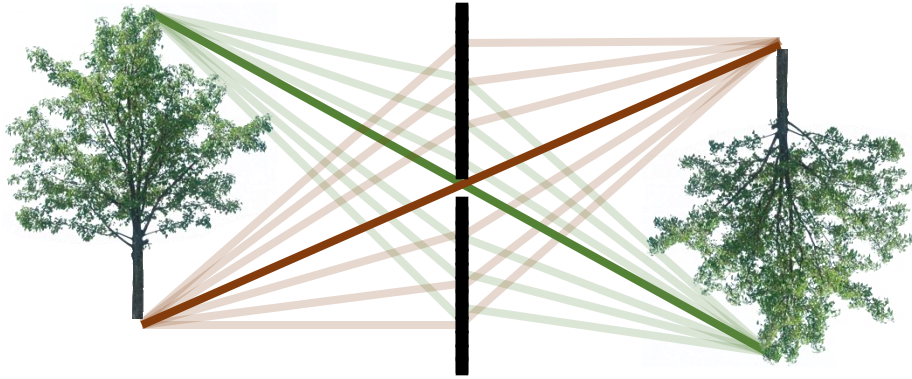
How to Increase the Depth of Field?



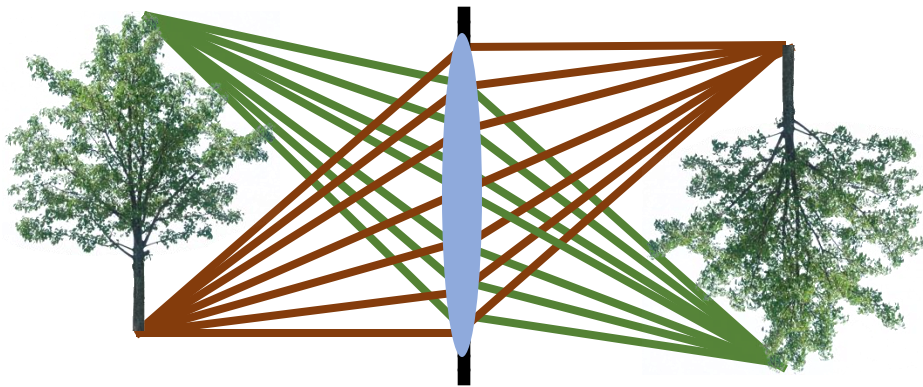
1. No defocus, everything is sharp regardless of its distance from the camera
2. Little light passes through the aperture resulting in a low signal-to-noise ratio

Pinhole Camera vs Lens Camera

Pinhole Camera

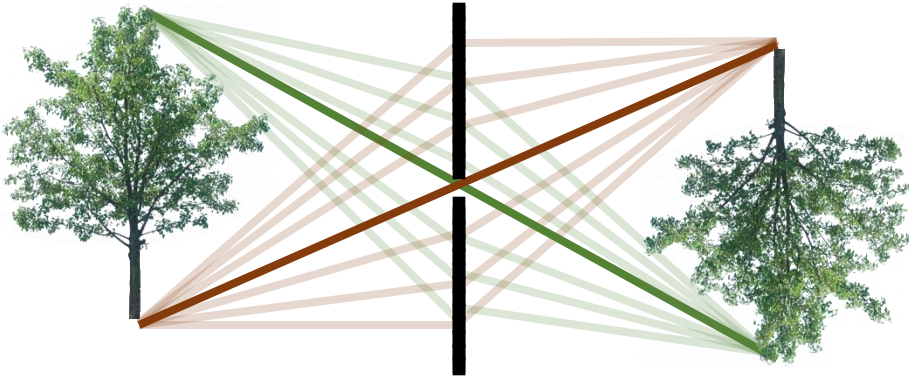


Lens Camera

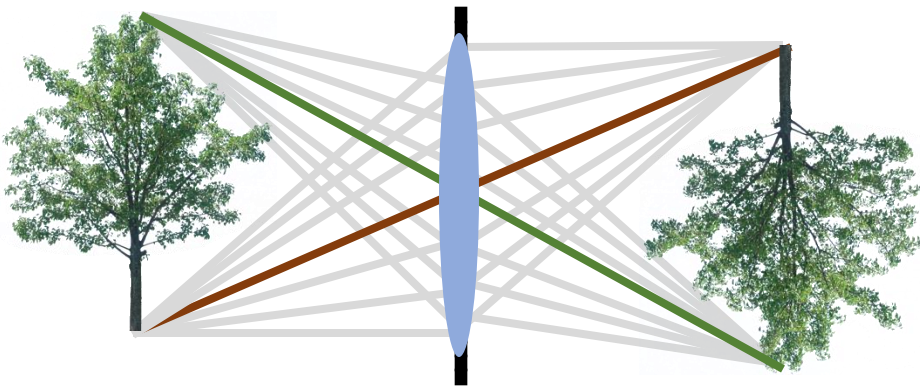


Pinhole Camera vs Lens Camera

Pinhole Camera



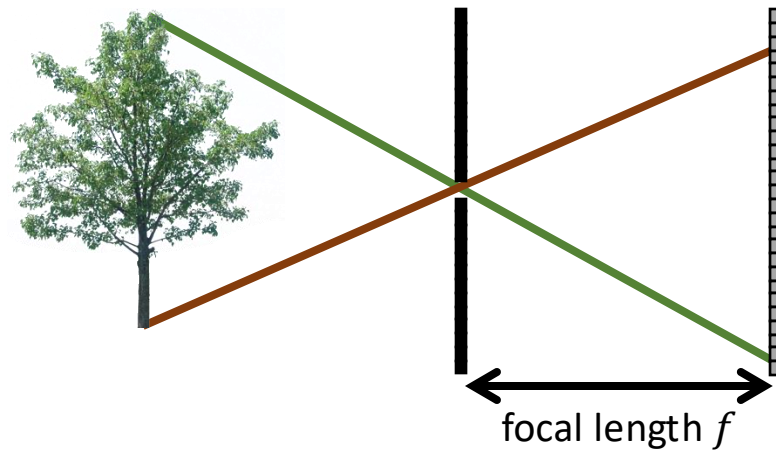
Lens Camera



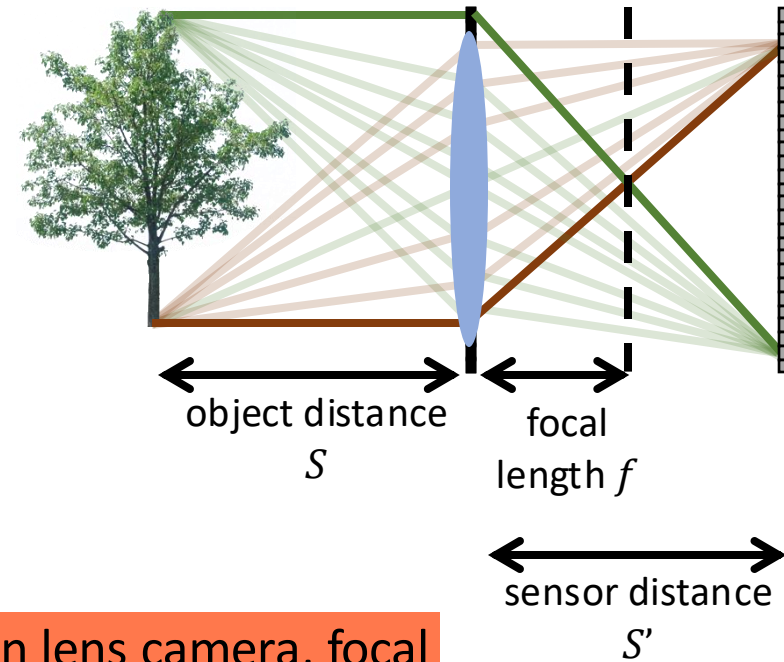
We can derive properties and descriptions that hold for both camera models if:

- We consider only central rays.
- We assume that everything of interest in the scene is within the depth of field.

Pinhole Cameras vs Lens Cameras



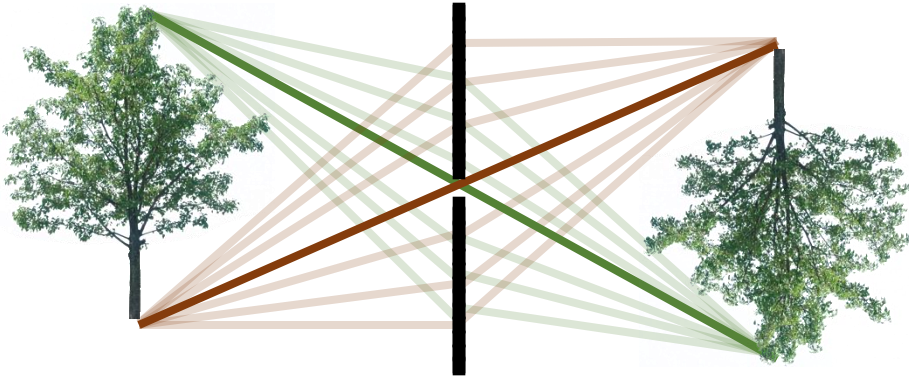
In pinhole camera,
focal length is the
distance of the sensor
from the pinhole



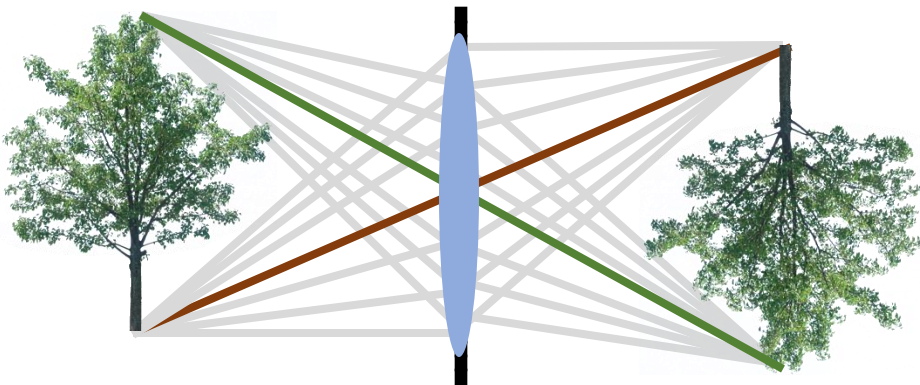
In lens camera, focal
length is the distance
where parallel rays
intersect

Pinhole Camera vs Lens Camera

Pinhole Camera



Lens Camera

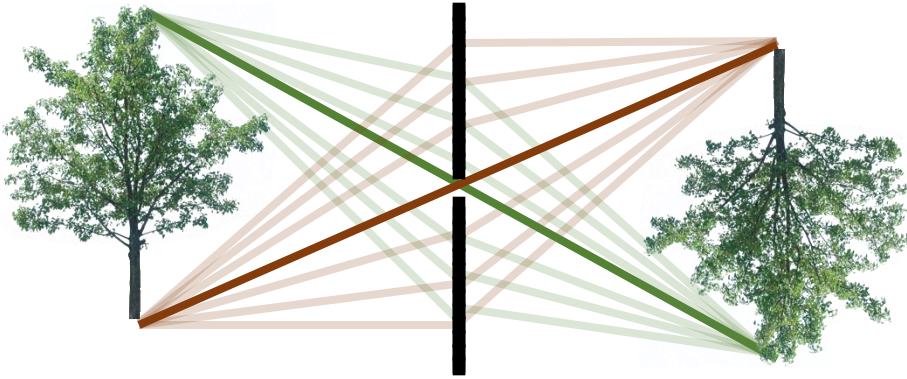


We can derive properties and descriptions that hold for both camera models if:

- We consider only central rays.
- We assume that everything of interest in the scene is within the depth of field.
- We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.

Pinhole Camera vs Lens Camera

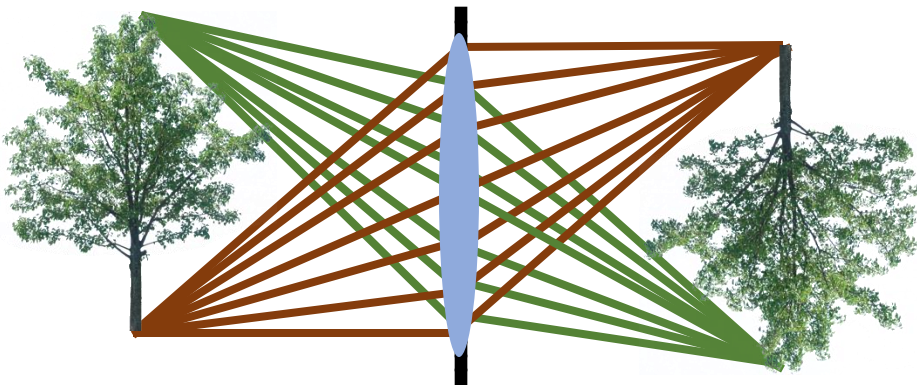
Pinhole Camera



Doubling the aperture diameter:

- Increases light throughput by four times.
- Increases circle of confusion for all planes by two times.

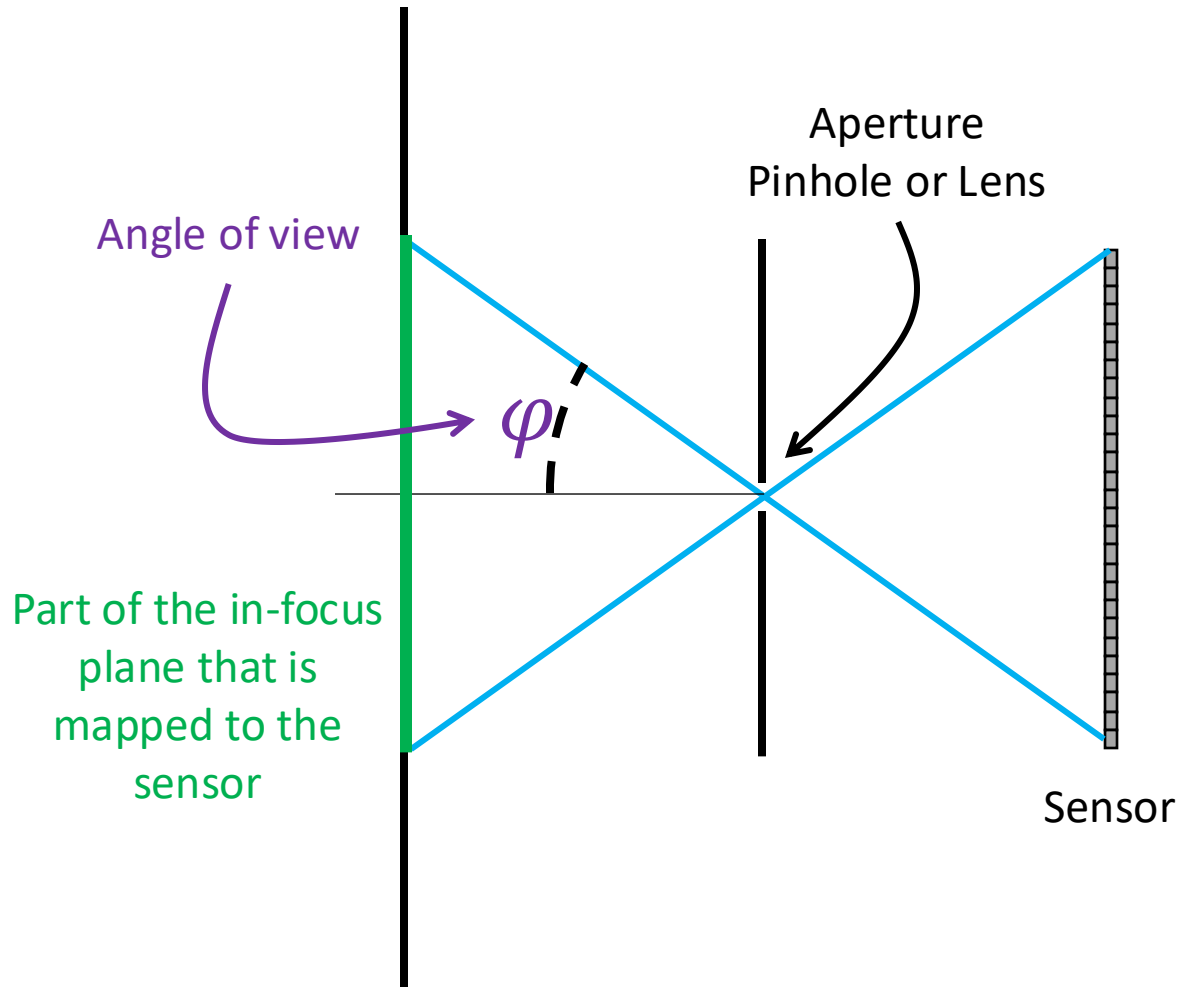
Lens Camera



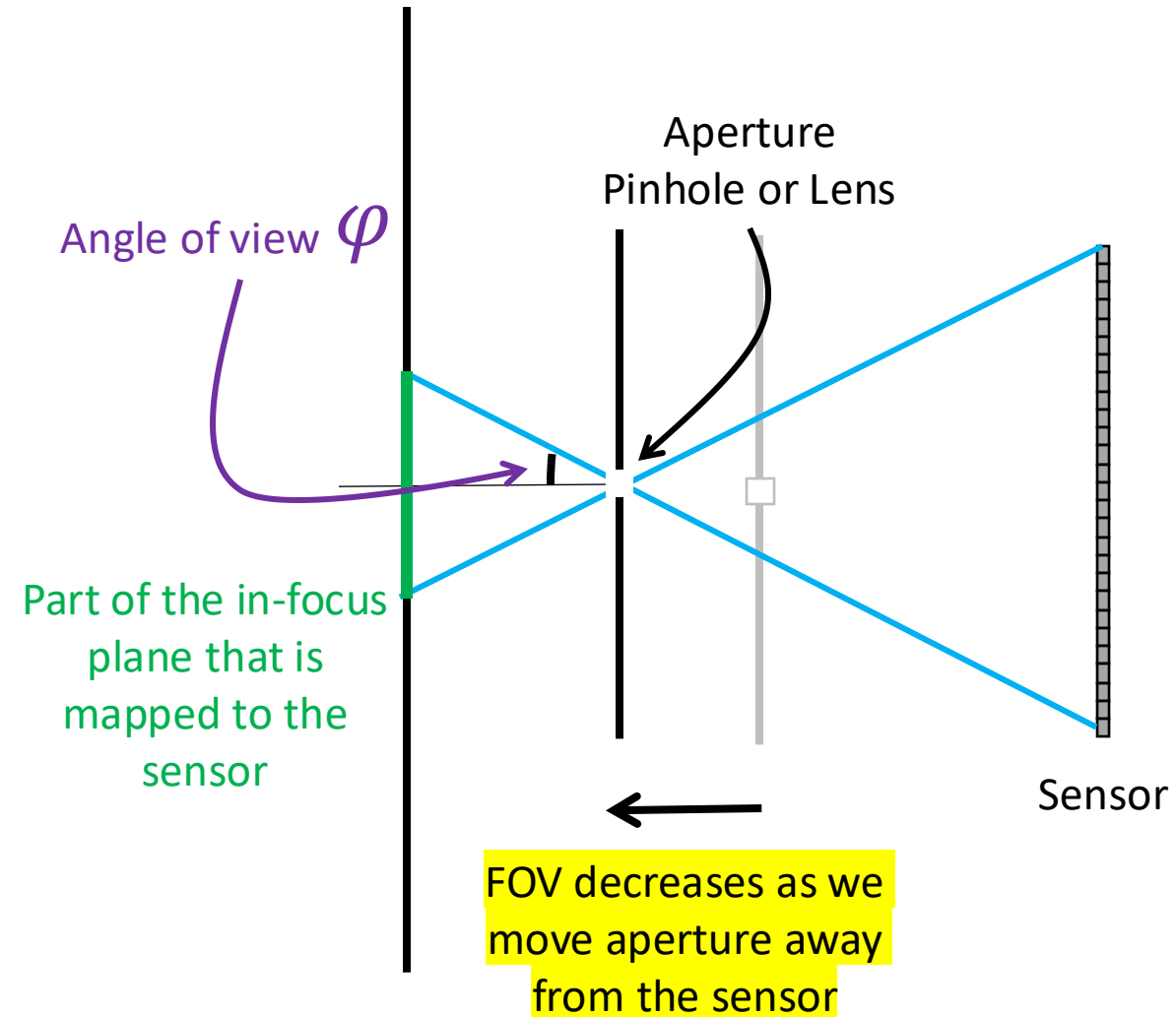
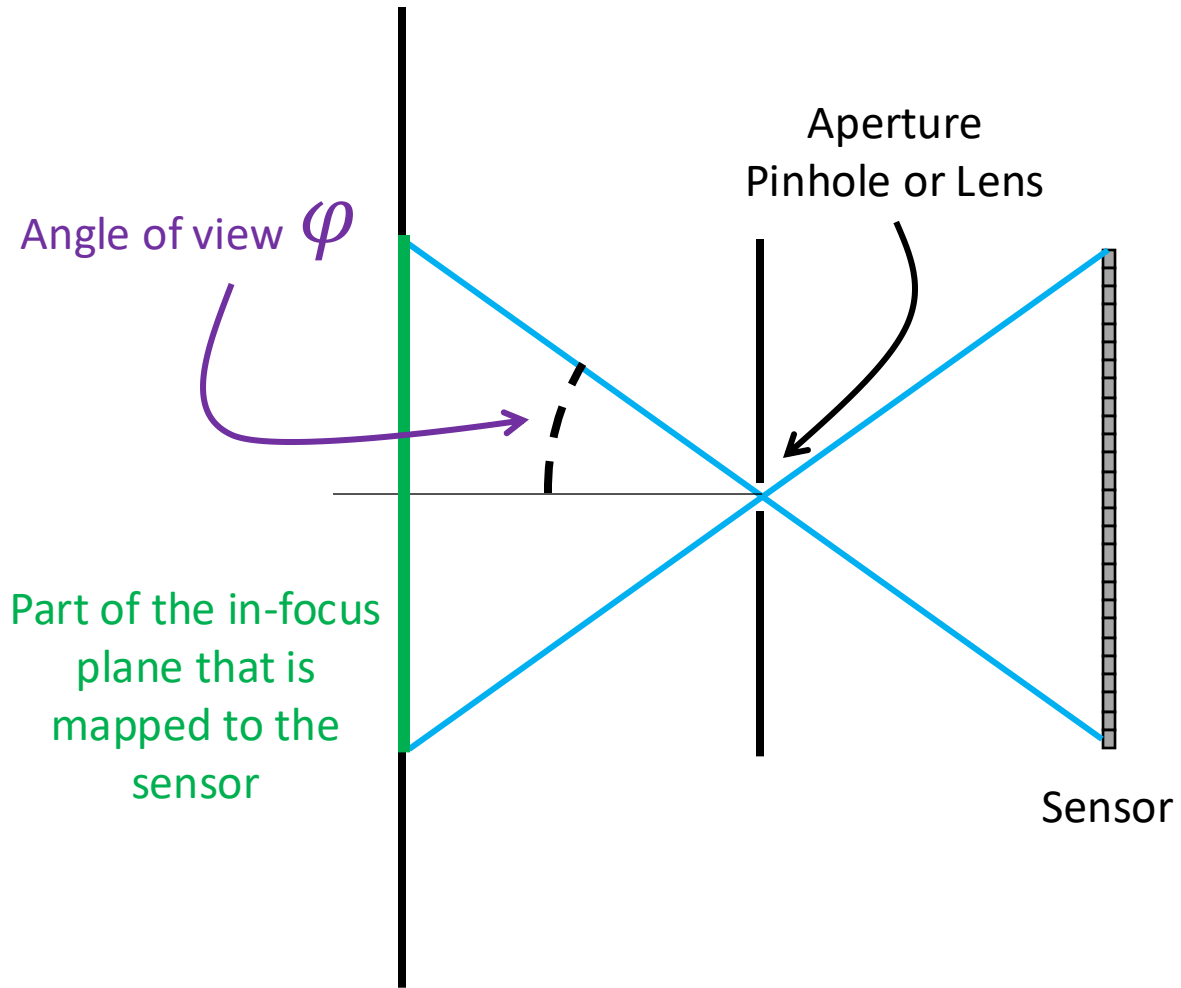
Doubling the aperture diameter:

- Increases light throughput by four times.
- Increases circle of confusion for out-of-focus plane by two times.
- Decreases depth of field by two times.

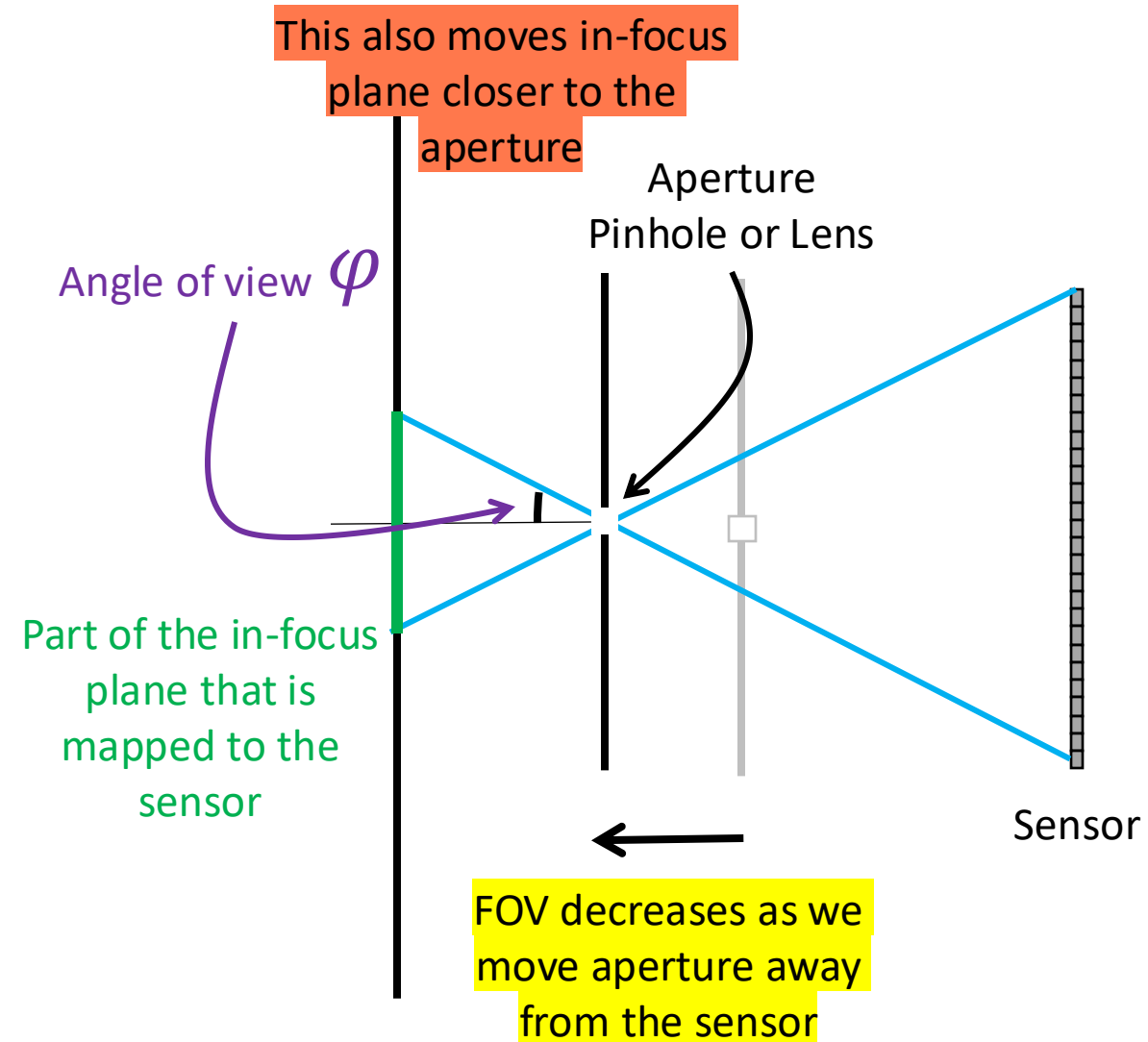
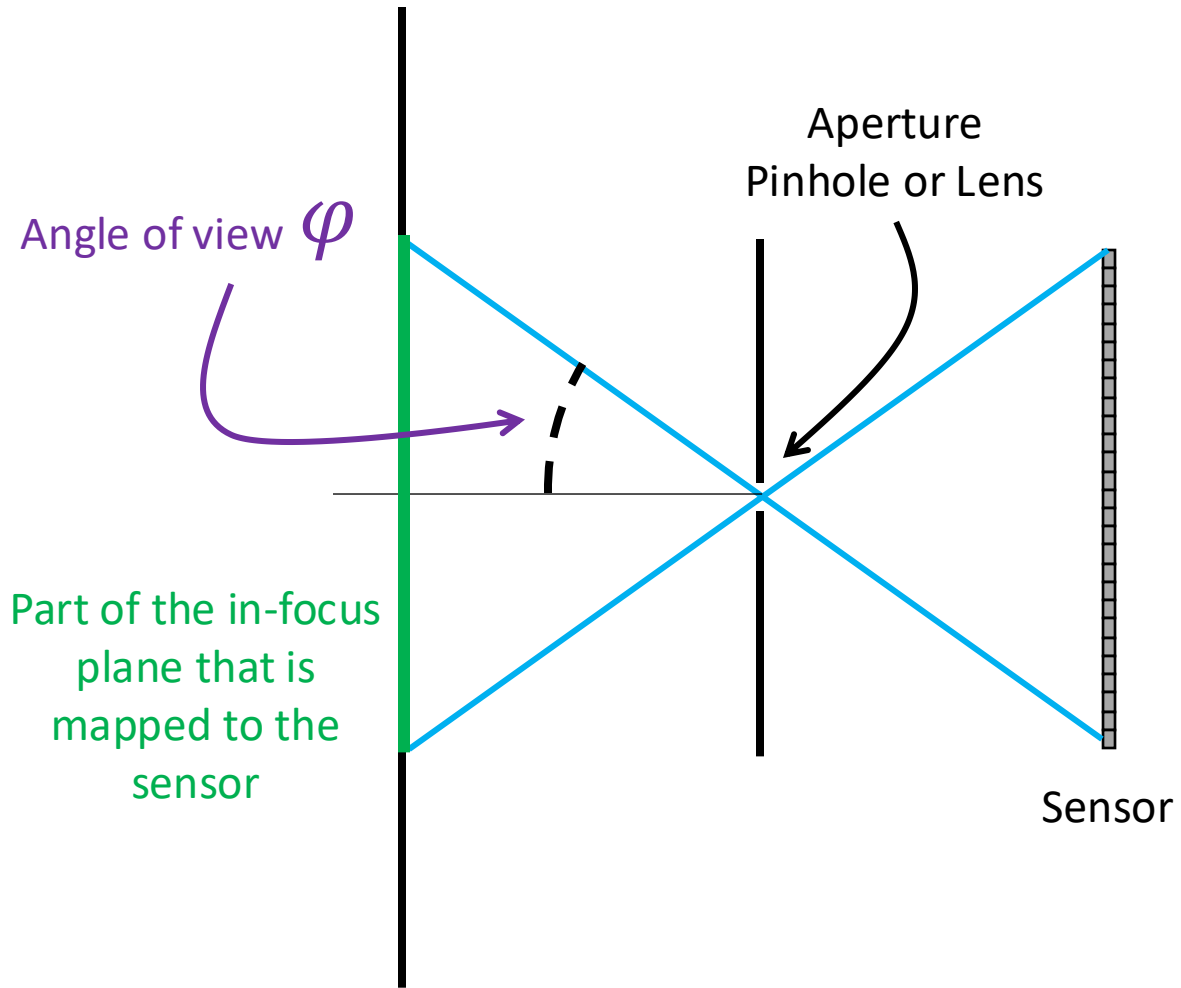
Field-of-View (FOV)



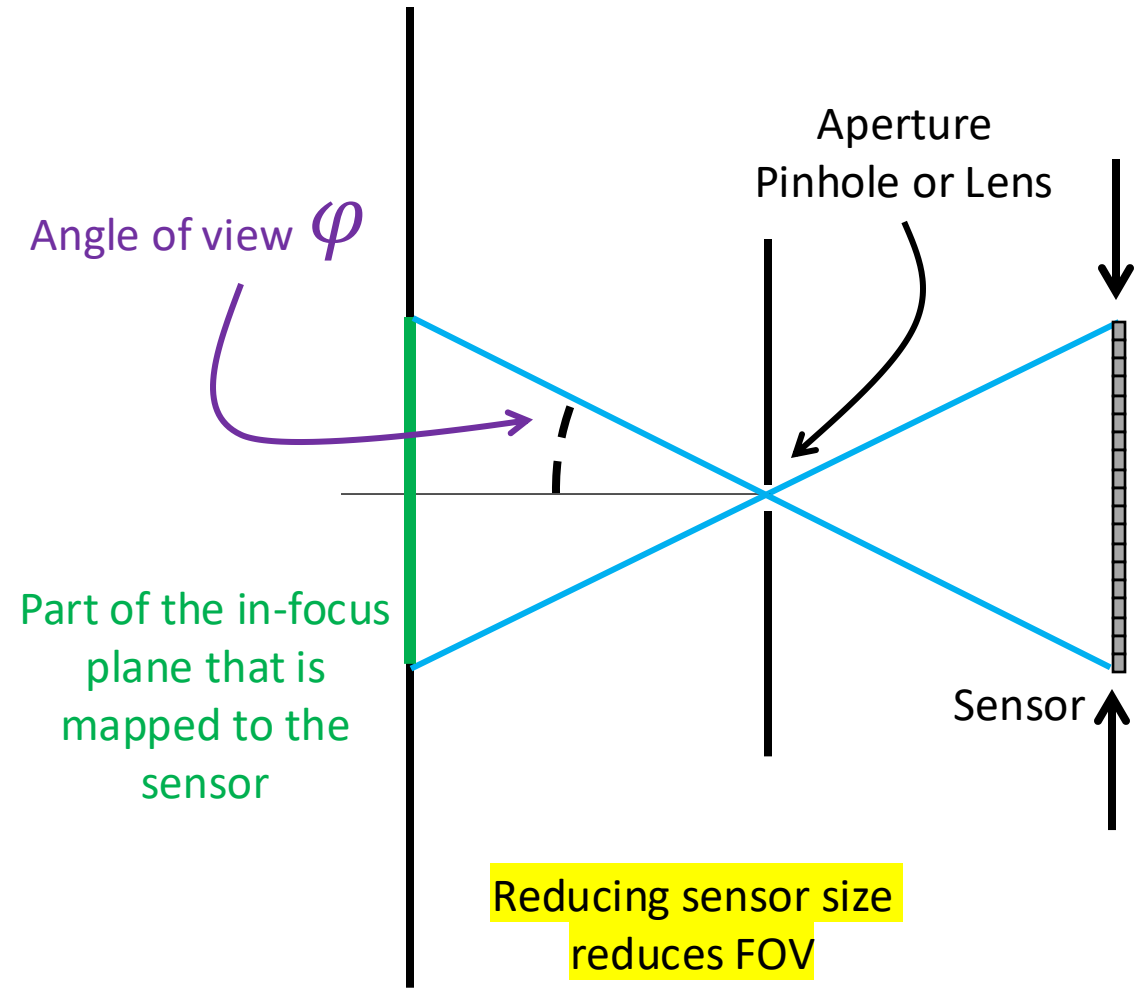
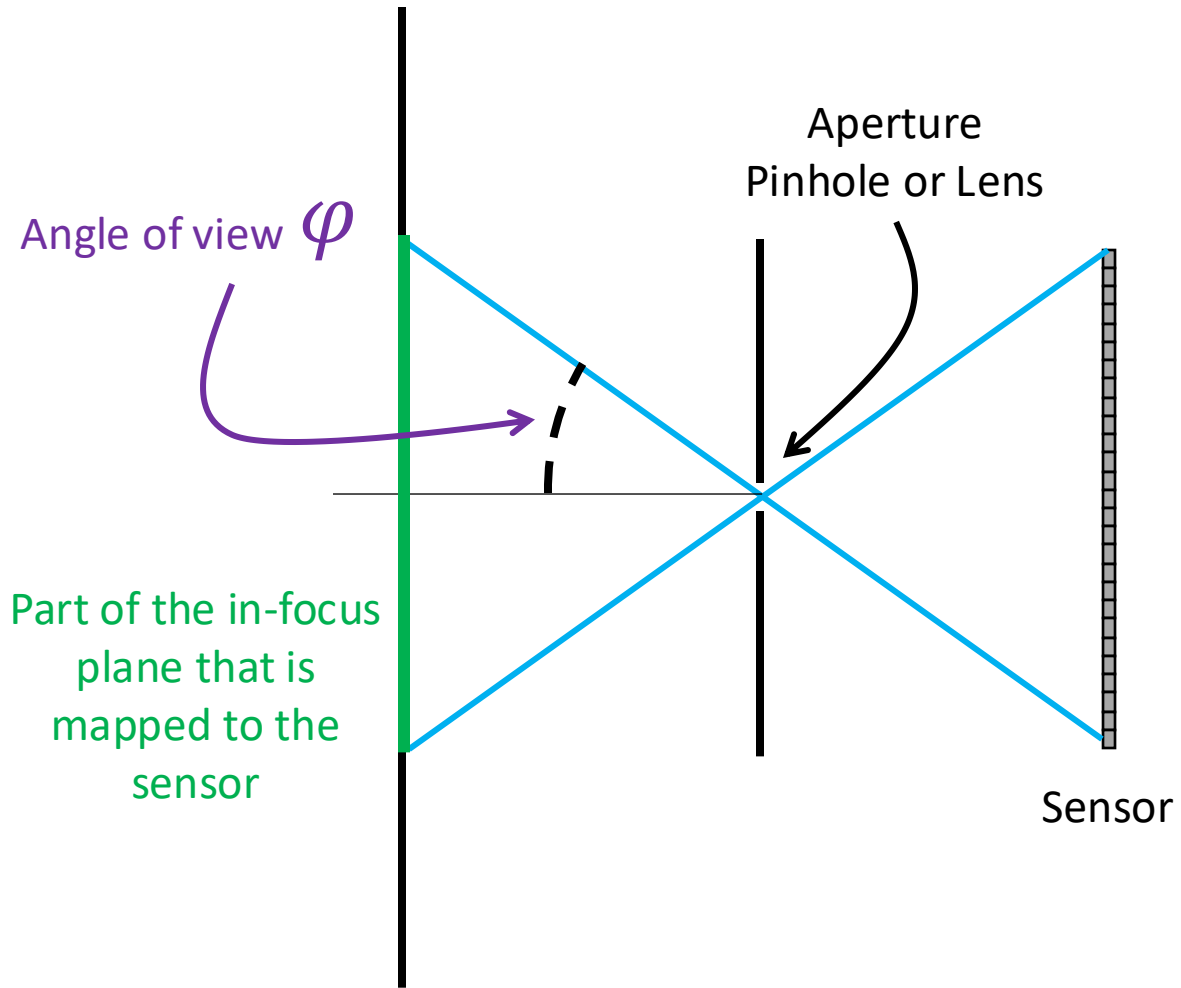
Field-of-View (FOV)



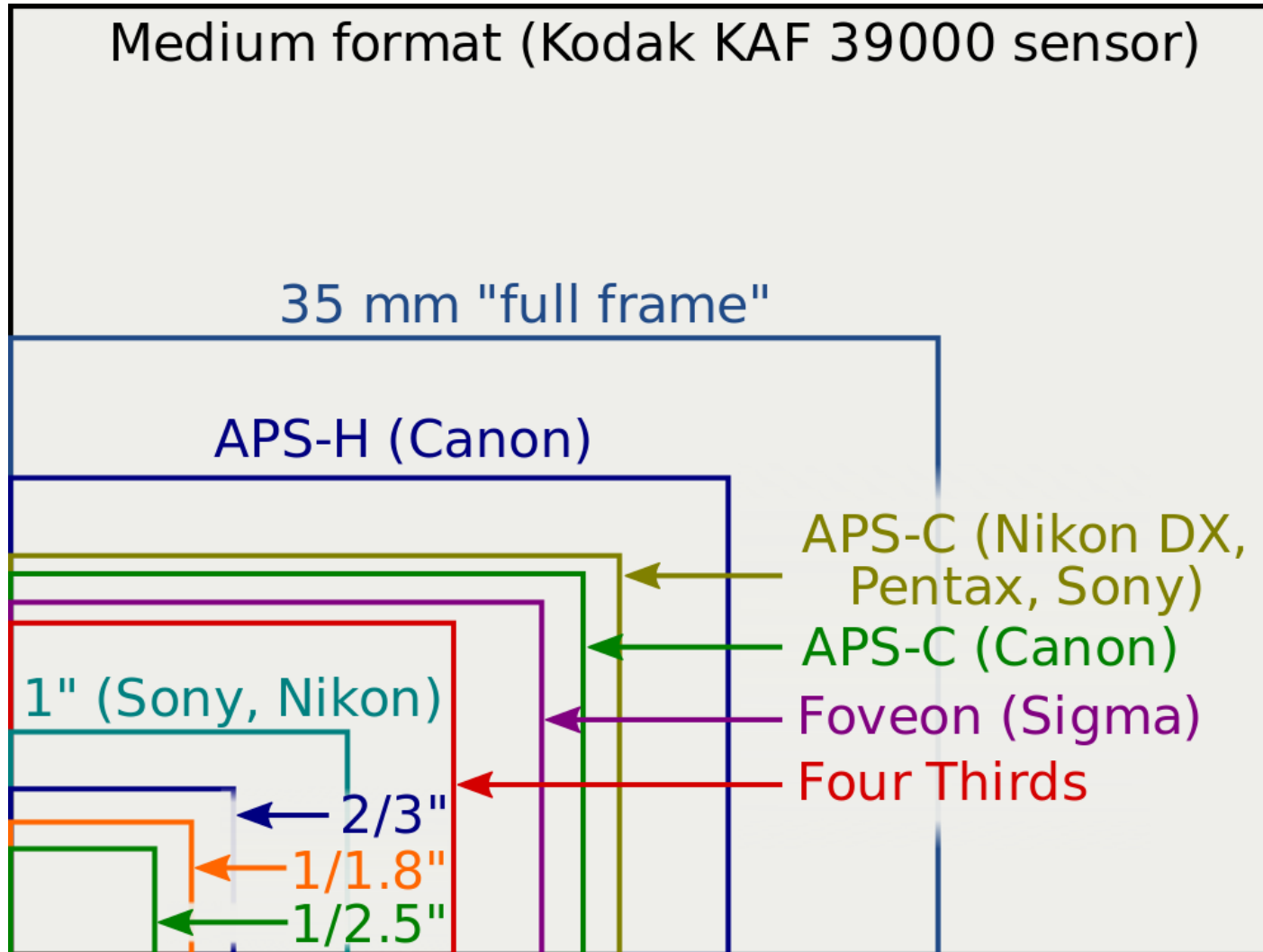
Field-of-View (FOV)



Field-of-View (FOV)

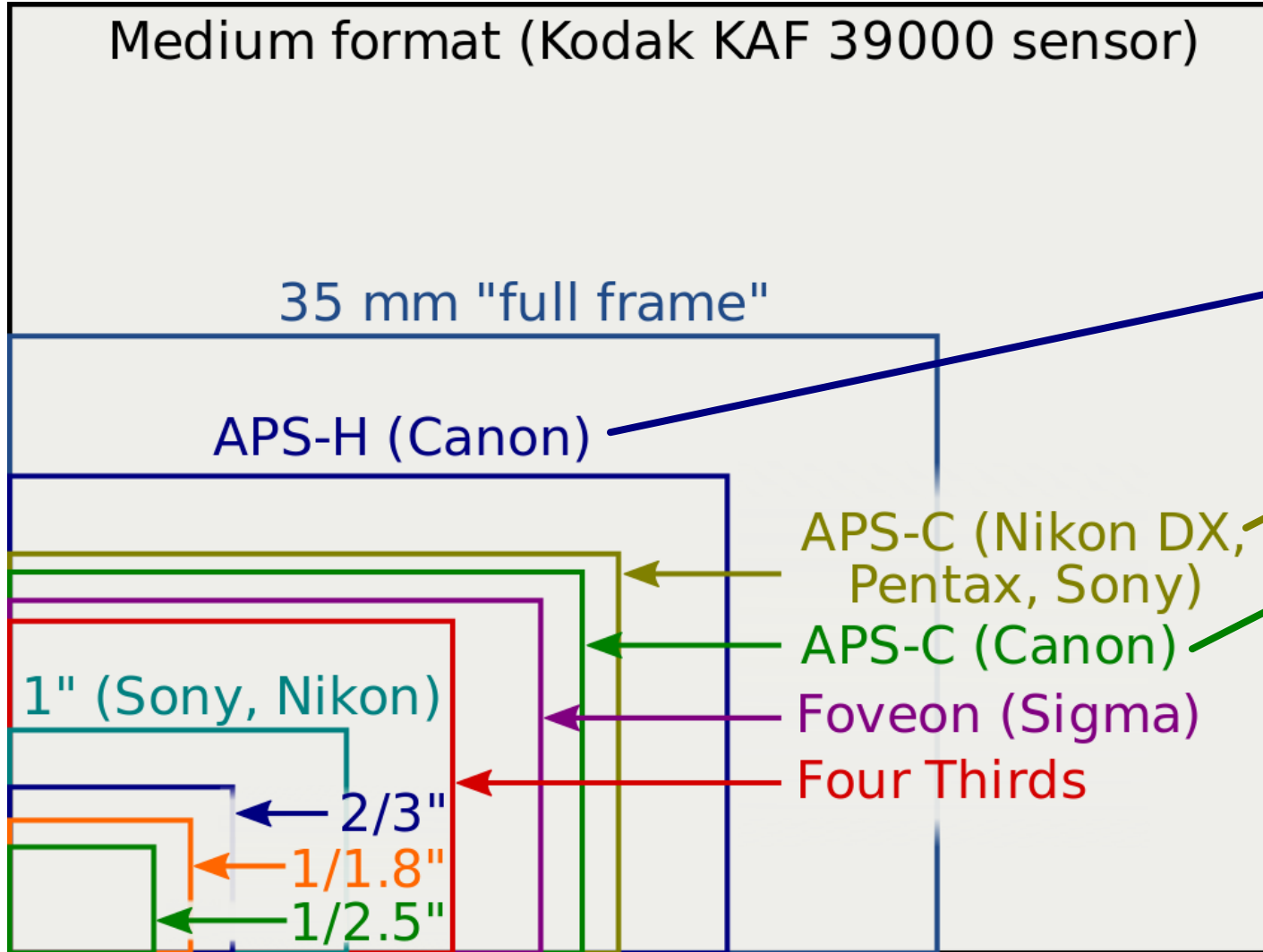


Sensor Sizes



- "Full frame" corresponds to standard film size.
- Digital sensors come in smaller formats due to manufacturing limitations (now mostly overcome).
- Lenses are often described in terms of field-of-view on film instead of focal length.
 - These descriptions are invalid when not using full-frame sensor.

Crop Factor



How much field of view is cropped when using a sensor smaller than full frame.

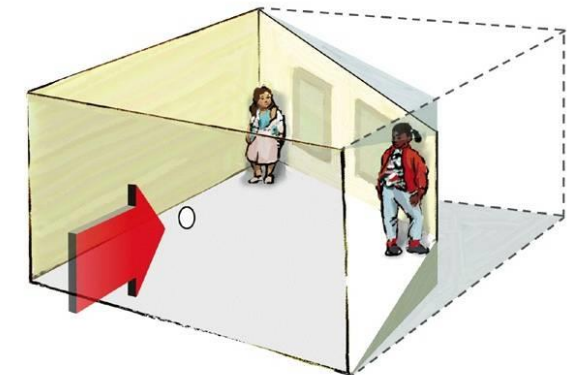
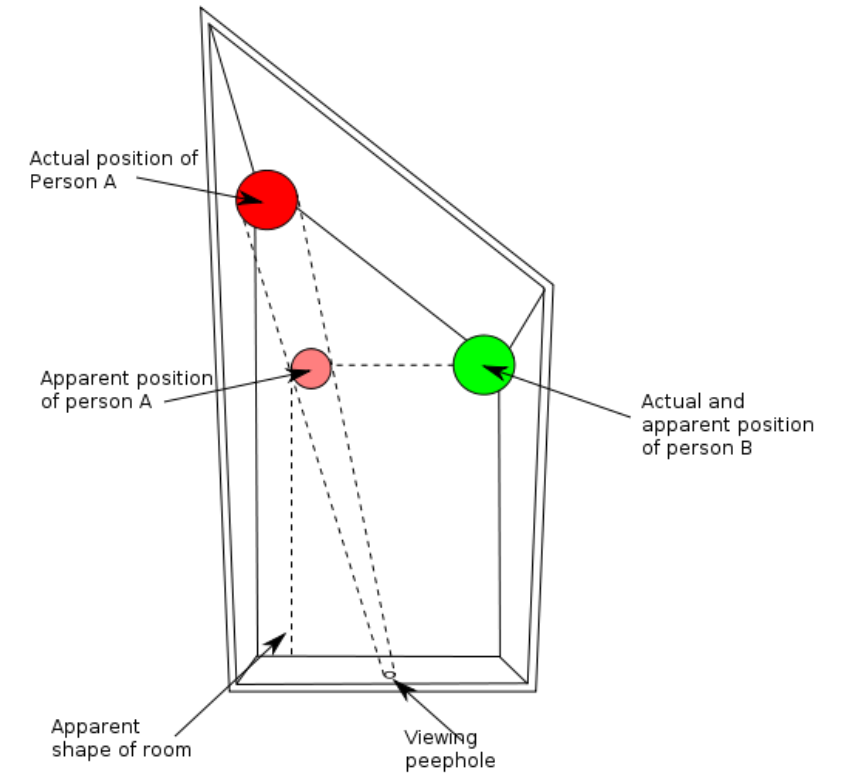
Perspective



The Ames Room Illusions



The Ames Room Illusions



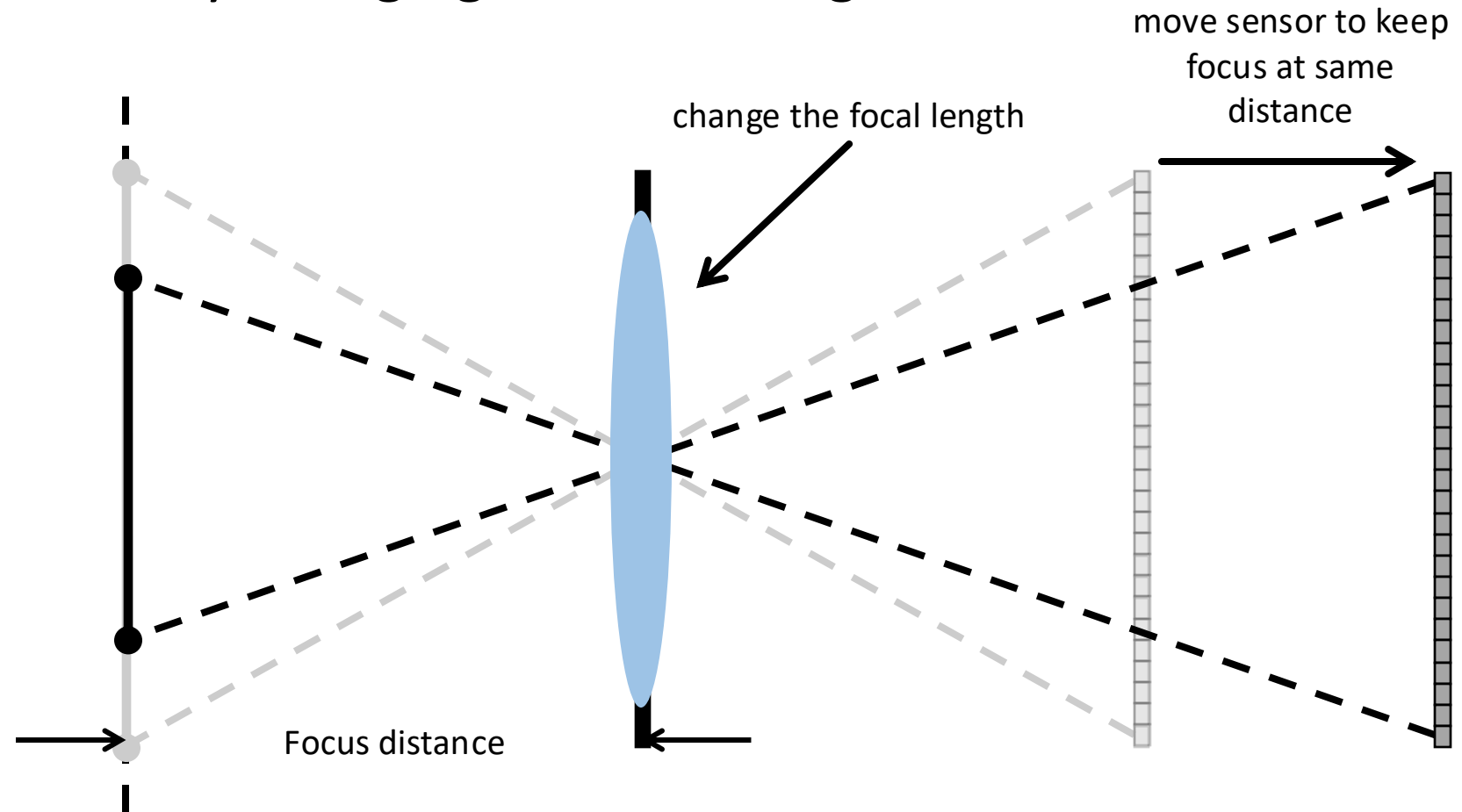
The Arrow Illusions

Prof. Kokichi Sugihara has many other amazing illusions involving perspective distortion, check them out on YouTube or on his website: <http://www.isc.meiji.ac.jp/~kokichis/>



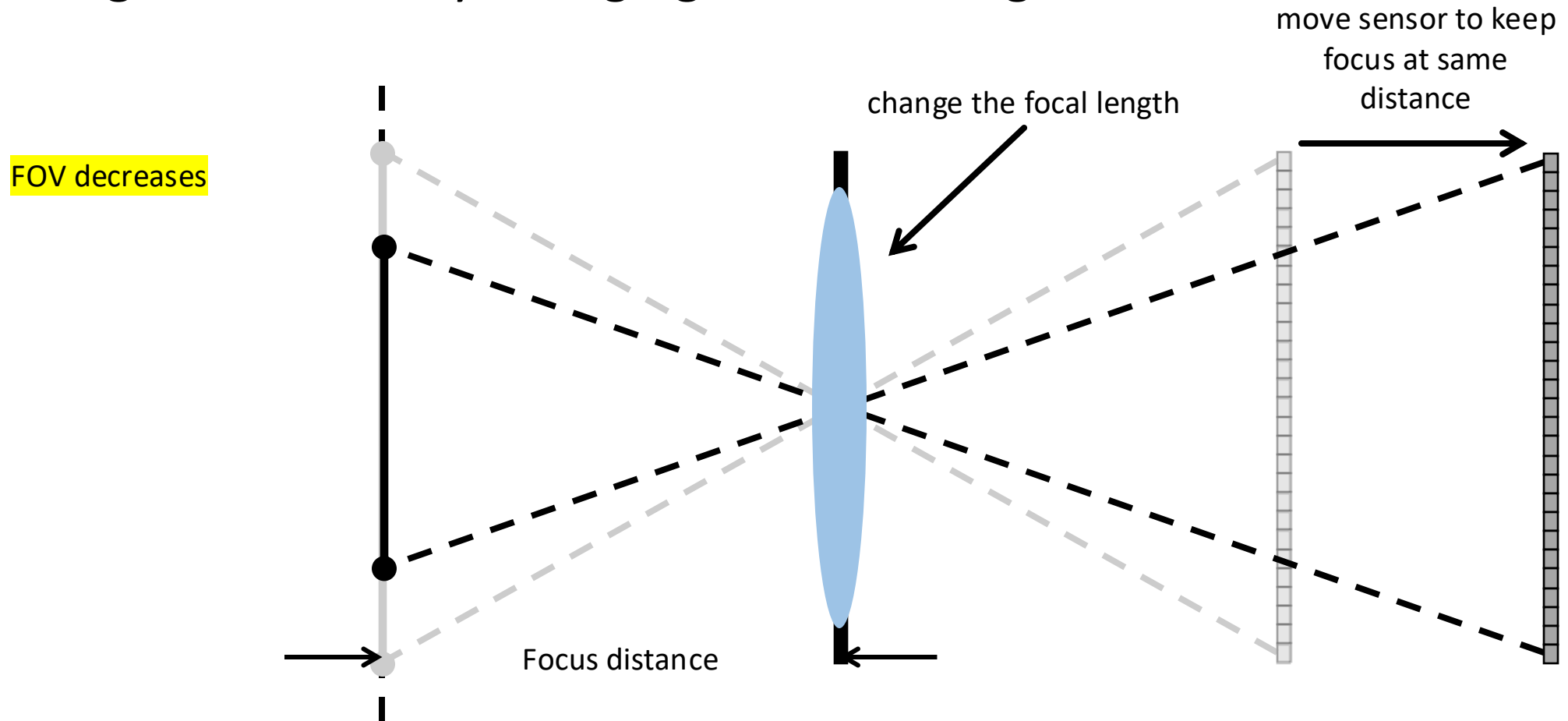
Zooming

- Zooming is achieved by changing the focal length



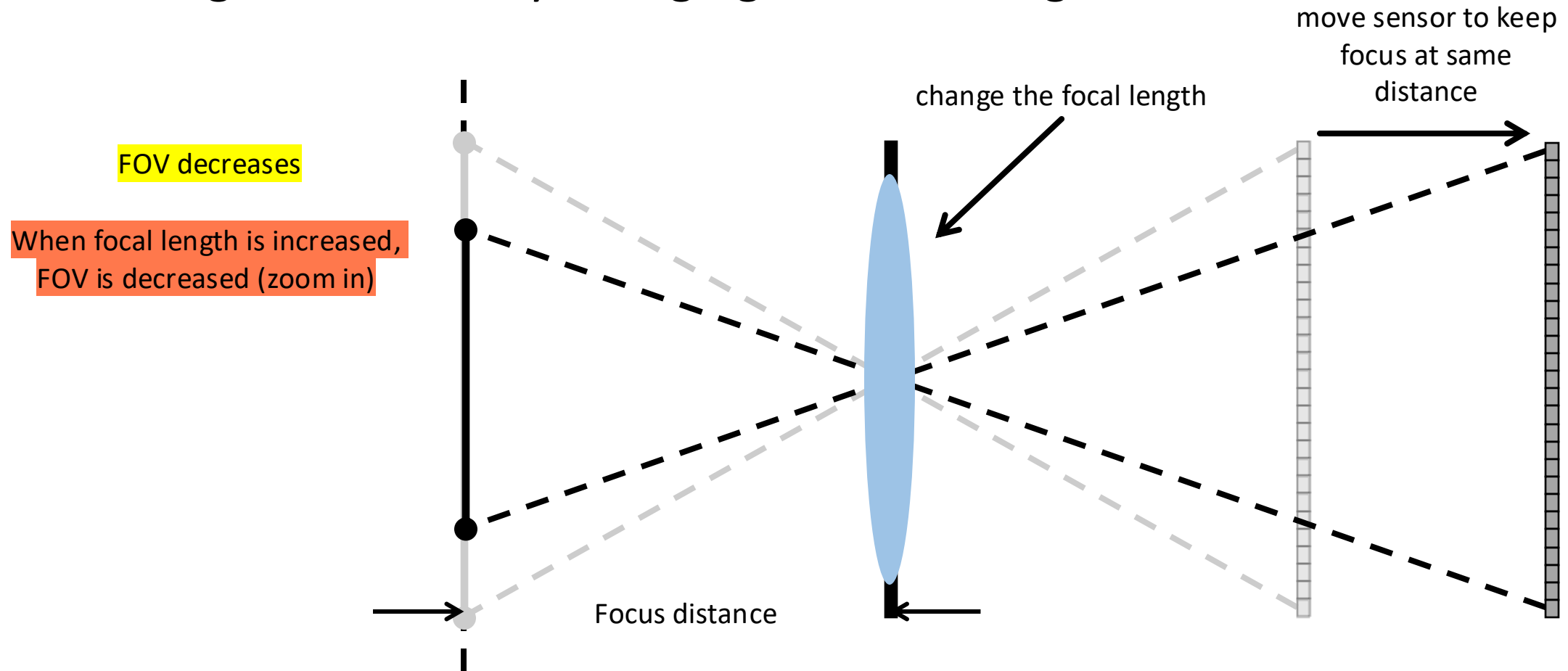
Zooming

- Zooming is achieved by changing the focal length



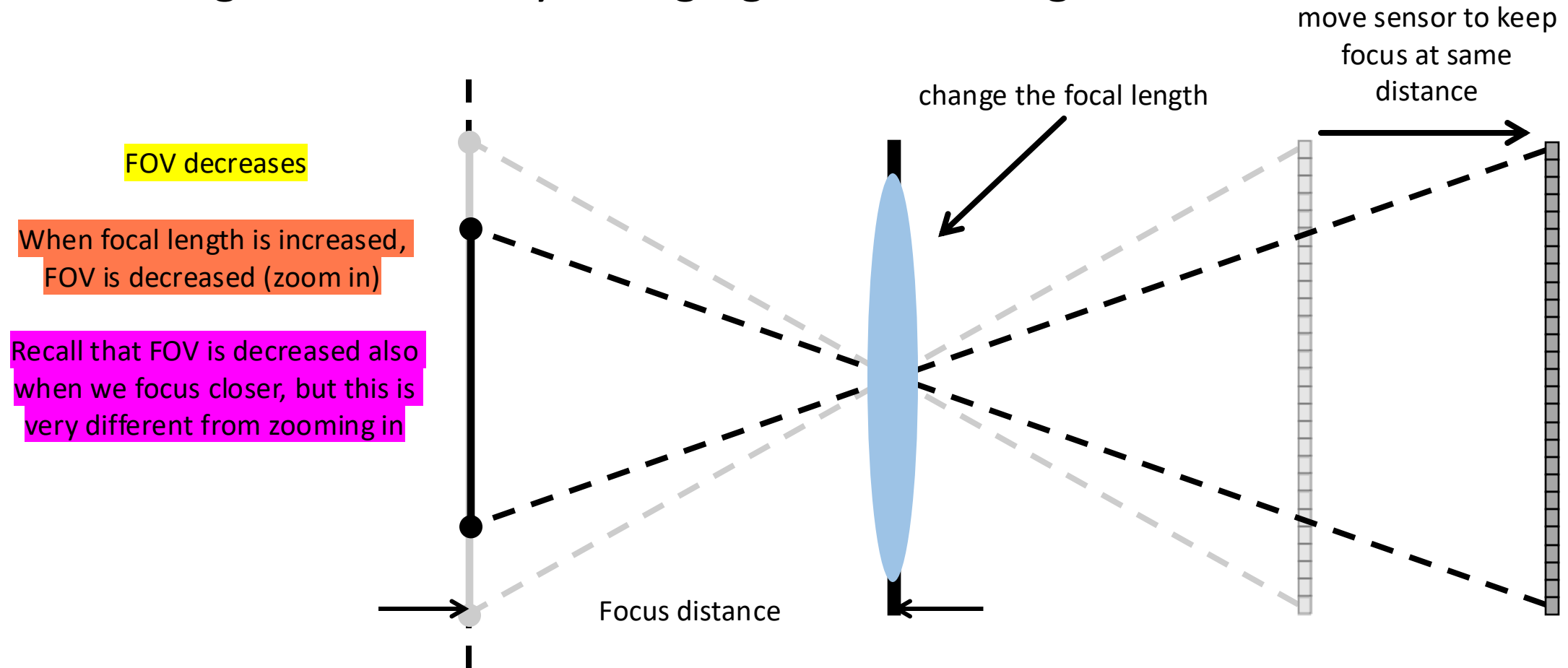
Zooming

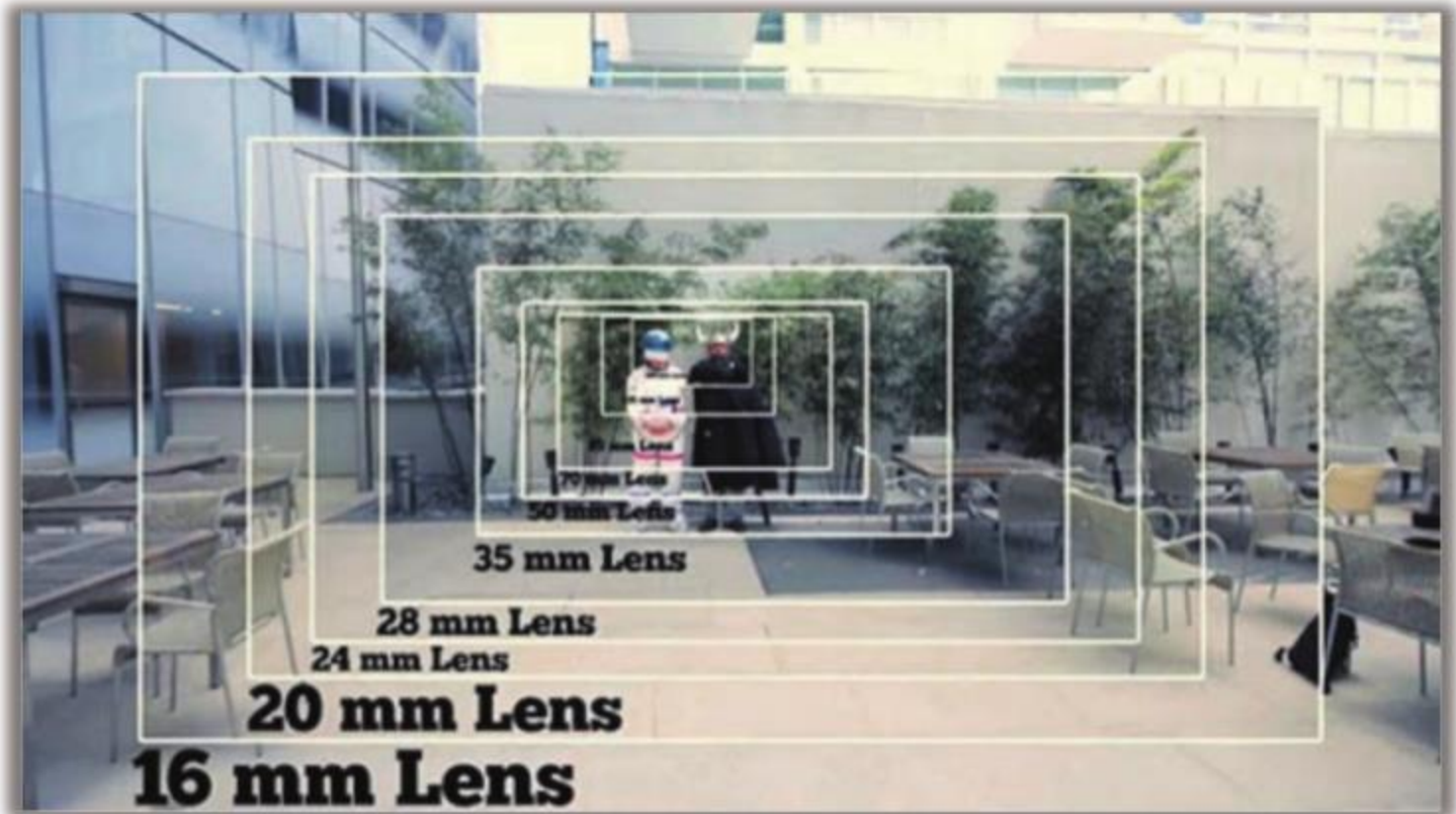
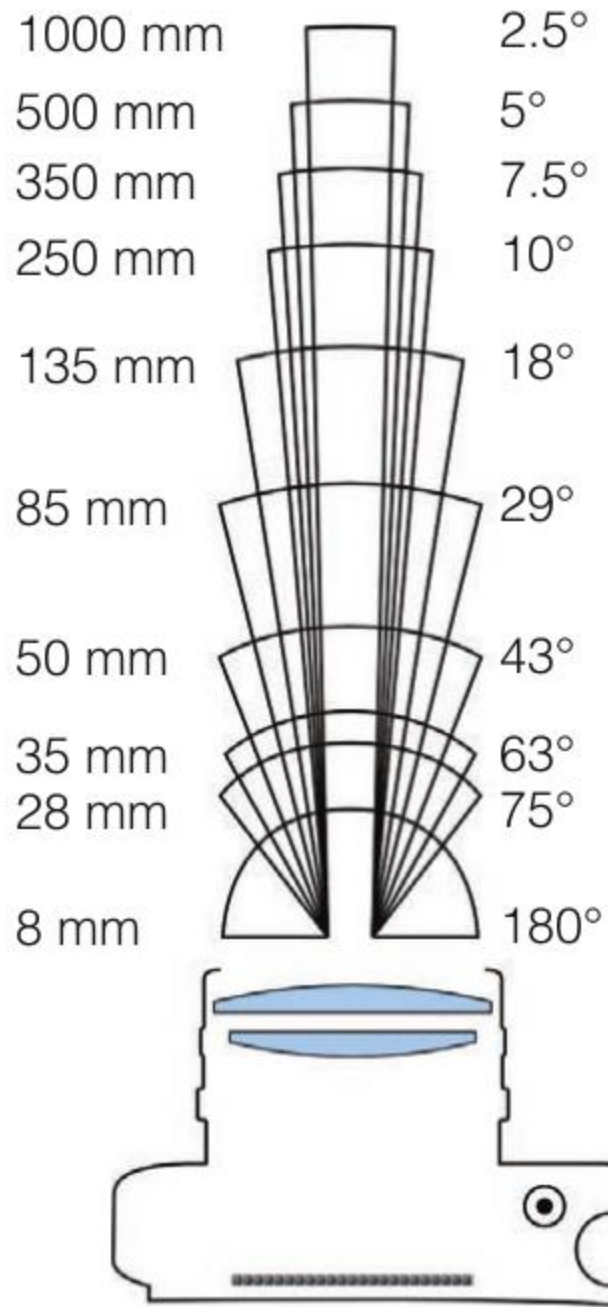
- Zooming is achieved by changing the focal length



Zooming

- Zooming is achieved by changing the focal length

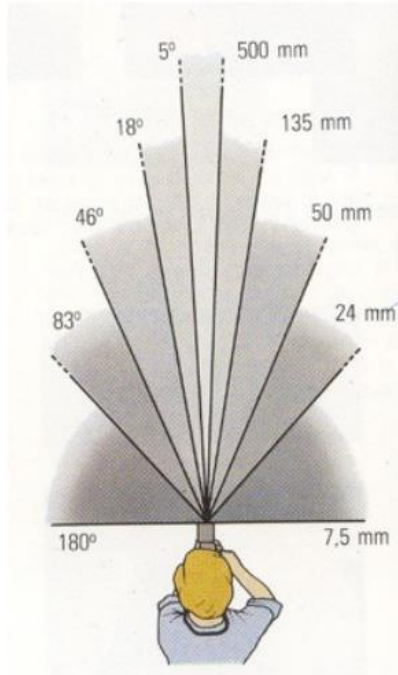




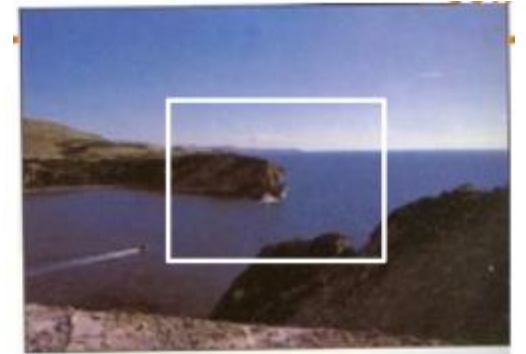
Andrew McWilliams

Field of View

- Increasing the lens focal length is similar to cropping
 - Is it *identical* cropping?



$f = 25 \text{ mm}$



$f = 50 \text{ mm}$



$f = 135 \text{ mm}$



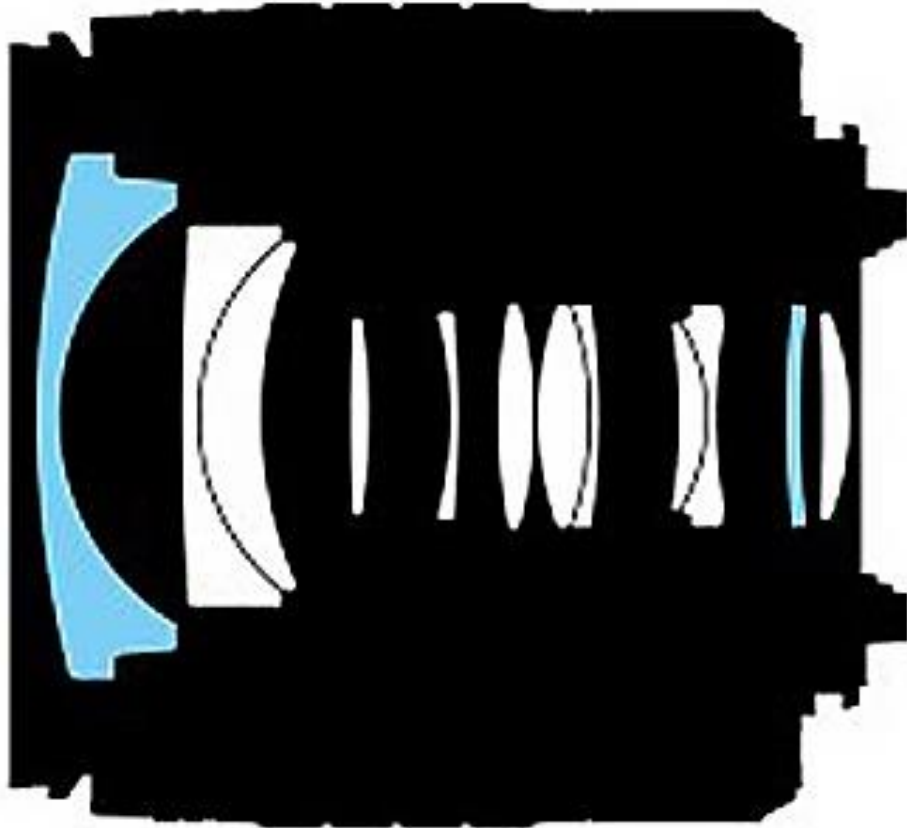
Zoom Lenses

Focus ring: controls distance of lens from sensor



Zoom ring: controls focal length of lens

Compound (Zoom) Lenses

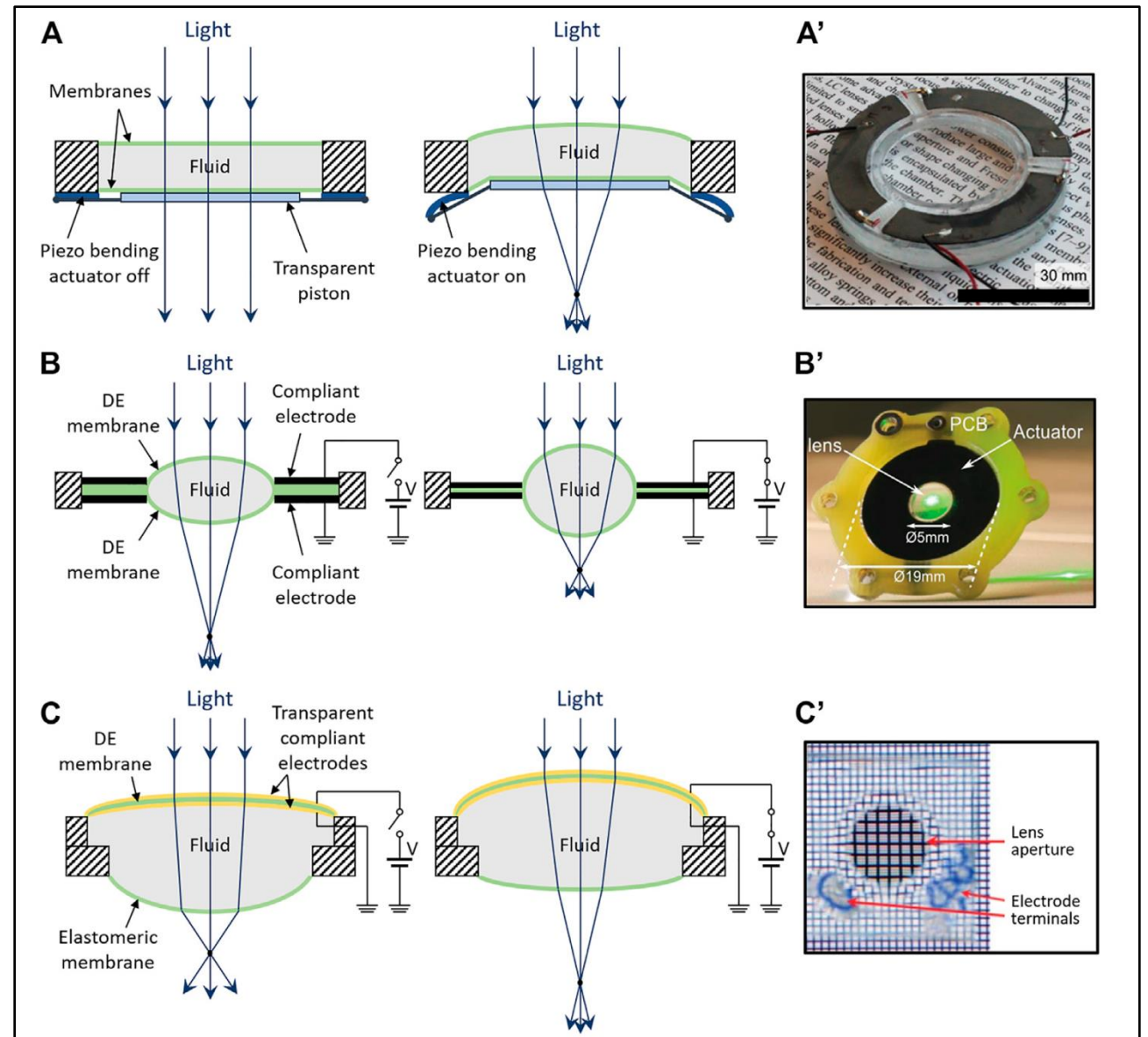


Cross-section of Nikon 18-55 mm lens

We can change the effective focal length of the overall compound lens by changing the relative placement of the individual lenses inside the lens tube.

Programmable Lenses

Use different processes (electric, magnetic, acoustic) to change the shape of a liquid lens (e.g., water droplet).



Focusing vs. Zooming

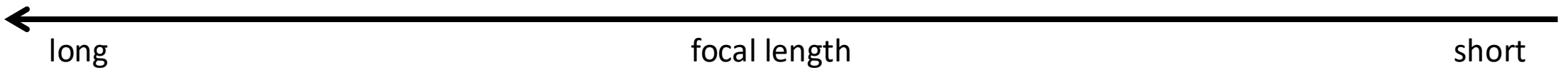
- When you turn the **focus** ring to bring lens further-away from the sensor:
 - The in-focus distance decreases (you need to get closer to object).
 - The field of view decreases (you see a smaller part of the object).
 - The magnification increases (same part of the object is bigger on sensor).
- When you turn the **zoom** ring to decrease the focal length of the lens:
 - The in-focus distance increases (you need to move away from the object).
 - The field of view increases (you see a larger part of the object).
 - The magnification decreases (same part of the object is smaller on sensor).

Focusing vs. Zooming

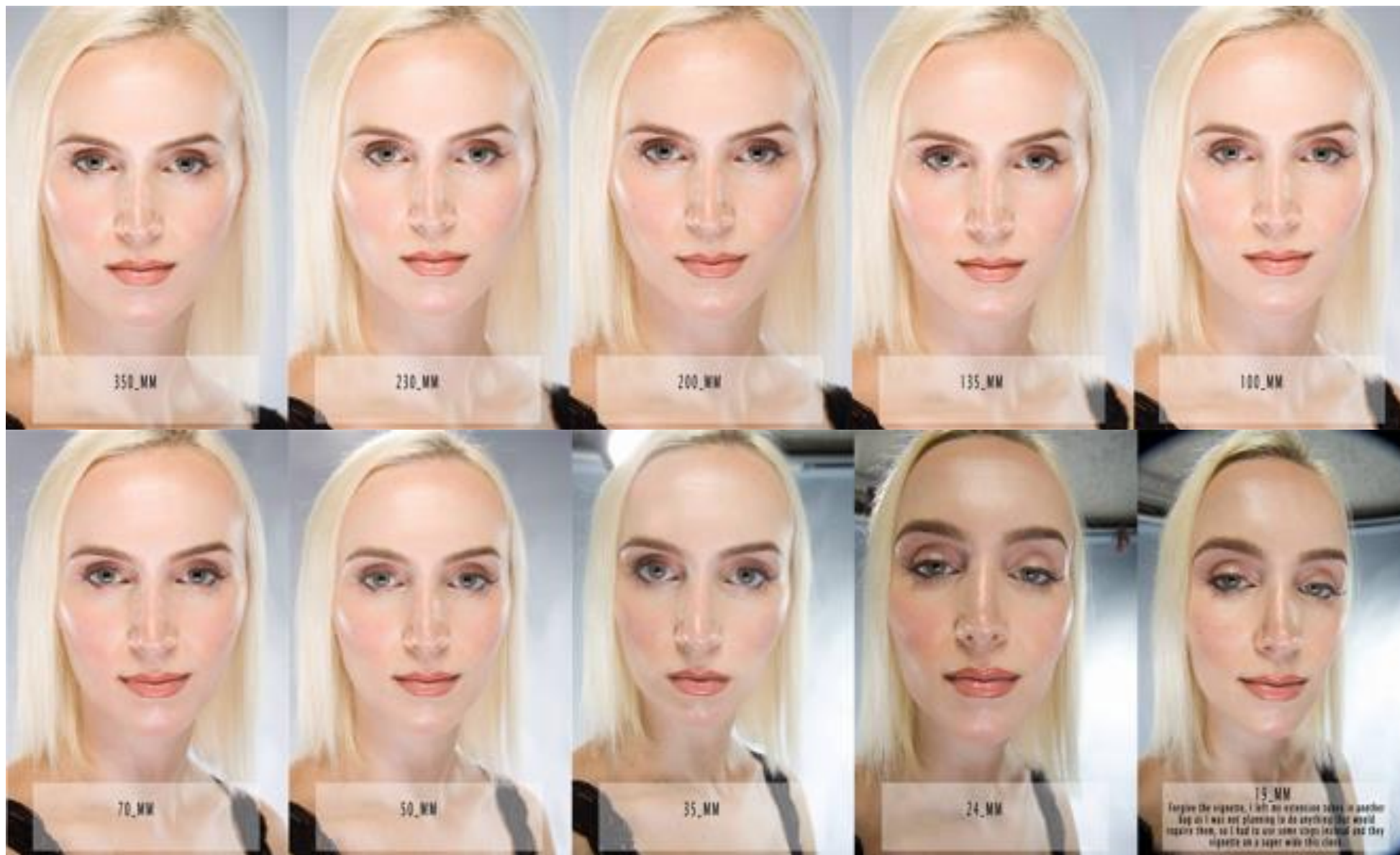
- When you turn the **focus** ring to bring lens further-away from the sensor:
 - The in-focus distance decreases (you need to get closer to object).
 - The field of view decreases (you see a smaller part of the object).
 - The magnification increases (same part of the object is bigger on sensor).
- When you turn the **zoom** ring to decrease the focal length of the lens:
 - The in-focus distance increases (you need to move away from the object).
 - The field of view increases (you see a larger part of the object).
 - The magnification decreases (same part of the object is smaller on sensor).

We can use both focus and zoom to cancel out their effects.

Perspective Distortion



Perspective Distortion

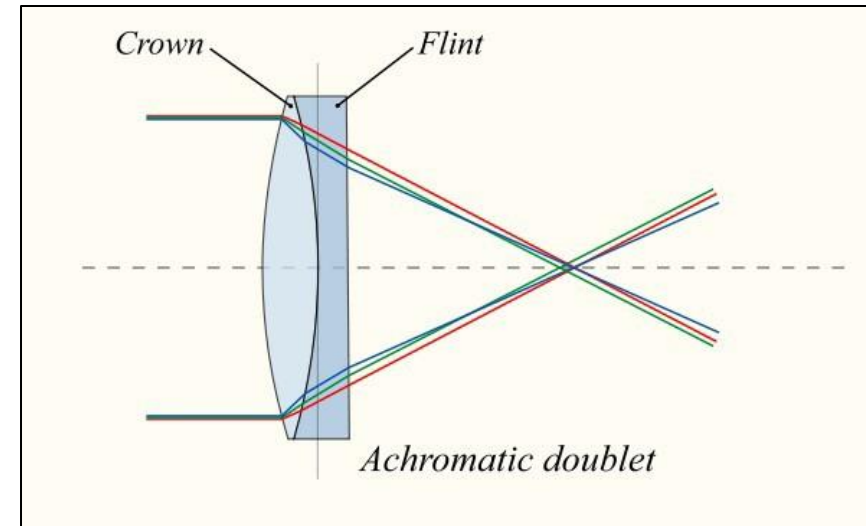
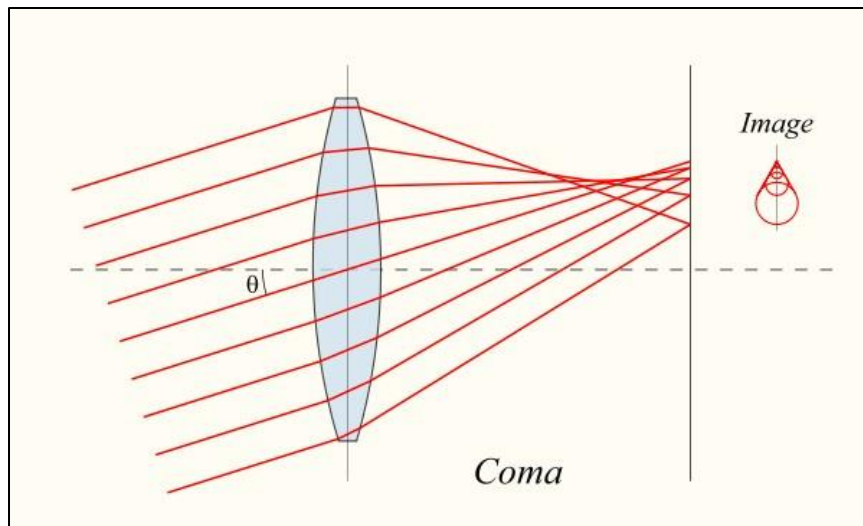
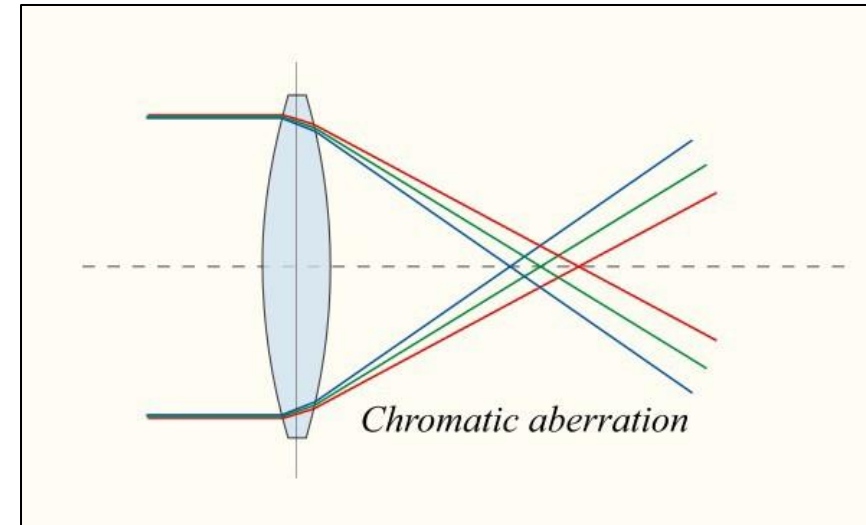
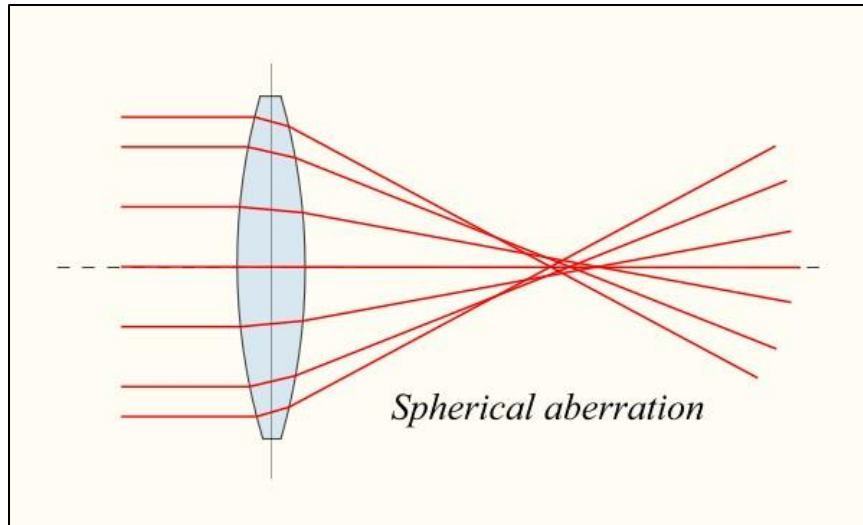


Vertigo Effect

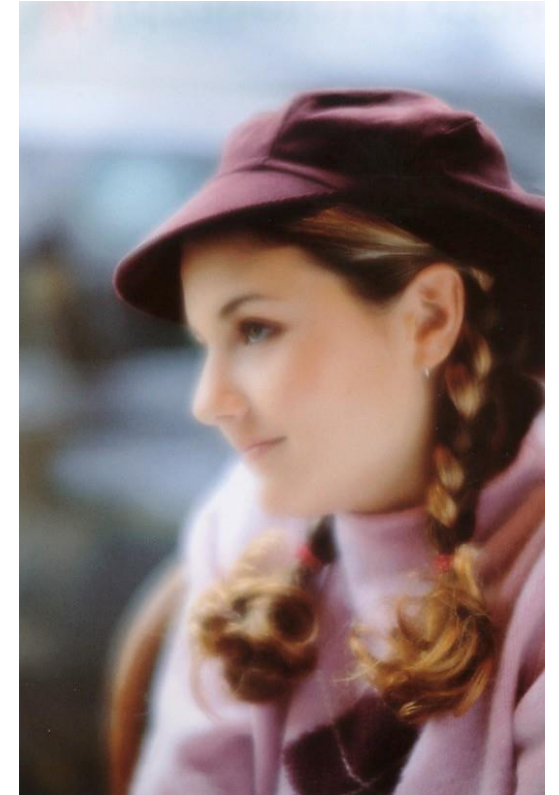
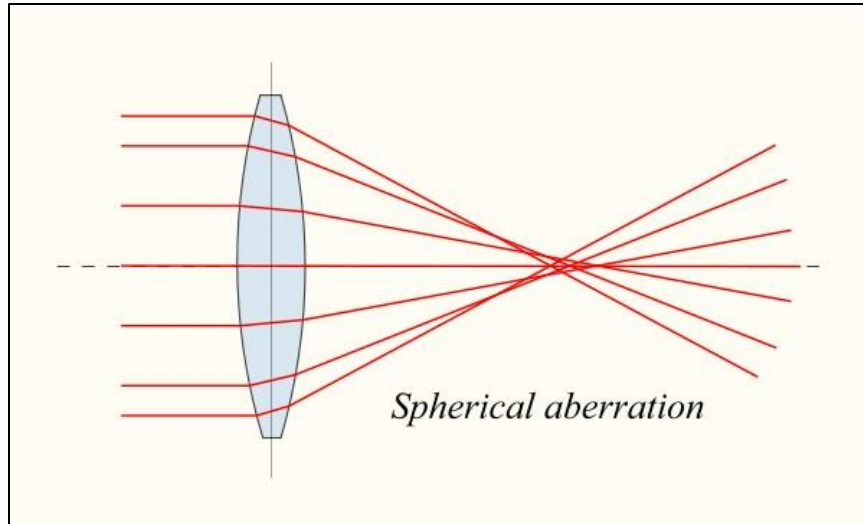
- Dolly Zoom – Named after Alfred Hitchcock's movie



Lens Aberrations



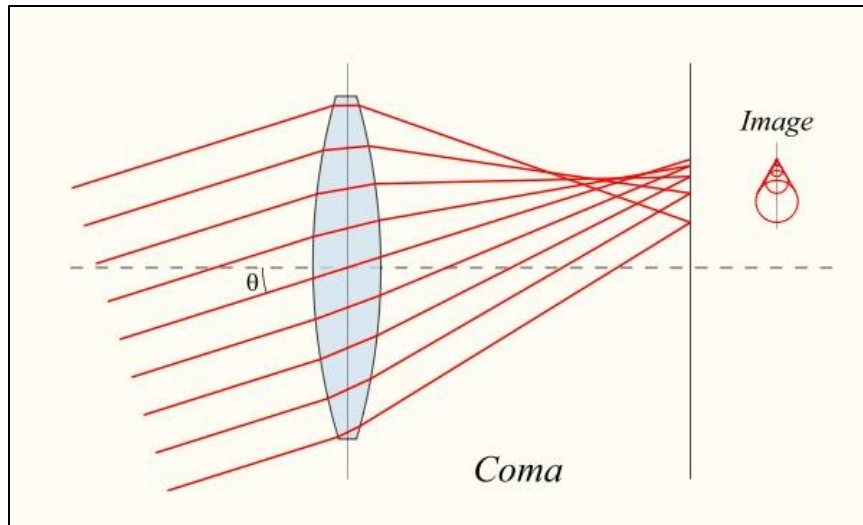
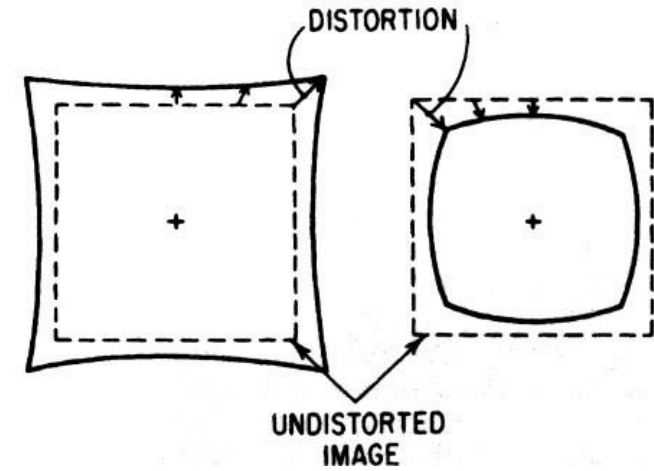
Lens Aberrations



Deviation from ideal thin-lens leads to
imperfect focus

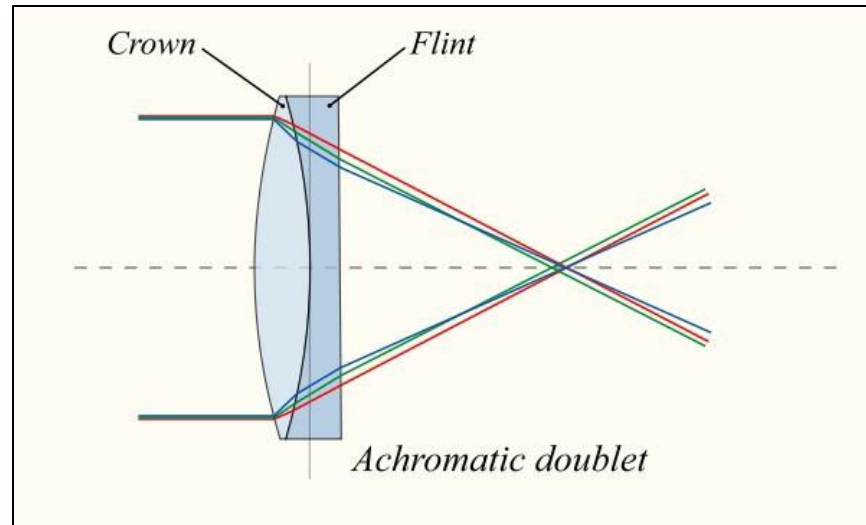
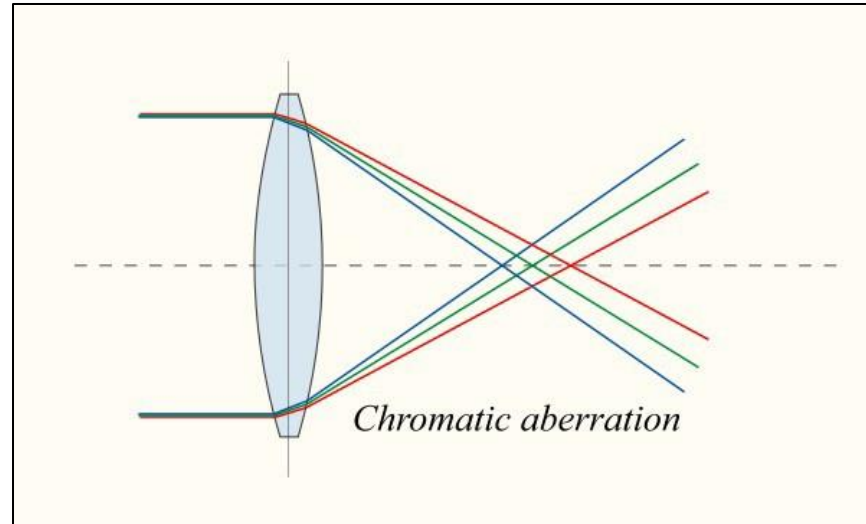
Lens Aberrations

Oblique aberrations that appear only as we move away from the center of the lens. Examples are astigmatism and field curvature.



Lens Aberrations

Focal length varies with wavelength, or one lens cancels out dispersion of the other lens. Also leads to imperfect focus.



Fastest Lens Ever Made?

- Zeiss 50 mm f/0.7 Planar lens
- Developed for NASA Apollo Missions
- Stanley Kubrick used it to shoot Barry Lyndon under only candlelight

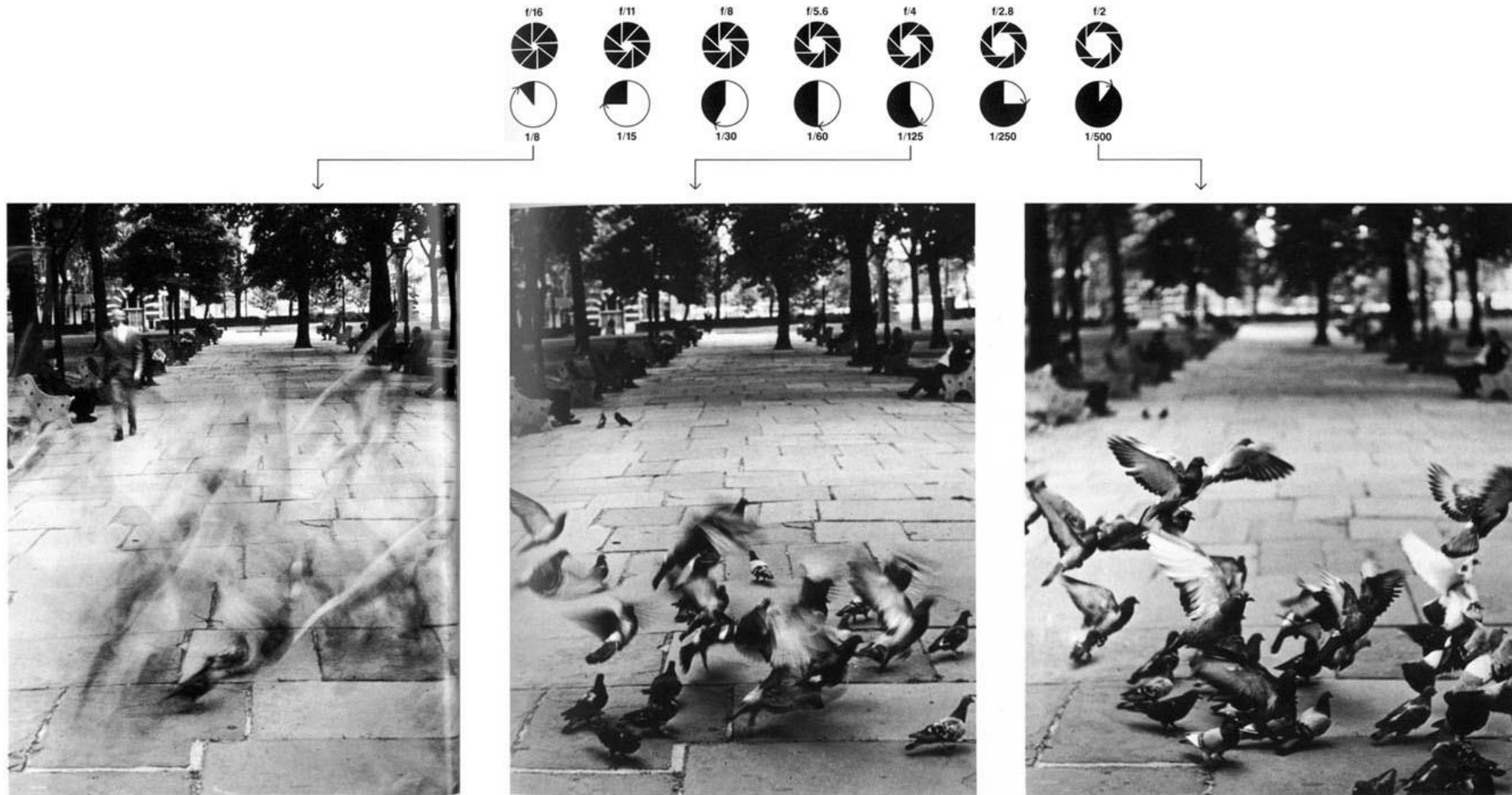


Summary

- Lenses
- Focal length
- Aperture size
- Depth of field
- Zooming
- Aberrations

Other Considerations in Image Formation

Depth-of-Field & Motion Blur



Exposure (Shutter Speed)

- Exposure = time the shutter remains open (e.g., 1/250 second)



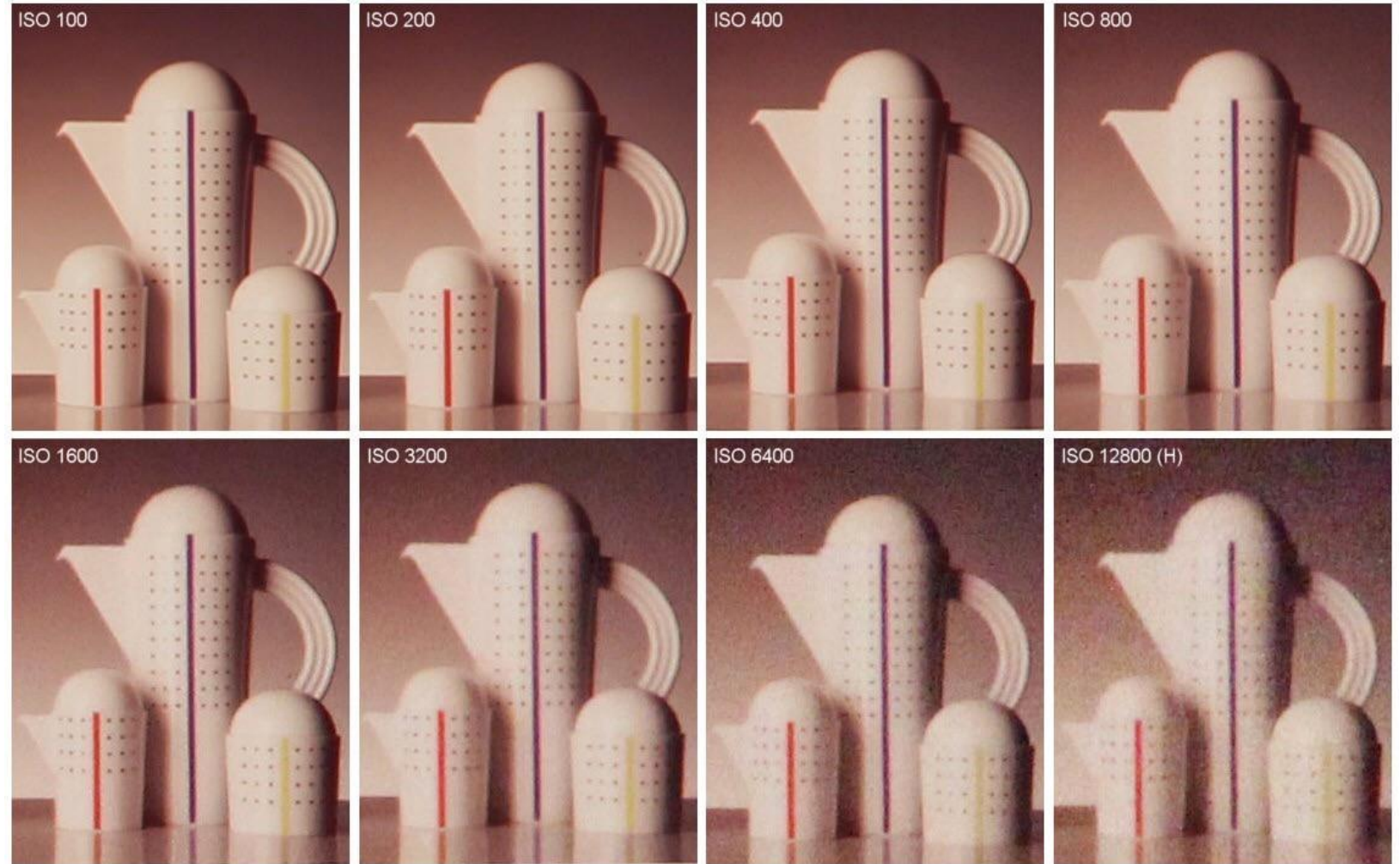
$\frac{1}{4}$ sec, f/3.3



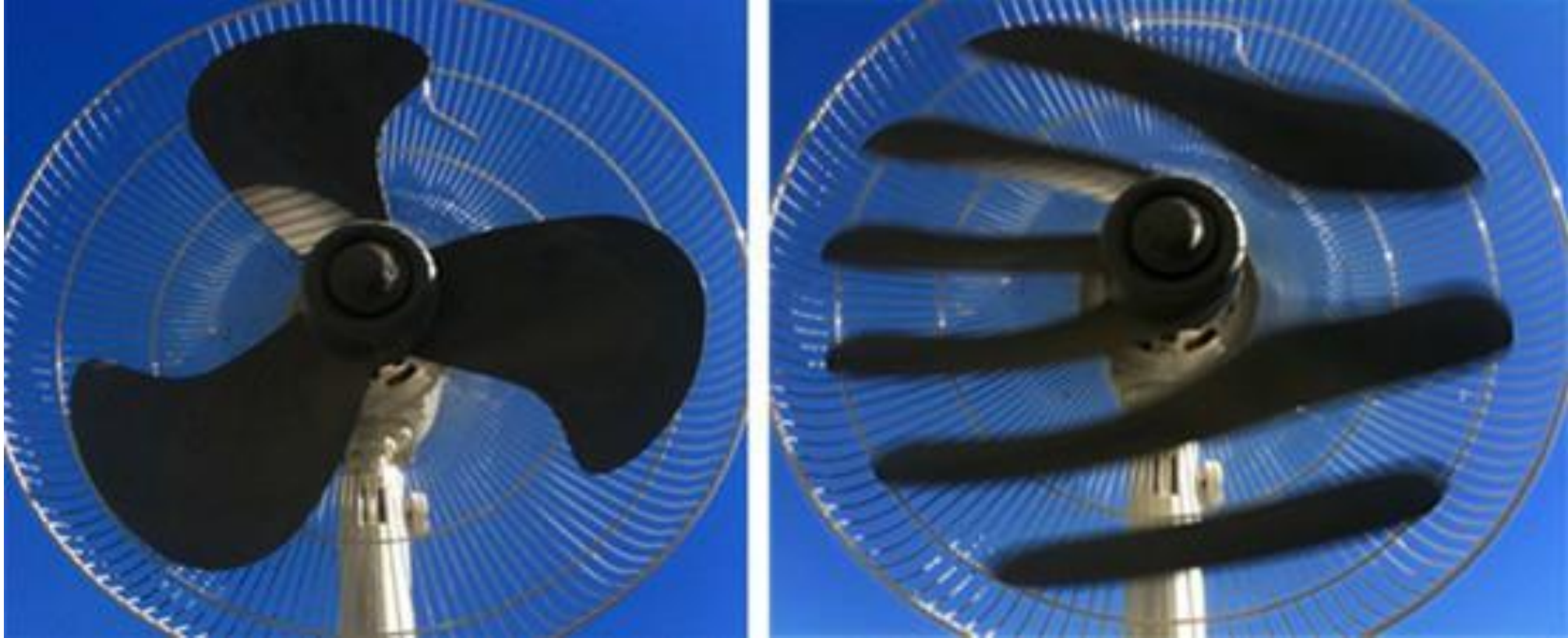
2 sec, f/6.3

ISO (Film Speed)

- Sensor sensitivity
- Analog gain applied before analog-to-digital conversion



Global vs. Rolling *Shutter*

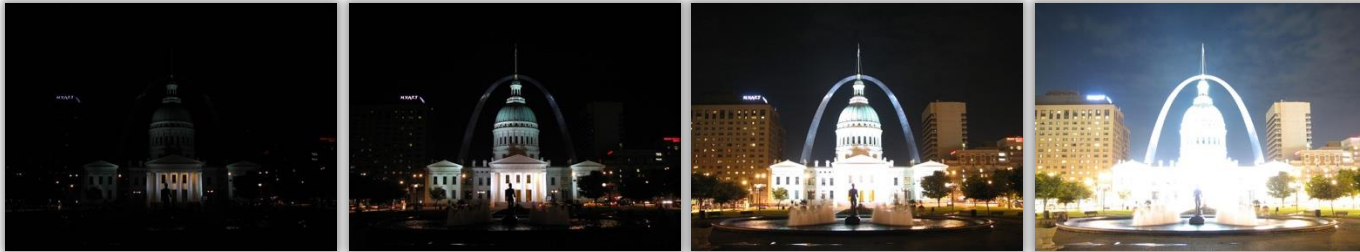


All sensor pixels are exposed at the same time

Row-by-row readout. Shorter pixel exposure times and motion artifacts

Dynamic Range

- Ratio between largest and smallest possible value
- Bit depth
 - The number of bits used to store raw pixels



Kevin McCoy

High-dynamic range

