Image Interpolation

Computational Photography (CSCI 3240U)

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How do we resize images?

Original

Upscaling

Downscaling

Let’s consider a 1D image

7	4	3

We want to increase its width by a factor of 2

Let’s consider a 1D image

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Upscaling

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Resampling pixel locations

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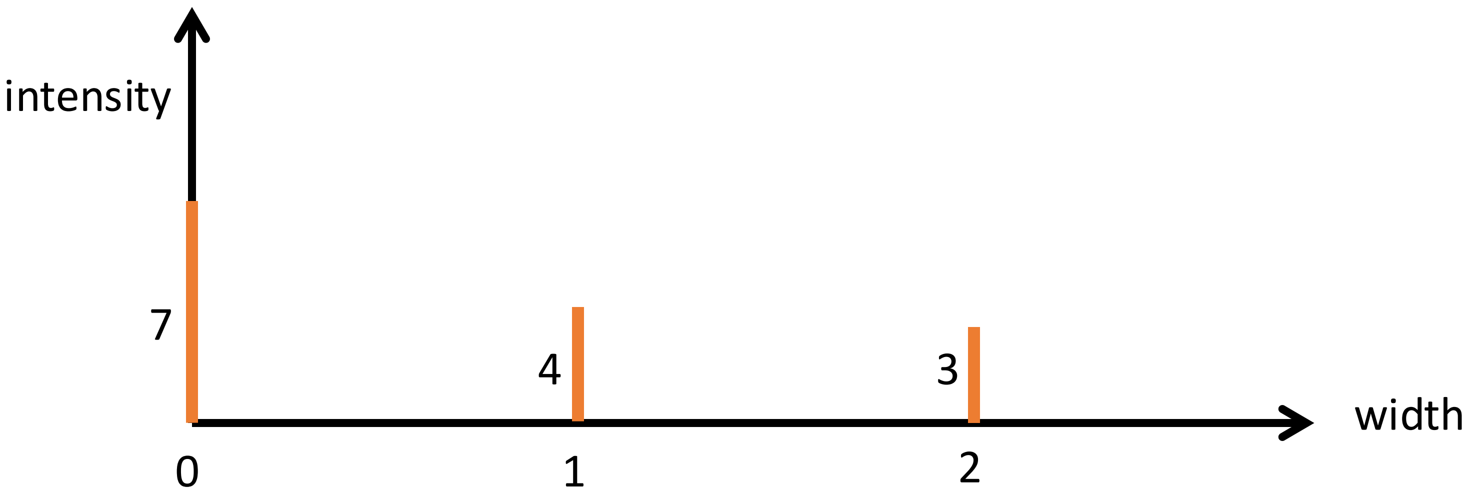
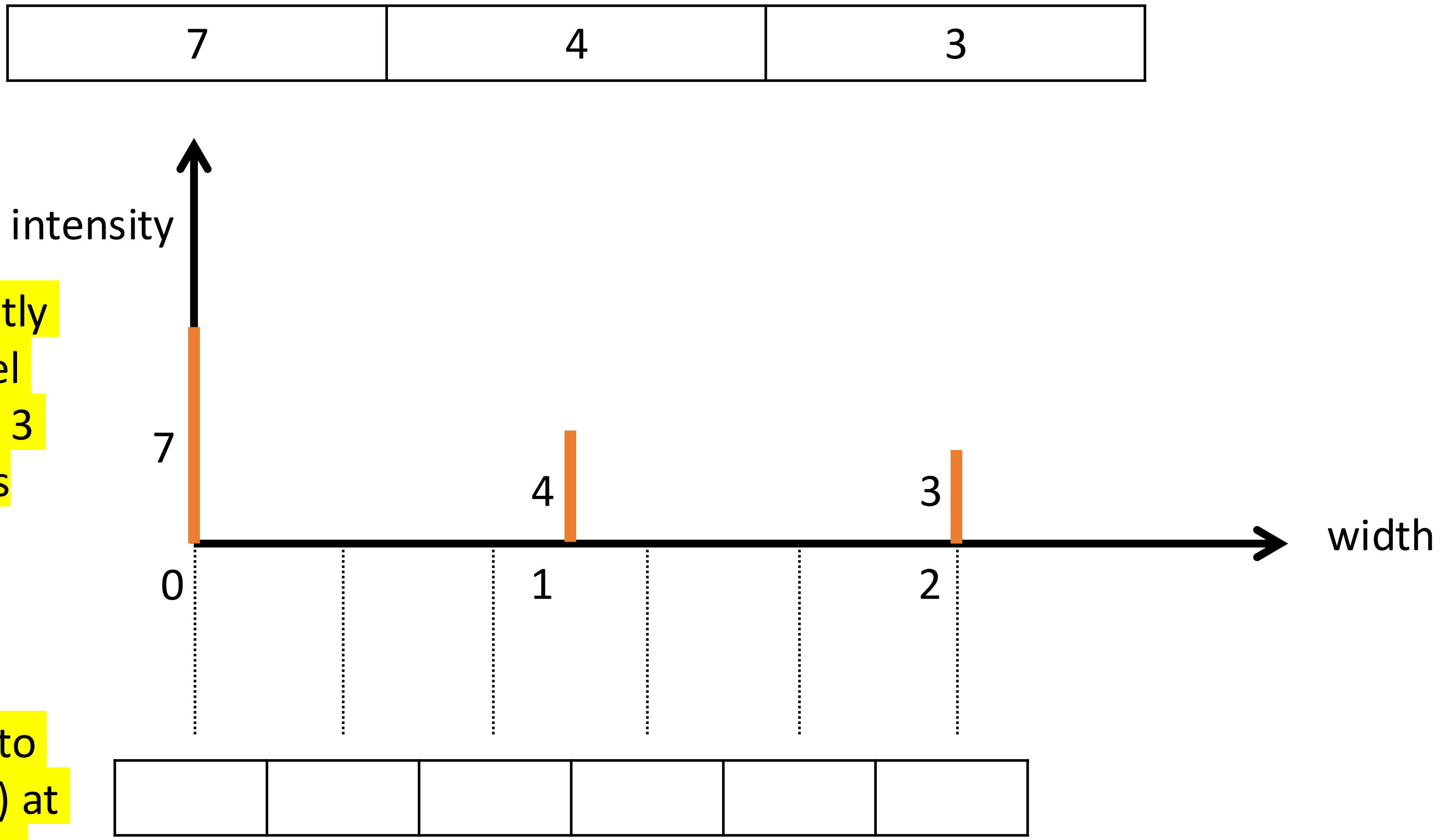


We currently have pixel values for 3 locations

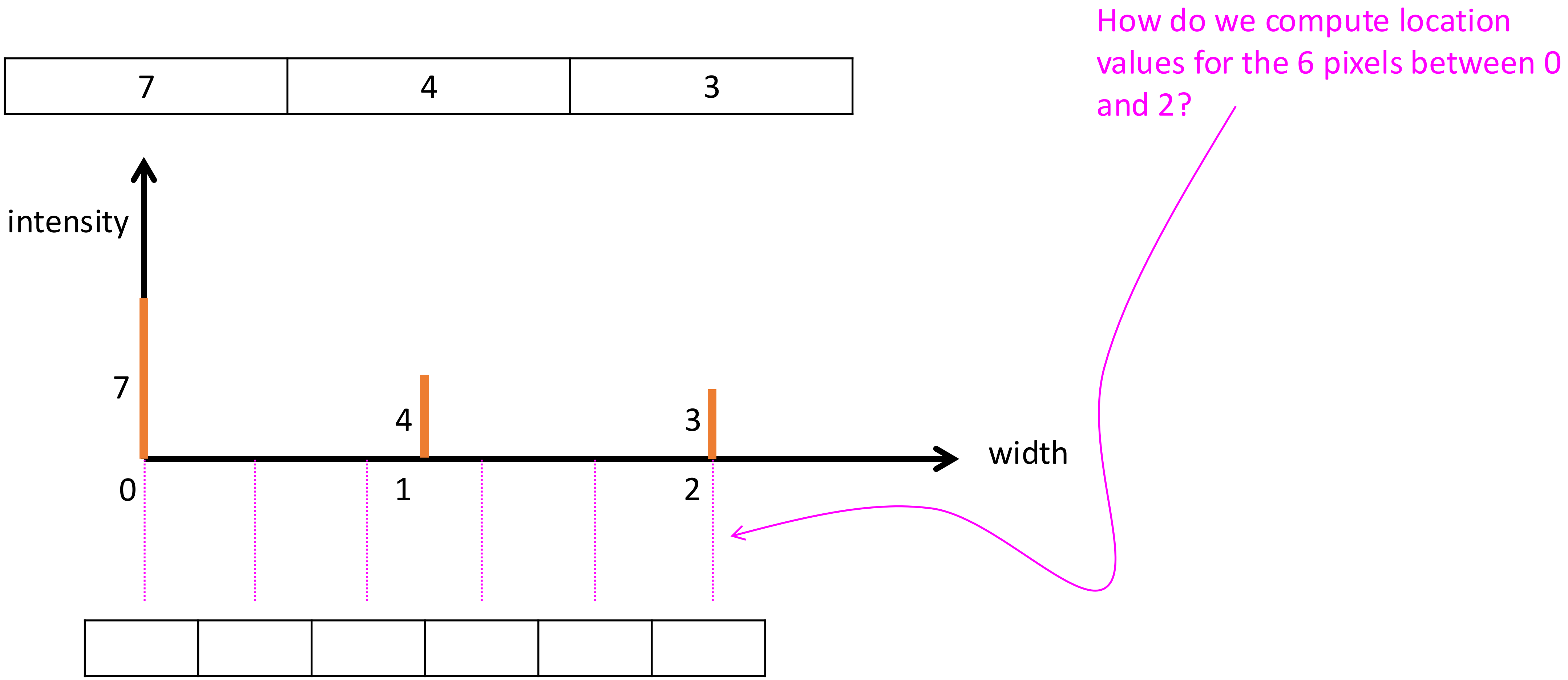
Resampling pixel locations



We currently have pixel values for 3 locations

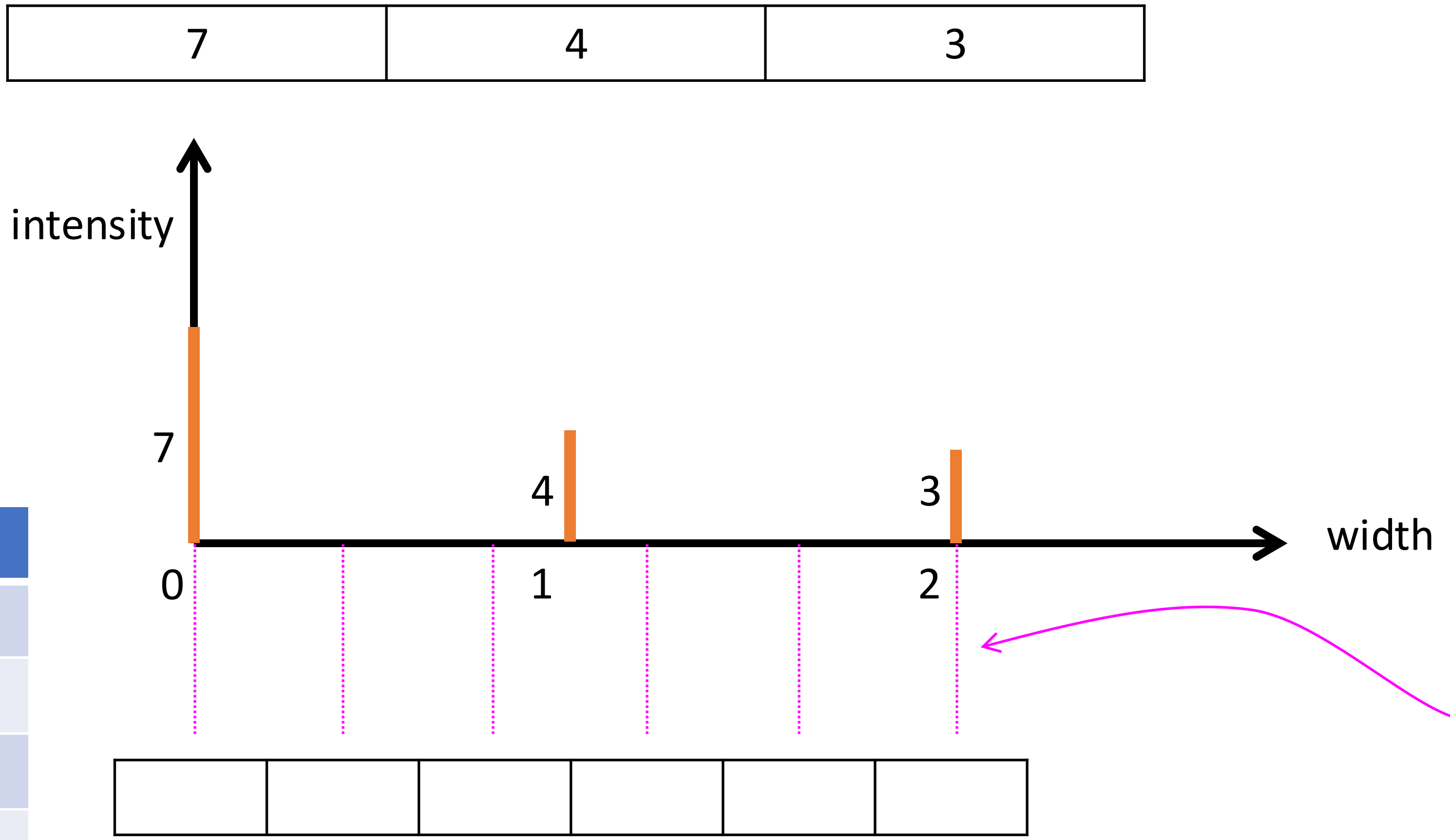
We need to (re-sample) at 6 locations

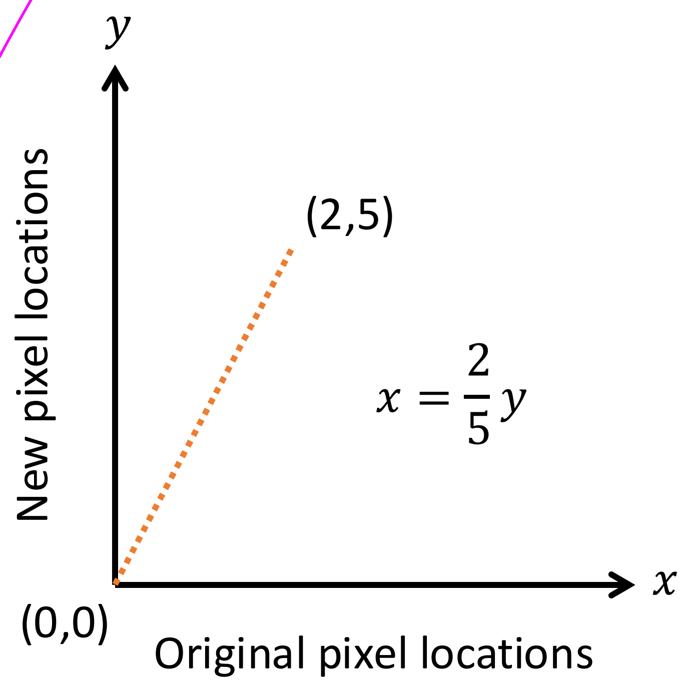
Resampling pixel locations



Resampling pixel locations

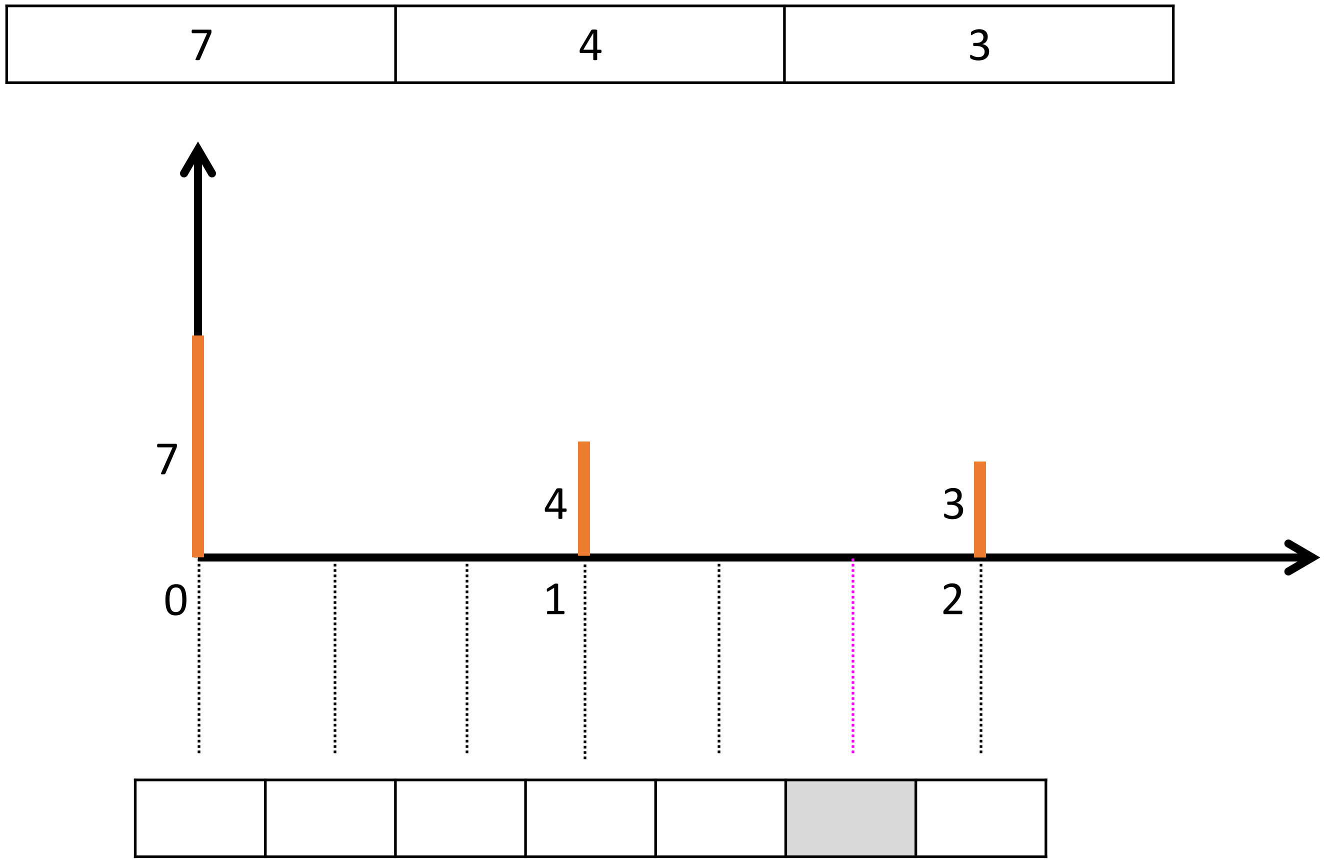
How do we compute location values for the 6 pixels between 0 and 2?





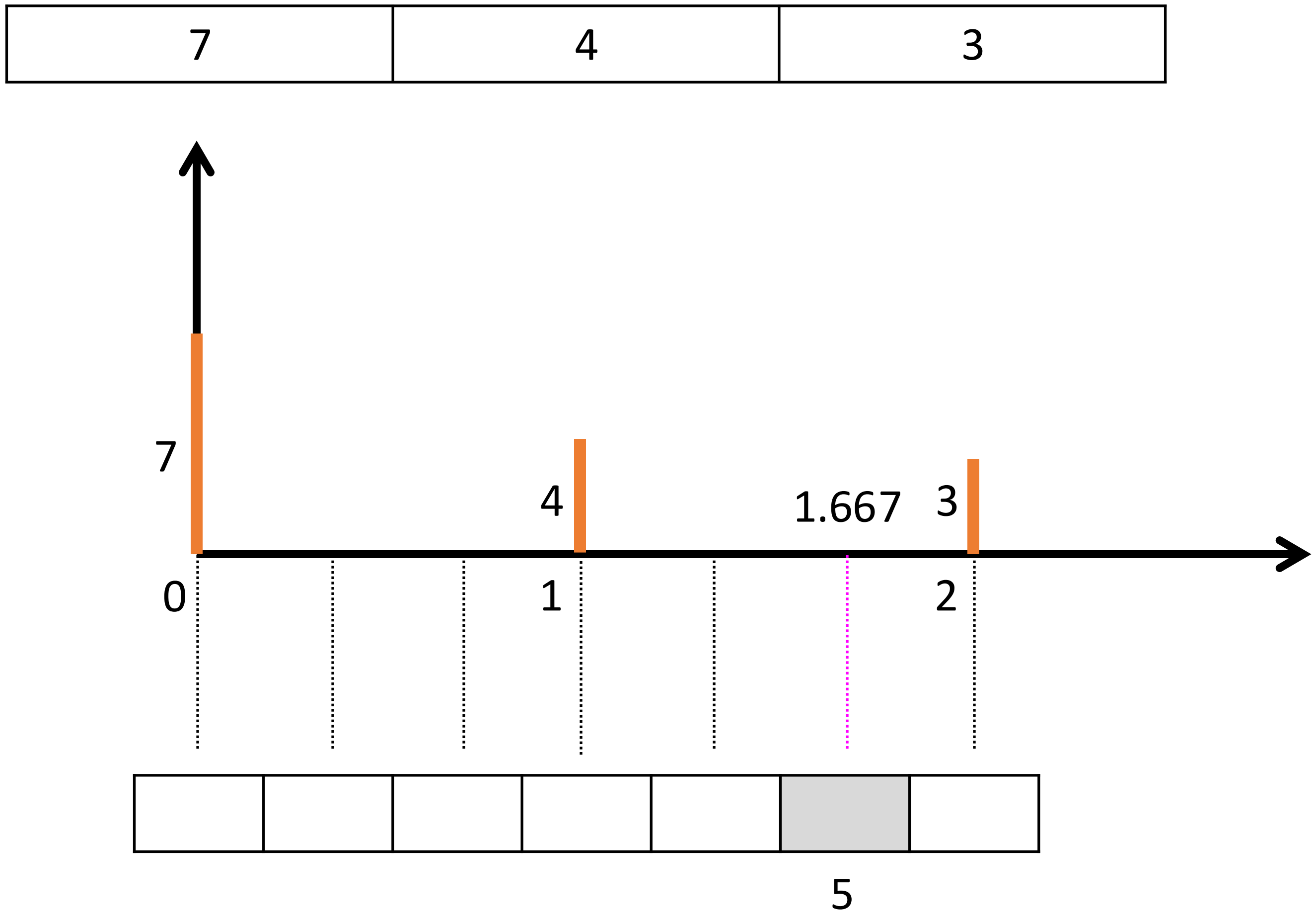
y	x
0	0
1	$2/5$
2	$4/5$
3	$6/5$
4	$8/5$
5	1

Resampling pixel locations



What is the location (between 0 and 2) of the shaded pixel?

Resampling pixel locations



What is the location (between 0 and 2) of the shaded pixel?

Given

Last pixel location in original image = 2
Last pixel location in resulting image = 6

Use the following relationship (that we developed in previous slides):

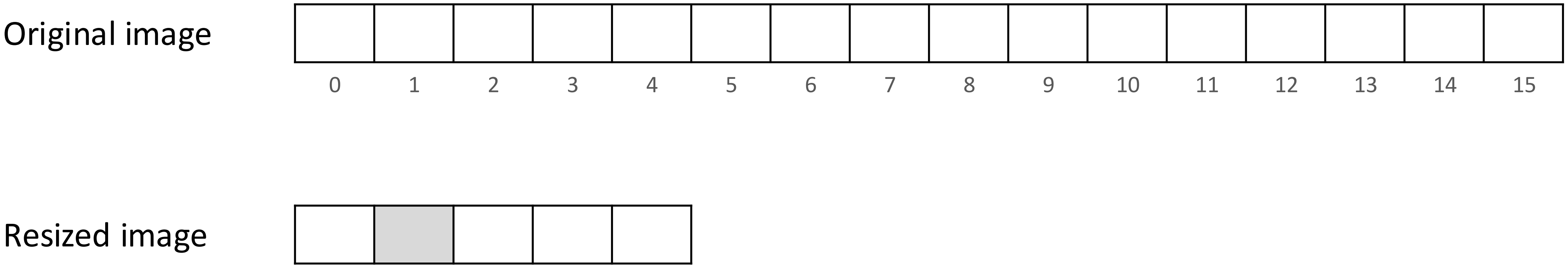
$$x = \frac{2}{6} y$$

Sample location is

$$x = \frac{2}{6}(5) = 1.667$$

Resampling pixel locations

Consider a 16-pixel 1D image. You are asked to resize it to a 5-pixel 1D image. What is the location of pixel 2 (between 0 and 15) of the new image?



Downscaling

Resampling pixel locations

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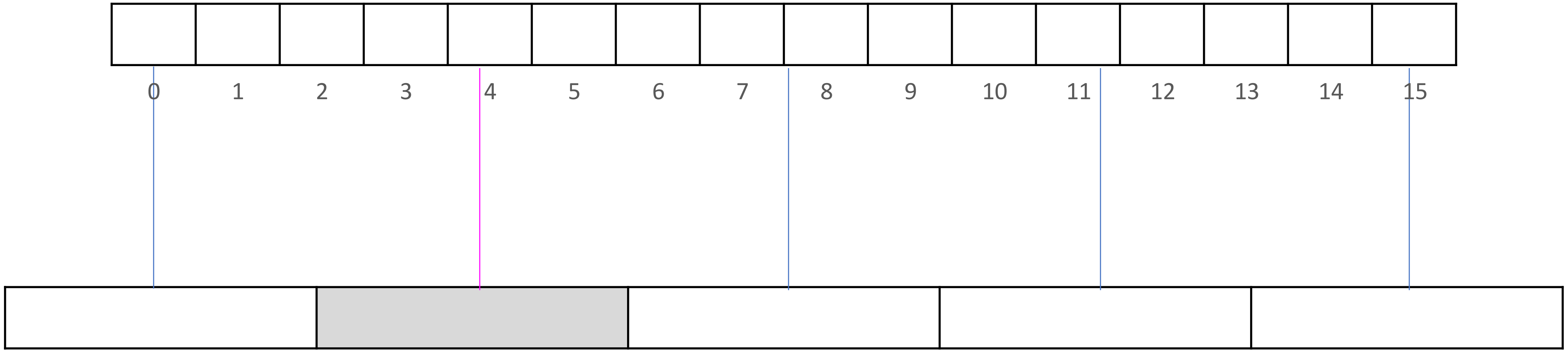


Given

Last pixel location in original image = 15
Last pixel location in resulting image = 4

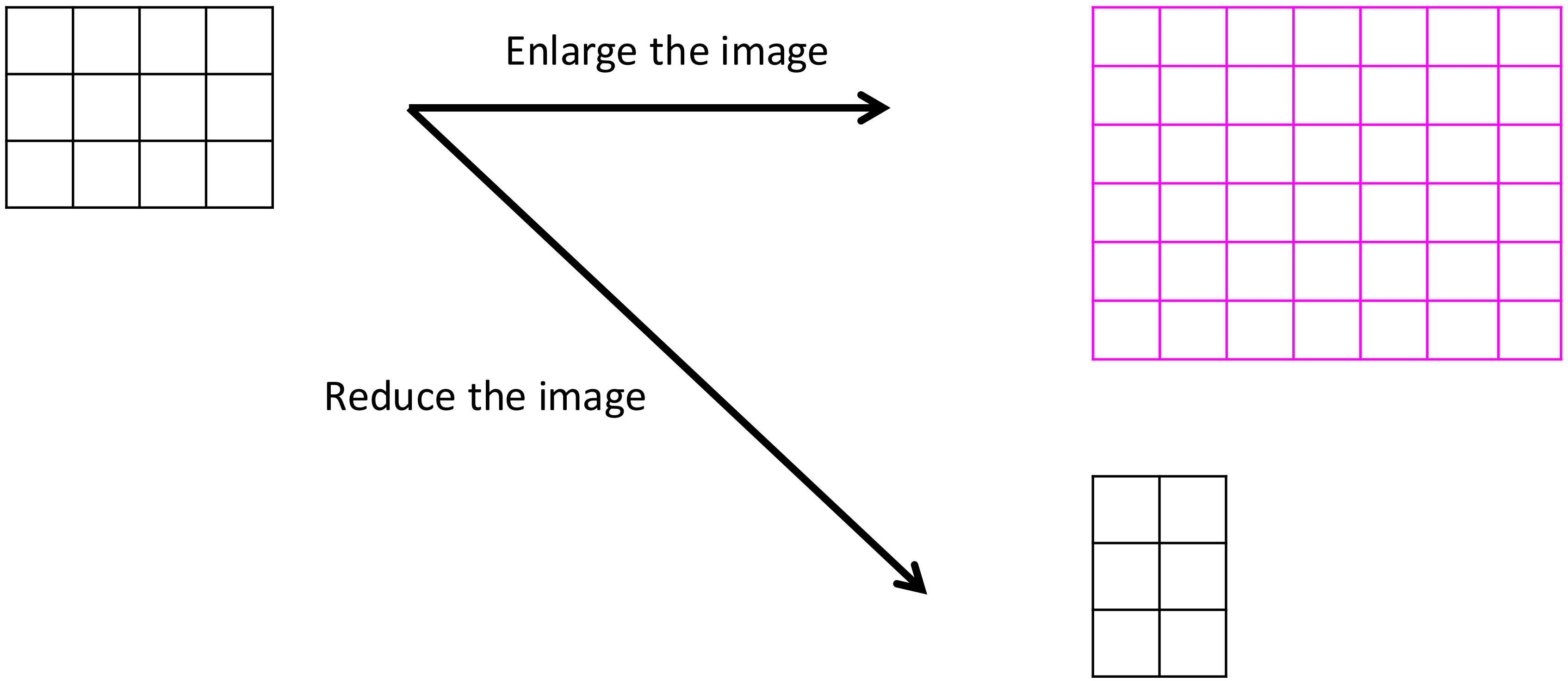
Use the relationship developed earlier

$$x = \frac{15}{4} y$$

Sample location is

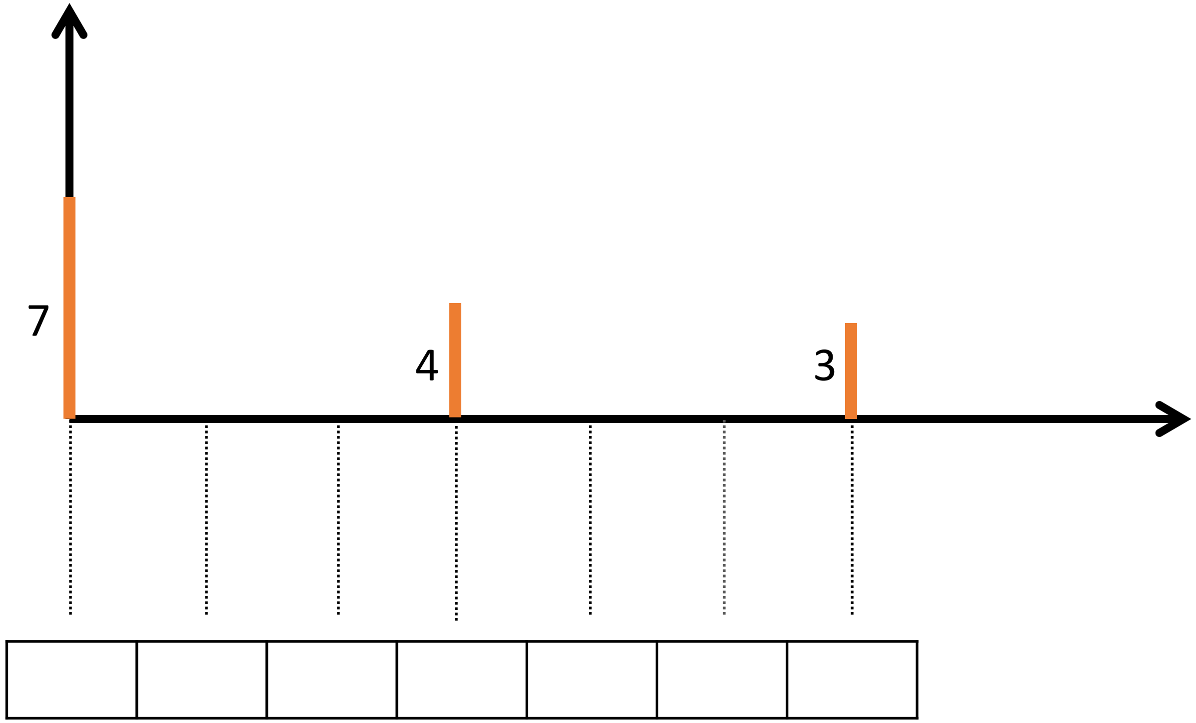
$$x = \frac{15}{4}(1) = 3.75 \approx 4$$

Resampling pixel locations (in 2D)



How to compute new pixel values?

7	4	3



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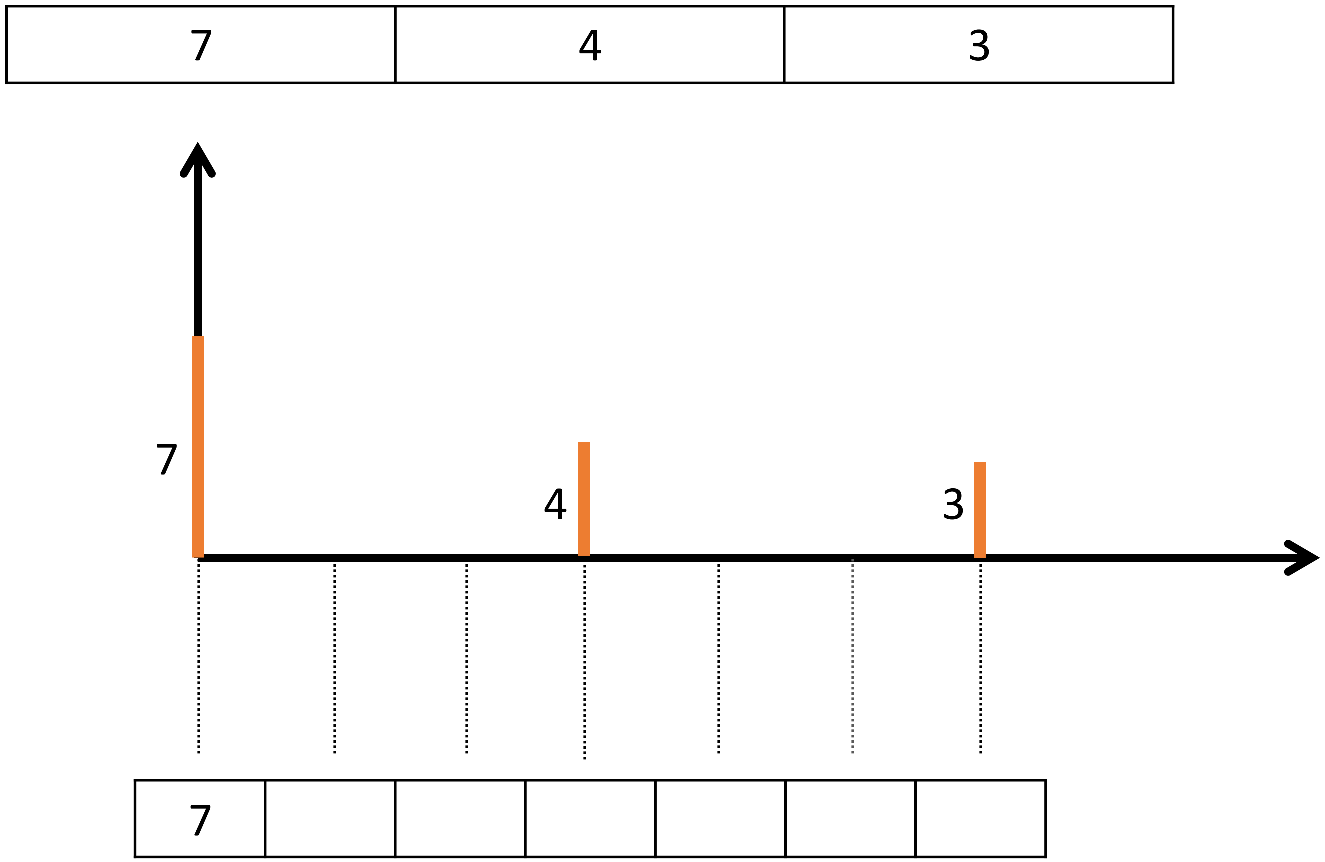
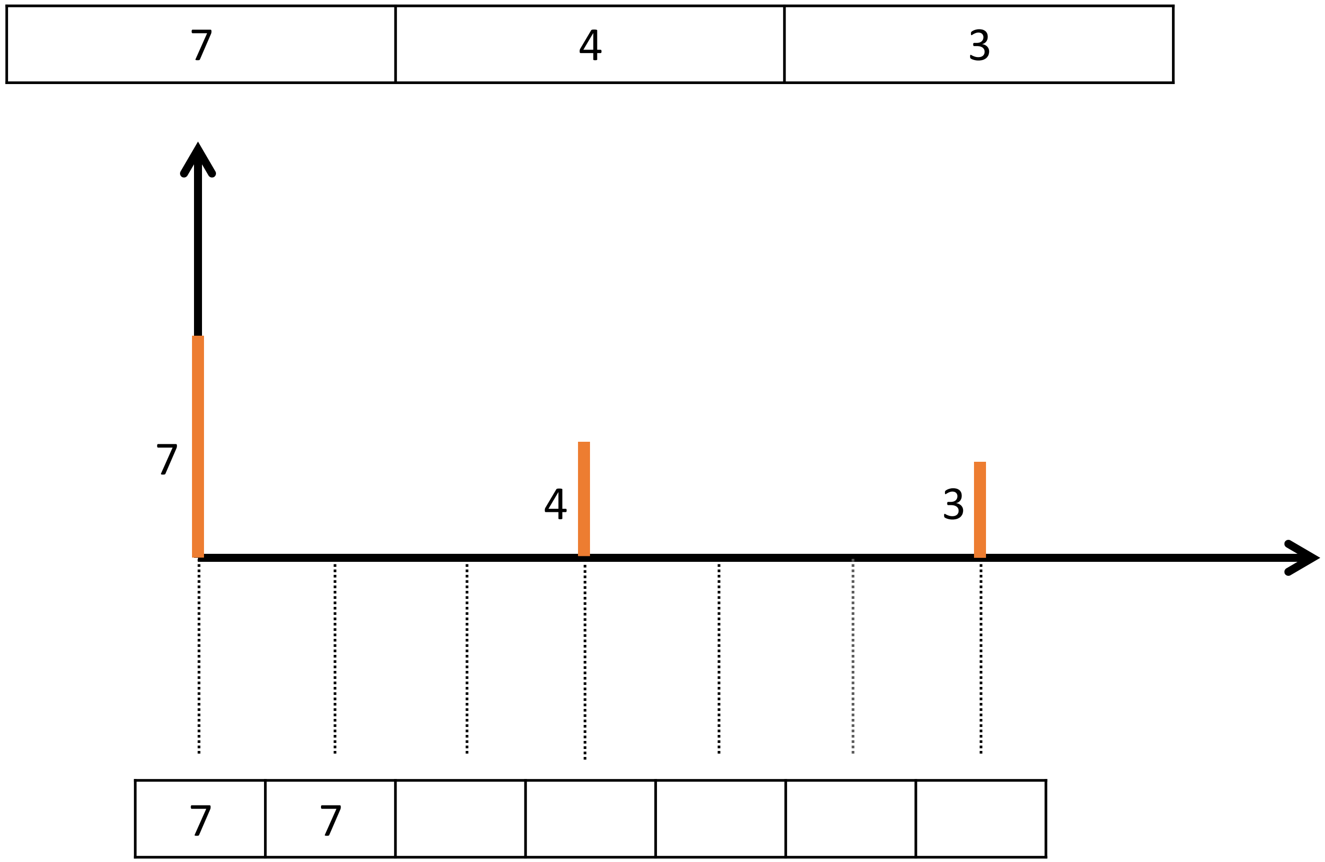


Nearest-Neighbor Interpolation

How to compute new pixel values?



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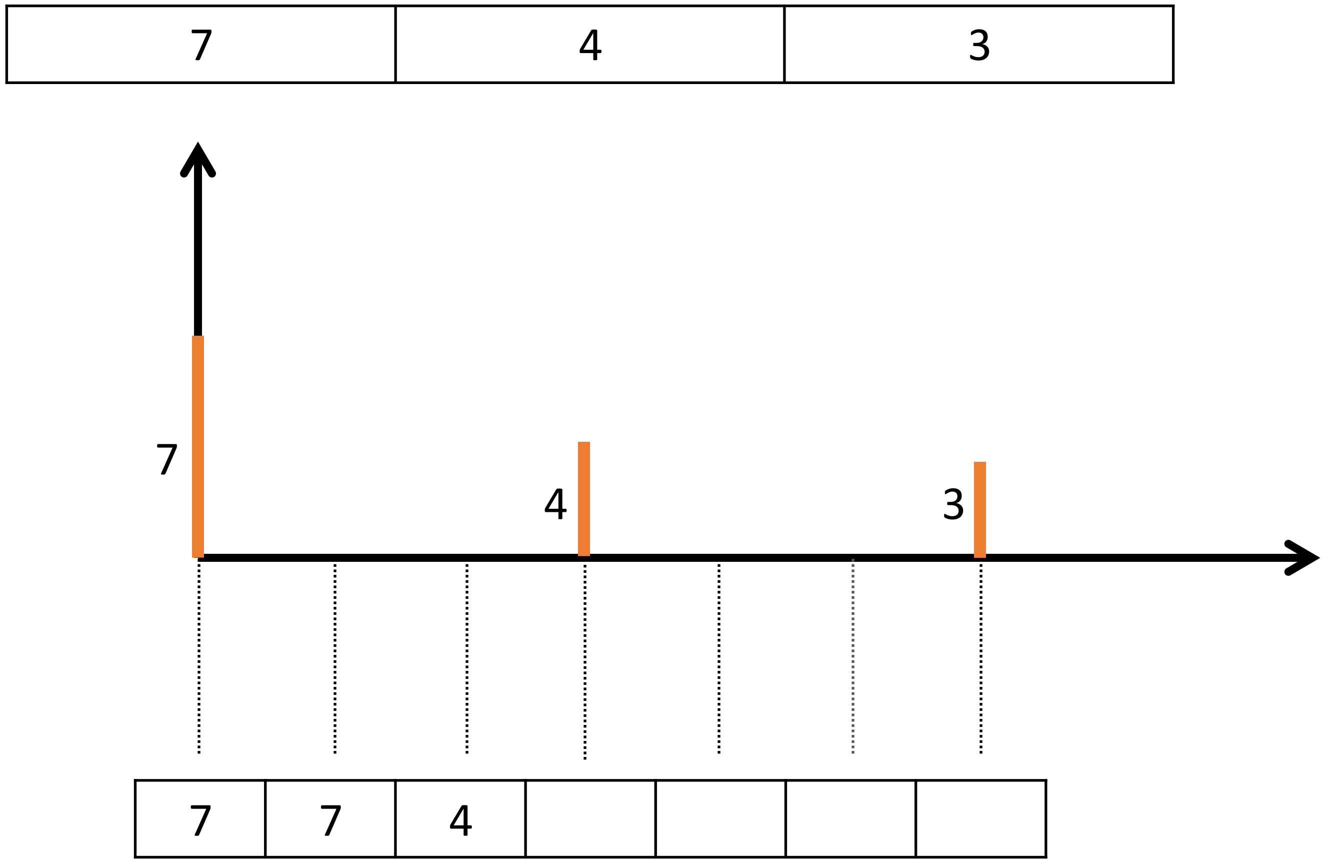

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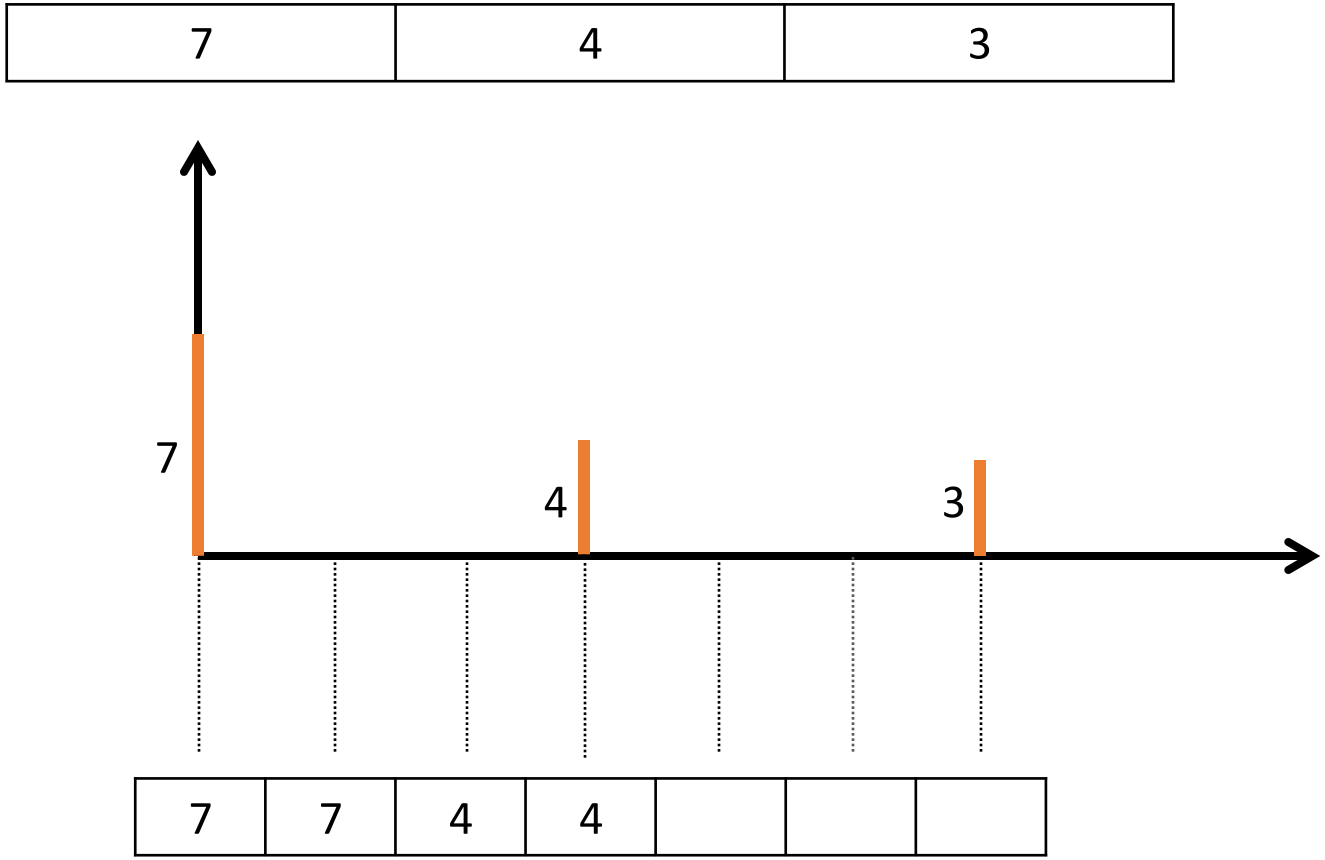

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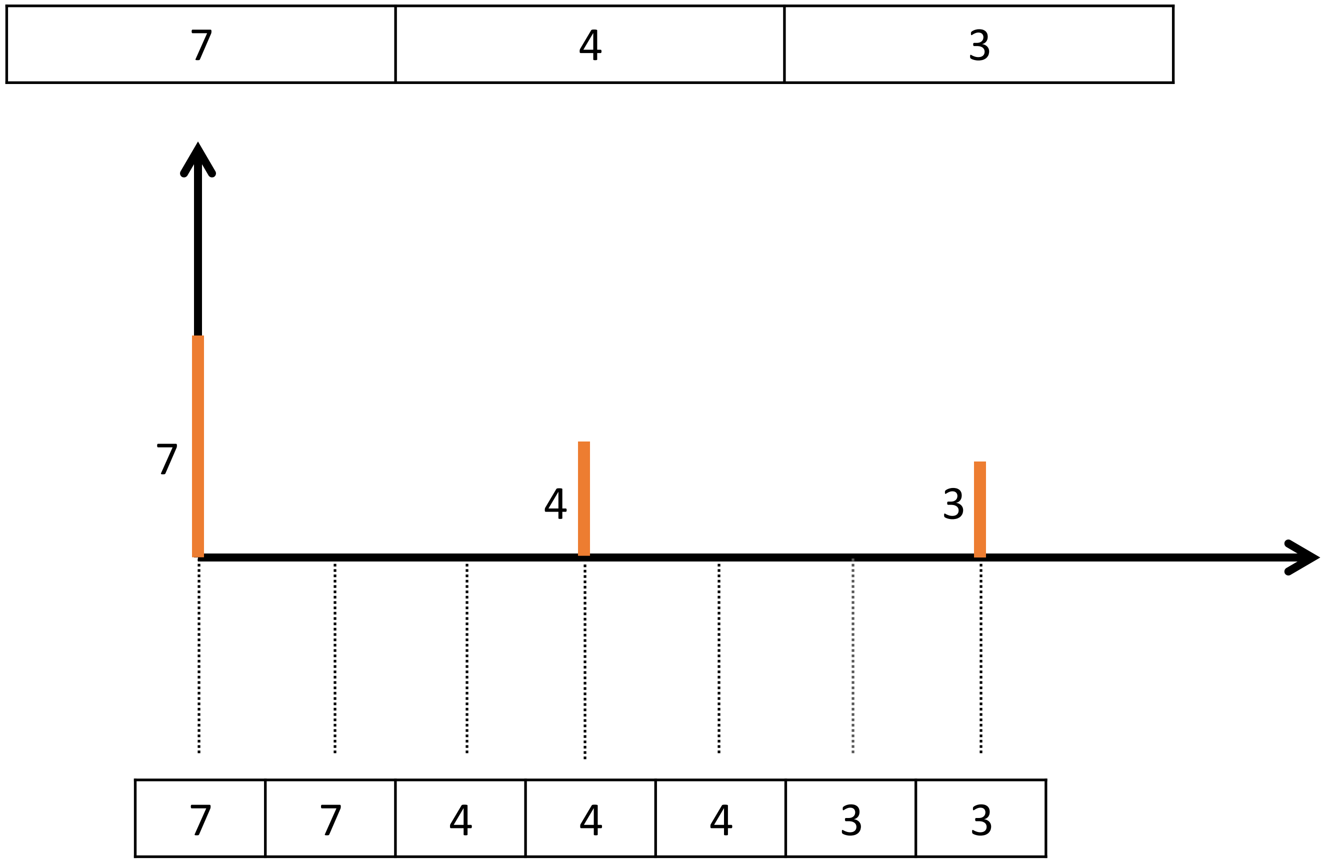


Nearest-Neighbor Interpolation

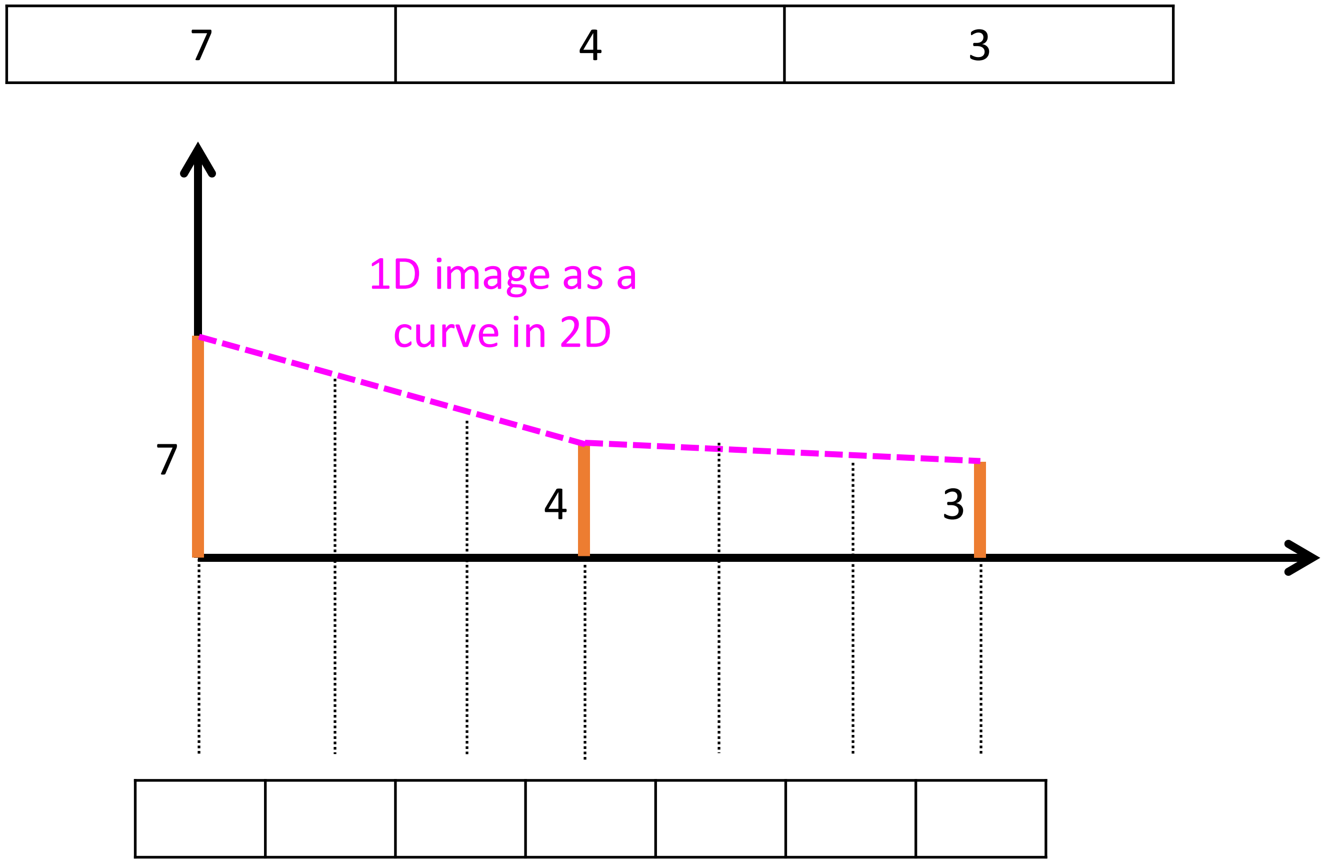
Nearest-Neighbor Interpolation

- Easy to implement.
- Results in blocky or pixelated results
- Does not consider neighboring pixels
- Losses details and smoothness
- Use other methods, e.g., bilinear, bicubic, etc., for higher-quality image resizing

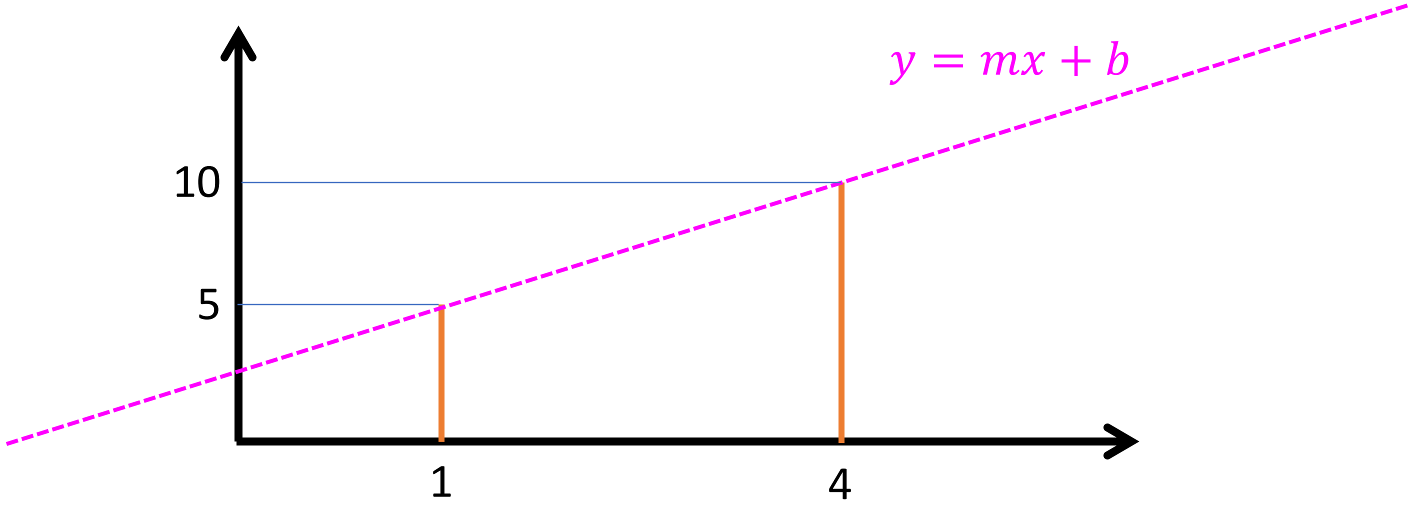
Nearest-Neighbor Interpolation

Nearest-Neighbor Interpolation

Linear Interpolation



2D Line Fitting

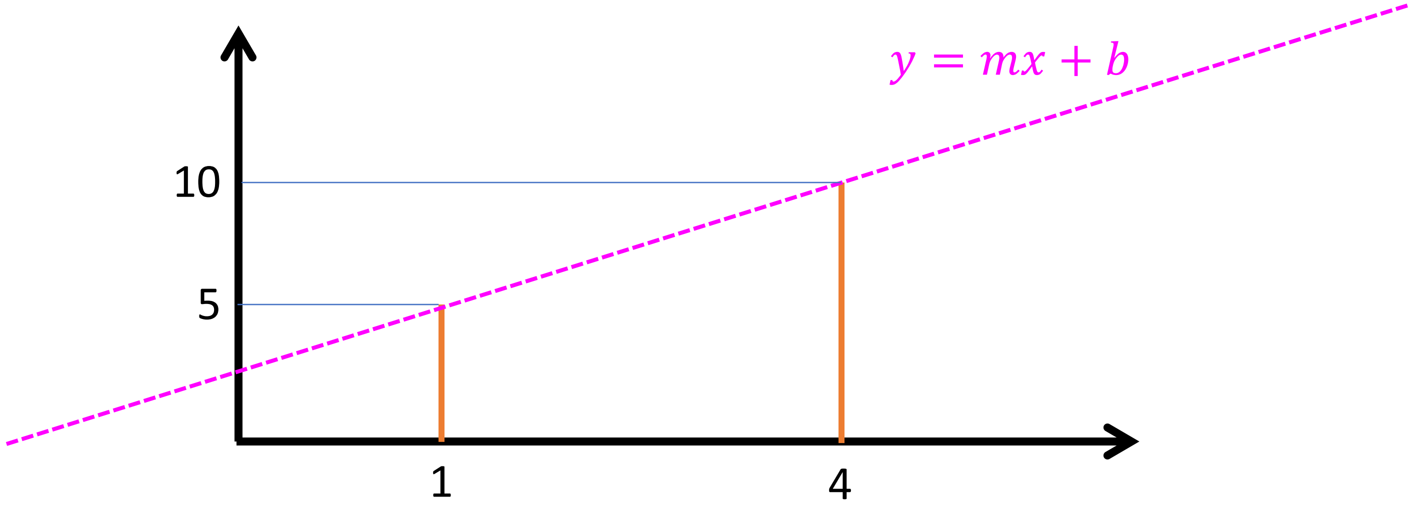


2D Line Fitting

A line between (x_1, y_1) and (x_2, y_2) is given by:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

2D Line Fitting



2D Line Fitting

A line between (x_1, y_1) and (x_2, y_2) is given by:

Matrix form

Re-write

$$x_1 m + b = y_1$$

$$x_2 m + b = y_2$$

as

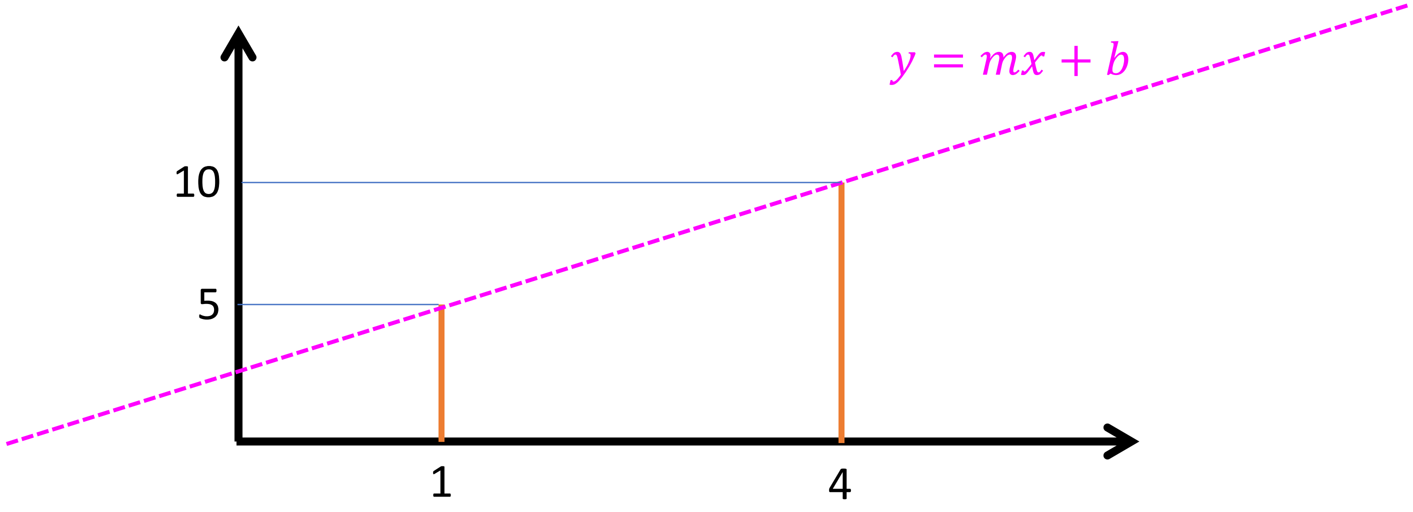
$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2D Line Fitting



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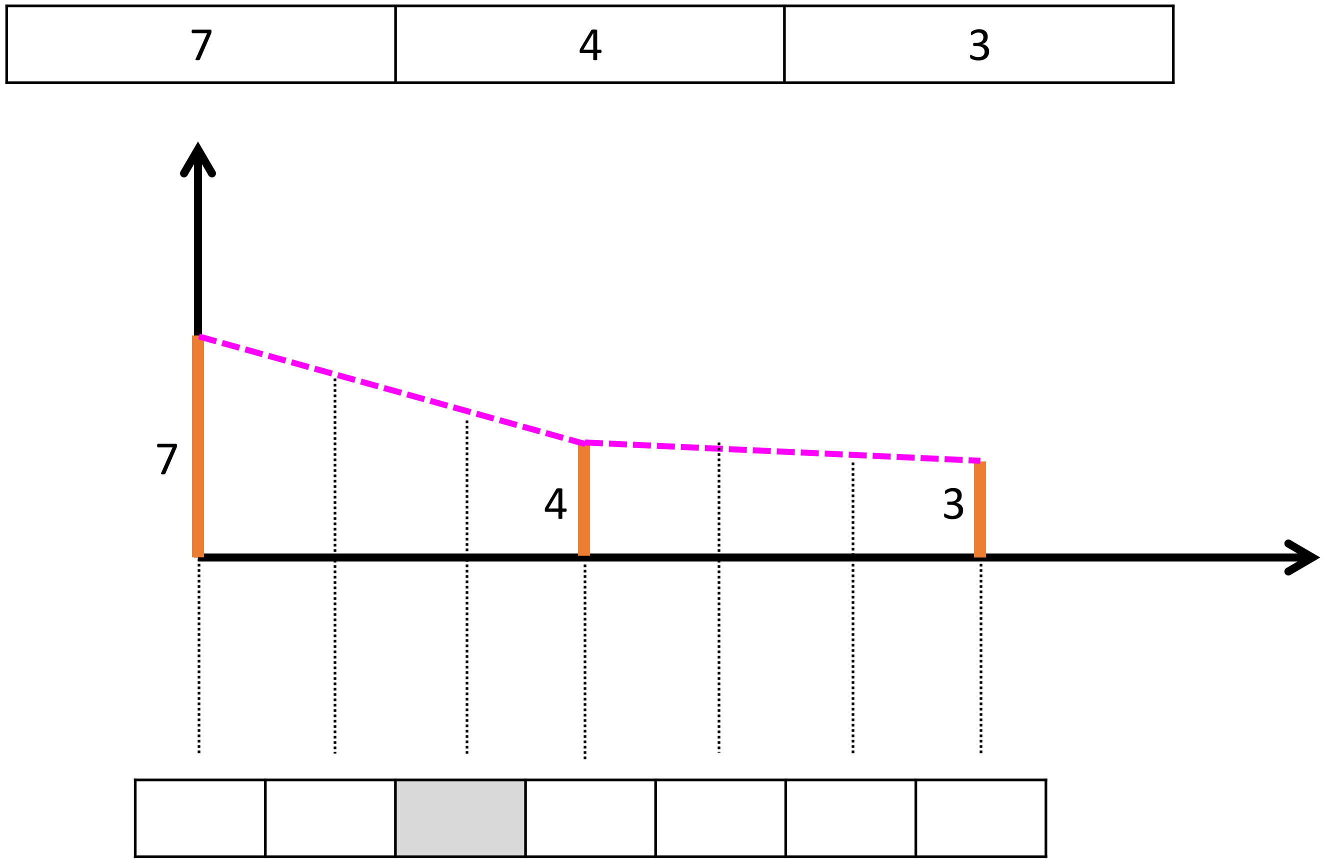
$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Practice Question

Estimate (fit) the dotted-line shown on the left.

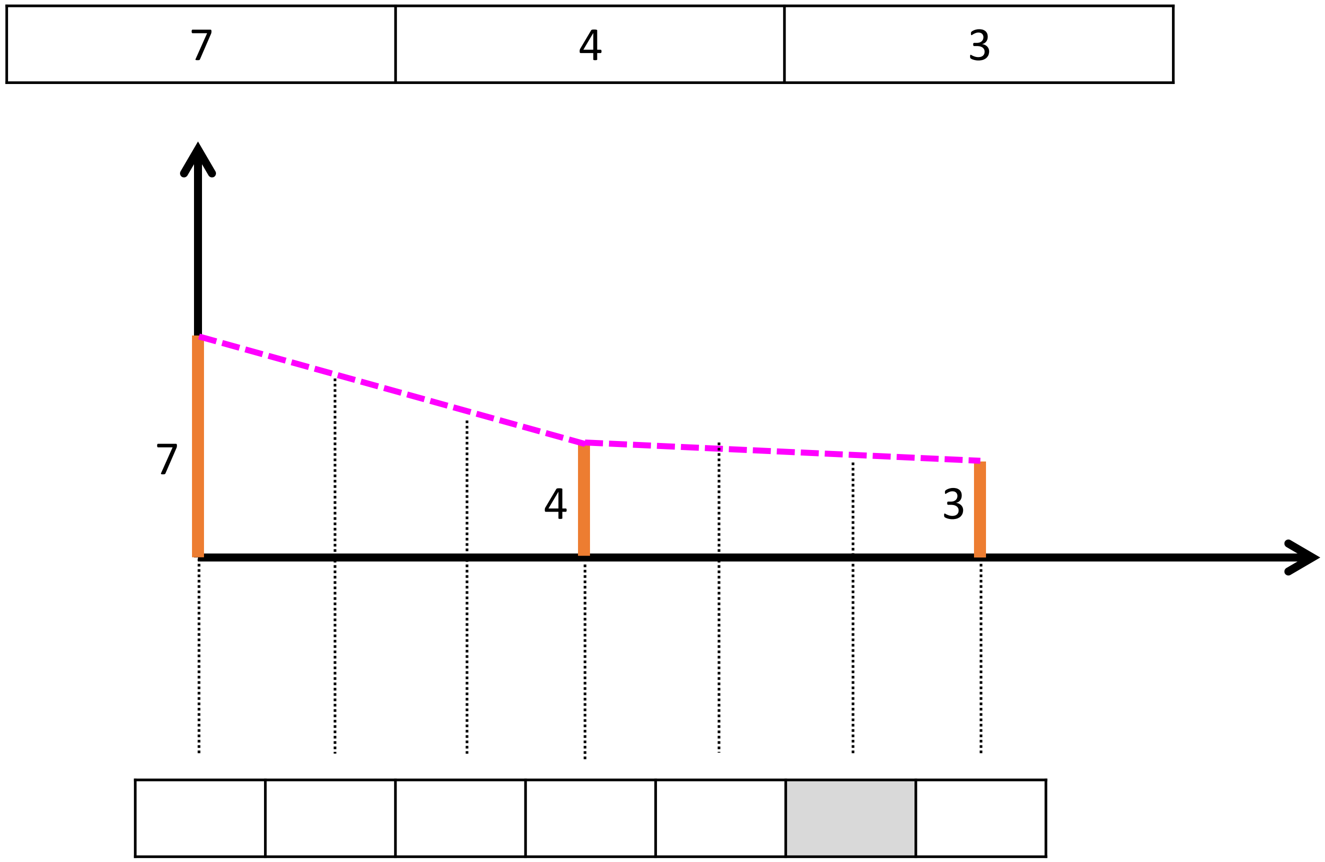
Linear Interpolation



Example

Compute the value for shaded pixel?

Linear Interpolation



Practice Question

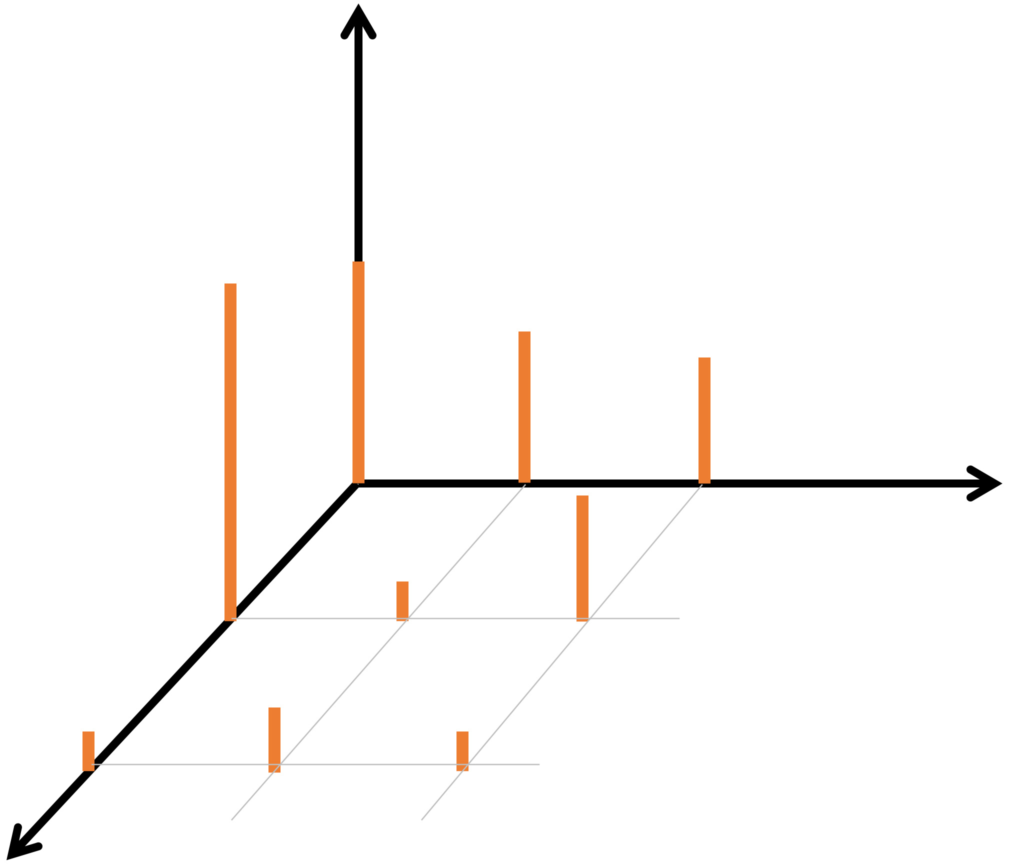
Compute the value for shaded pixels?

Images as surfaces

Images are not just 1D. How do we deal with a 2D image?

Consider the following 3x3 2D image.

7	4	3
9	1	3
1	2	1



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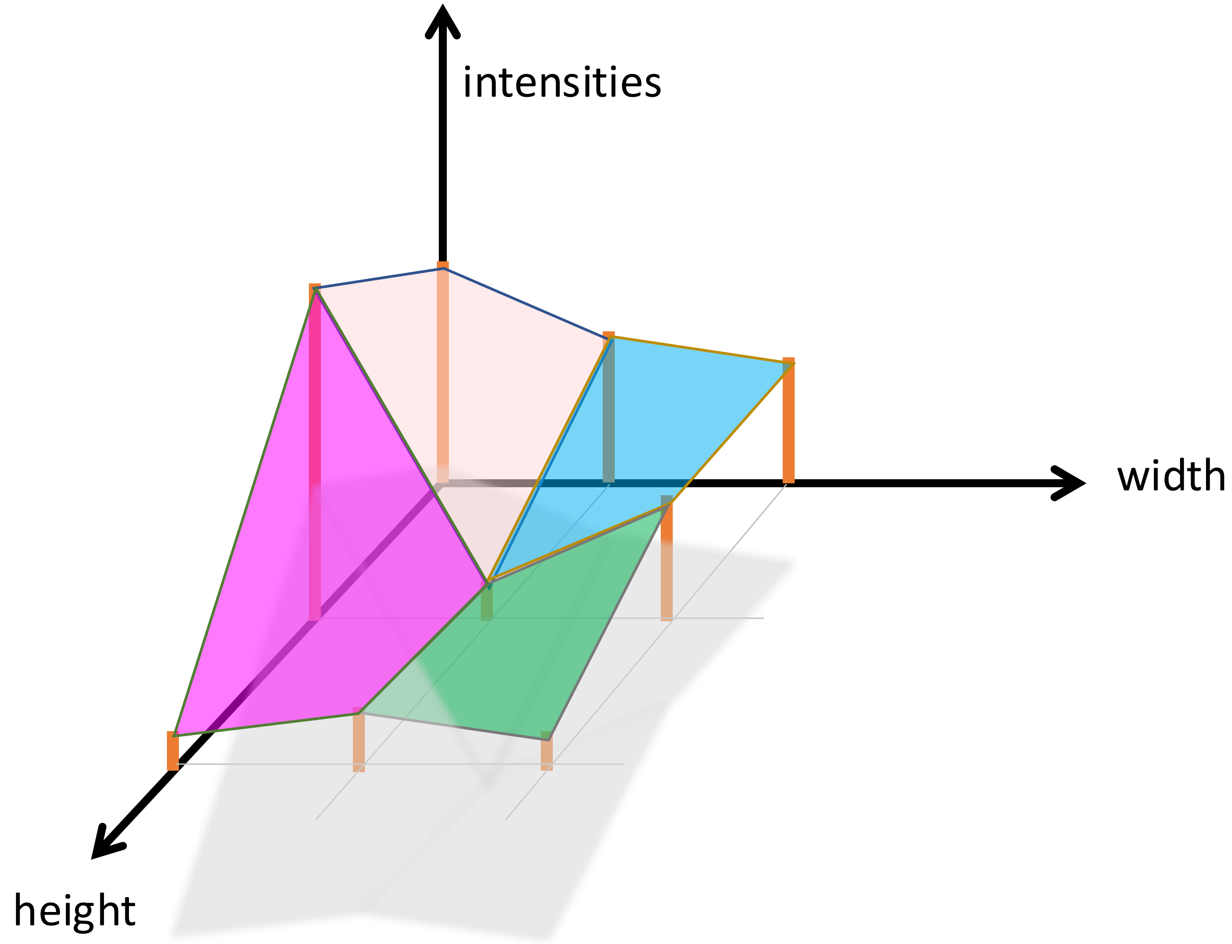

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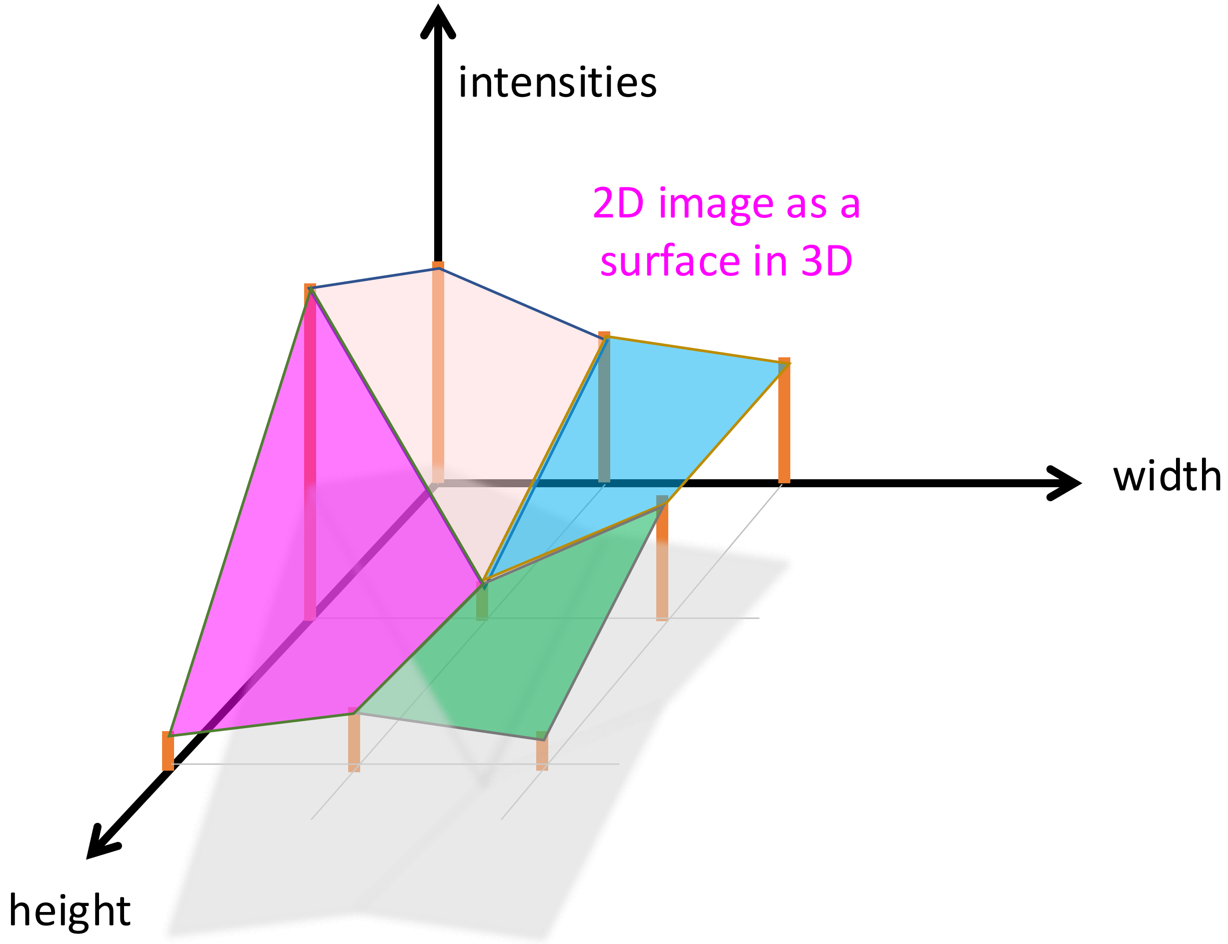

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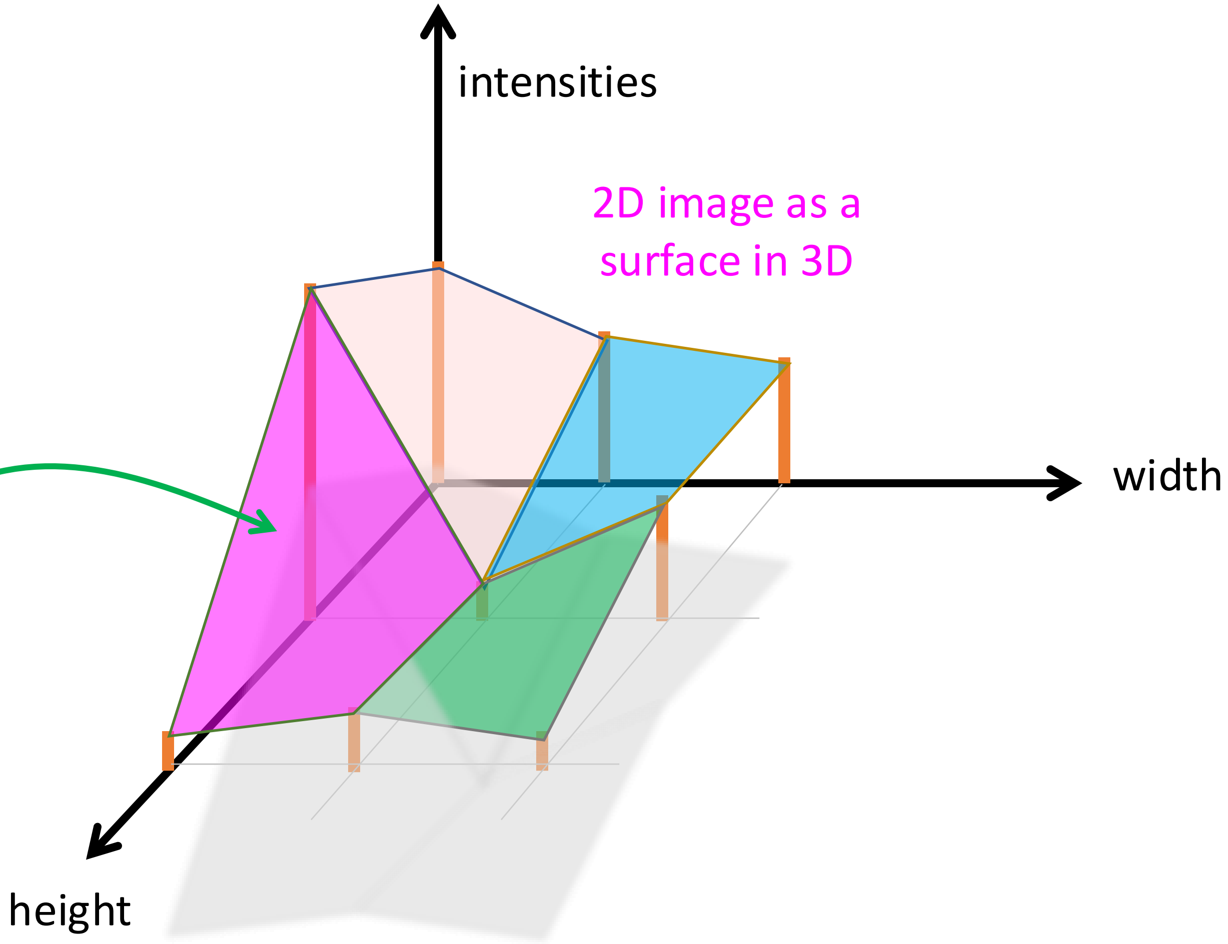
Are these planar patches?



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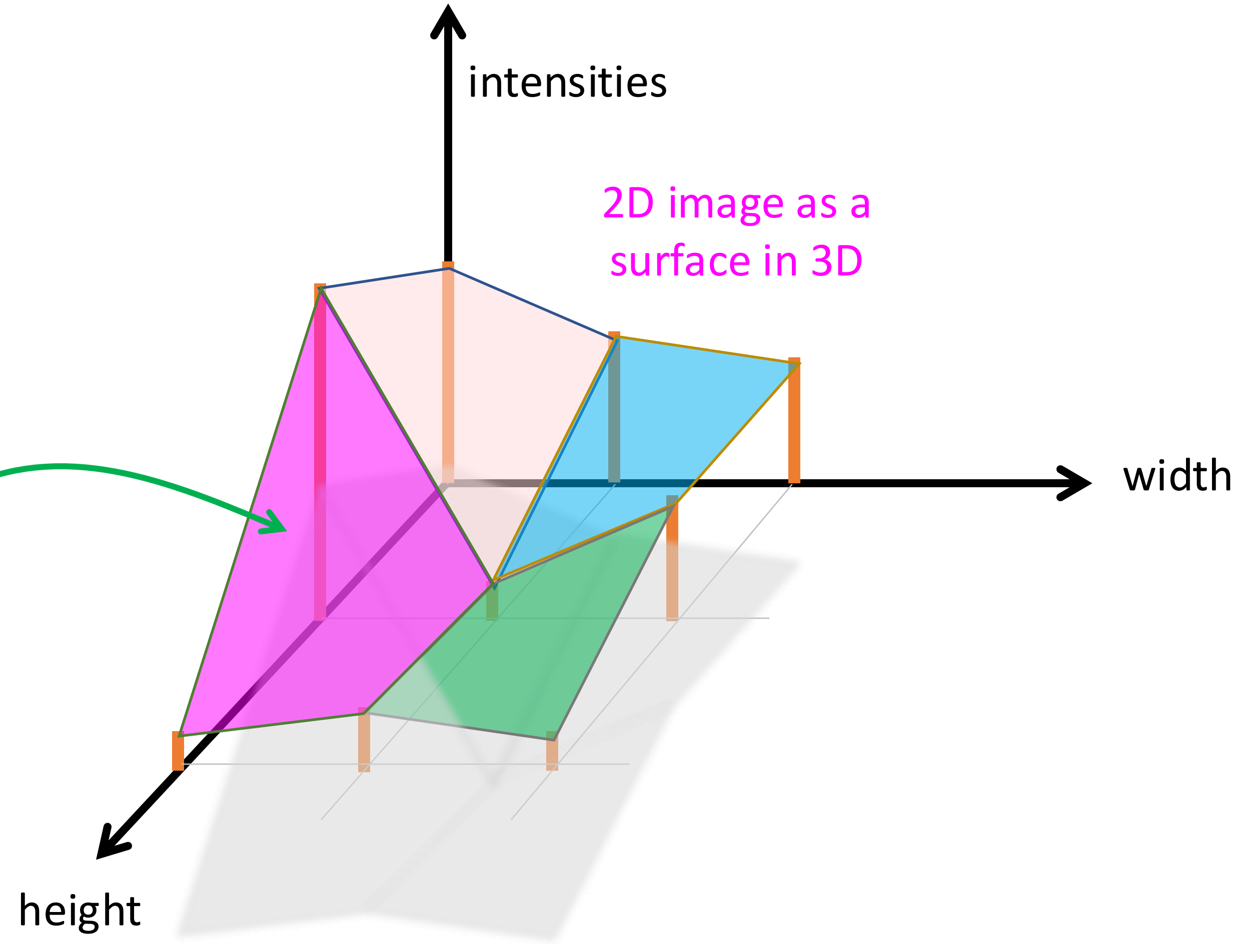
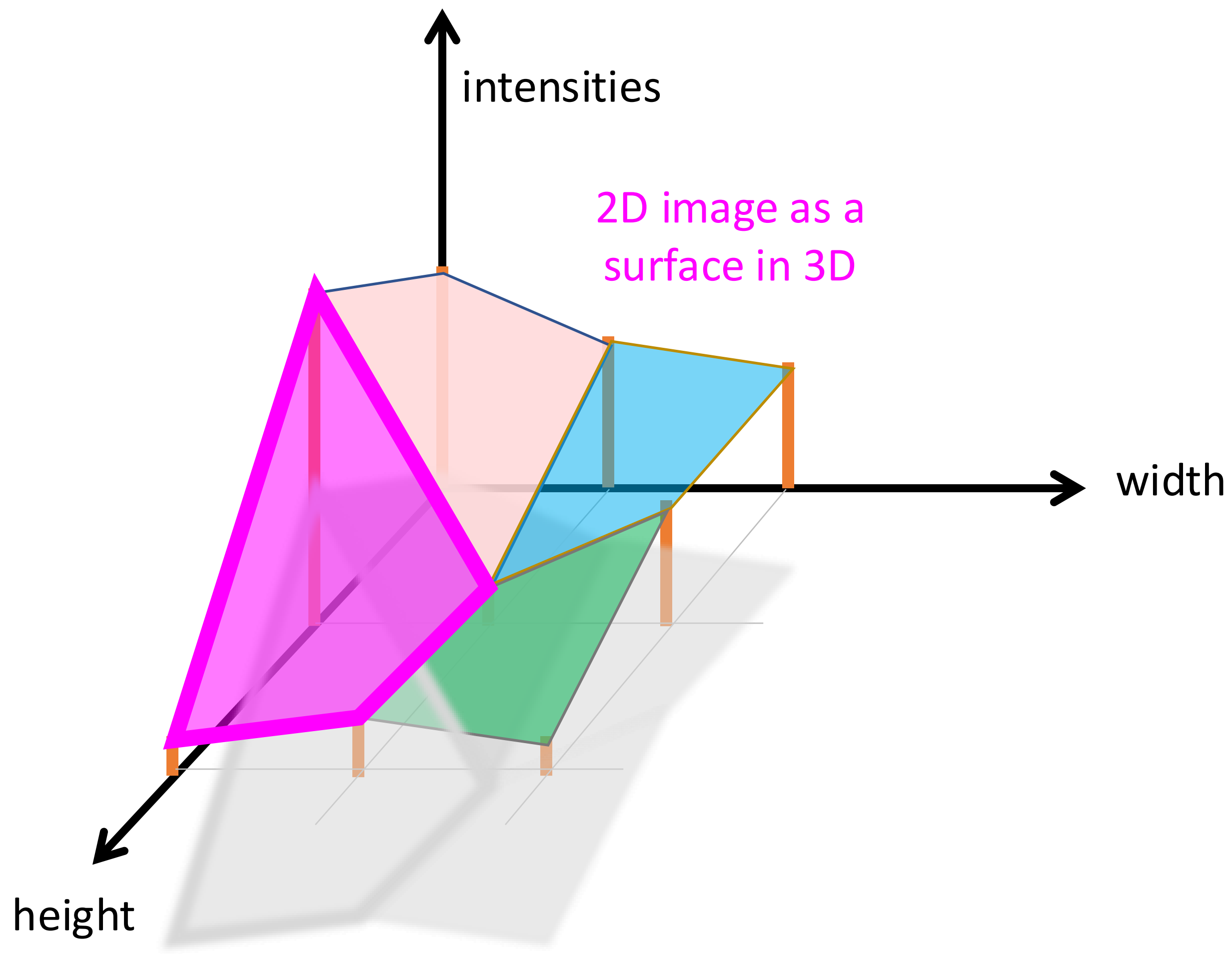
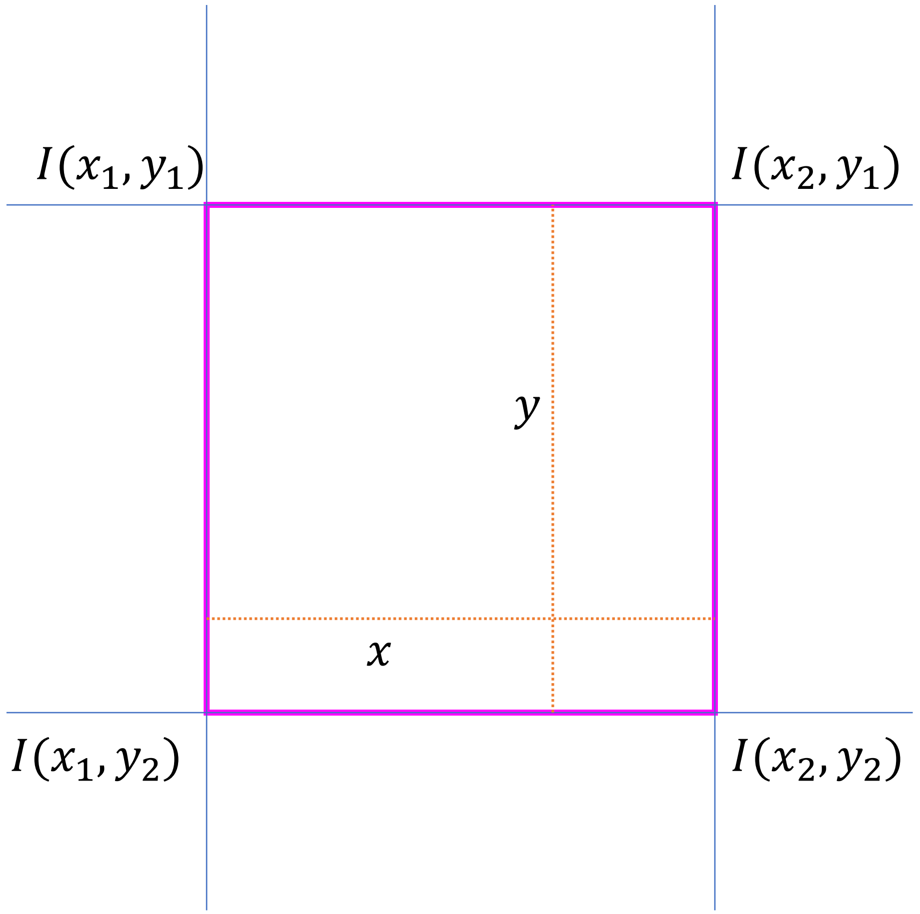


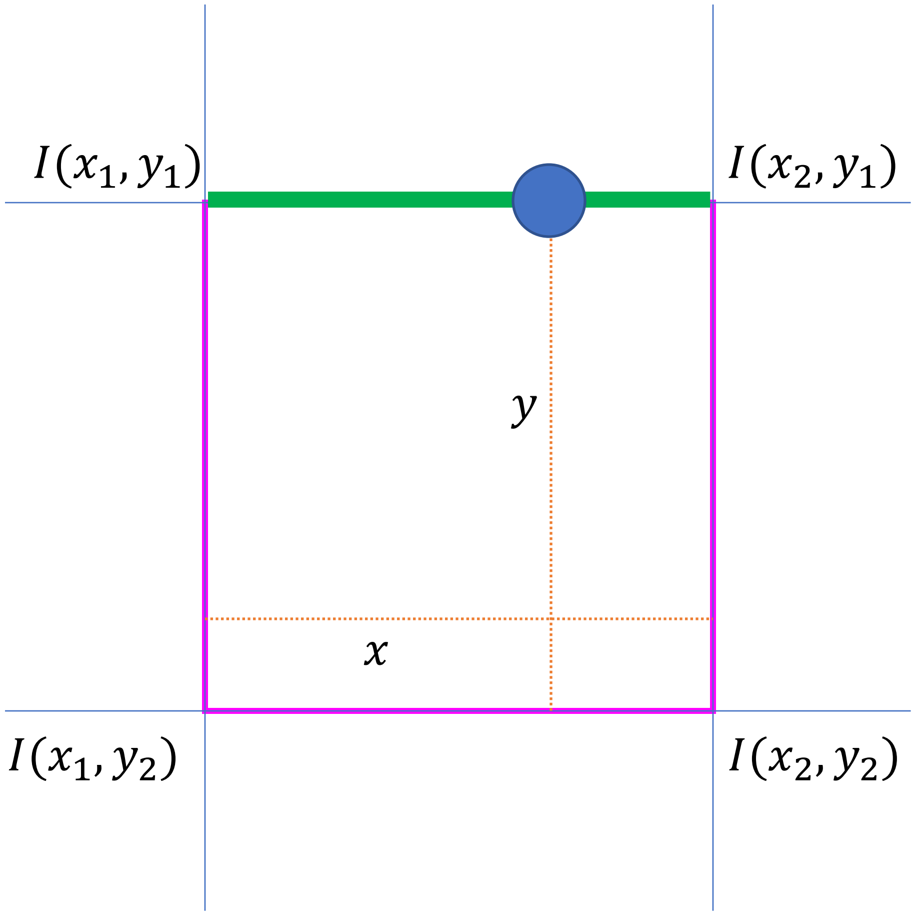
Are these planar patches? **No**

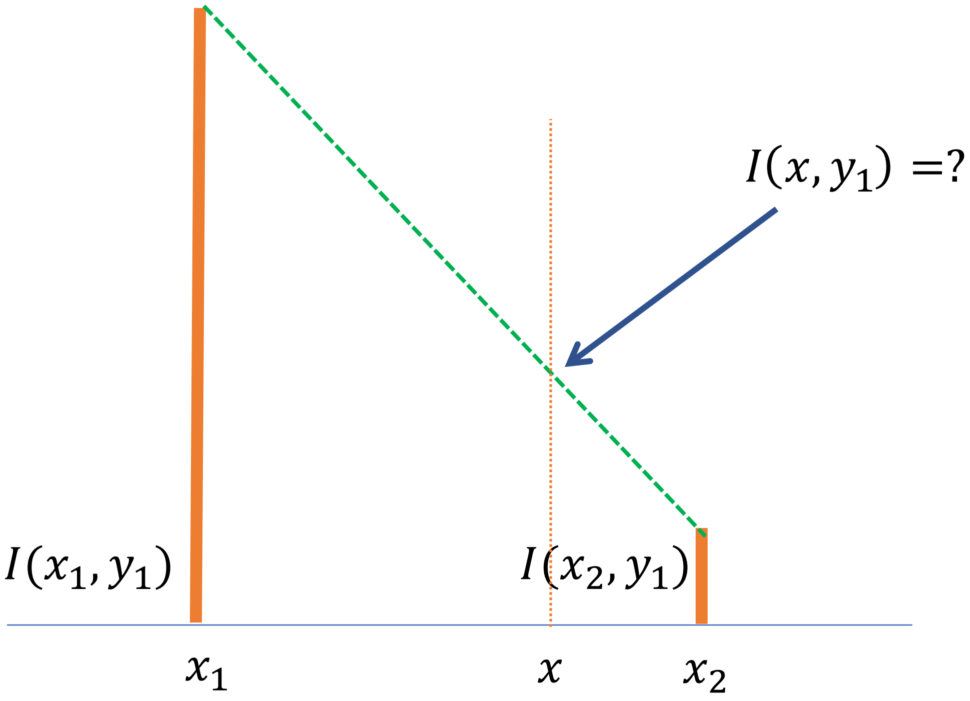
Images as surfaces





Bi-linear interpolation





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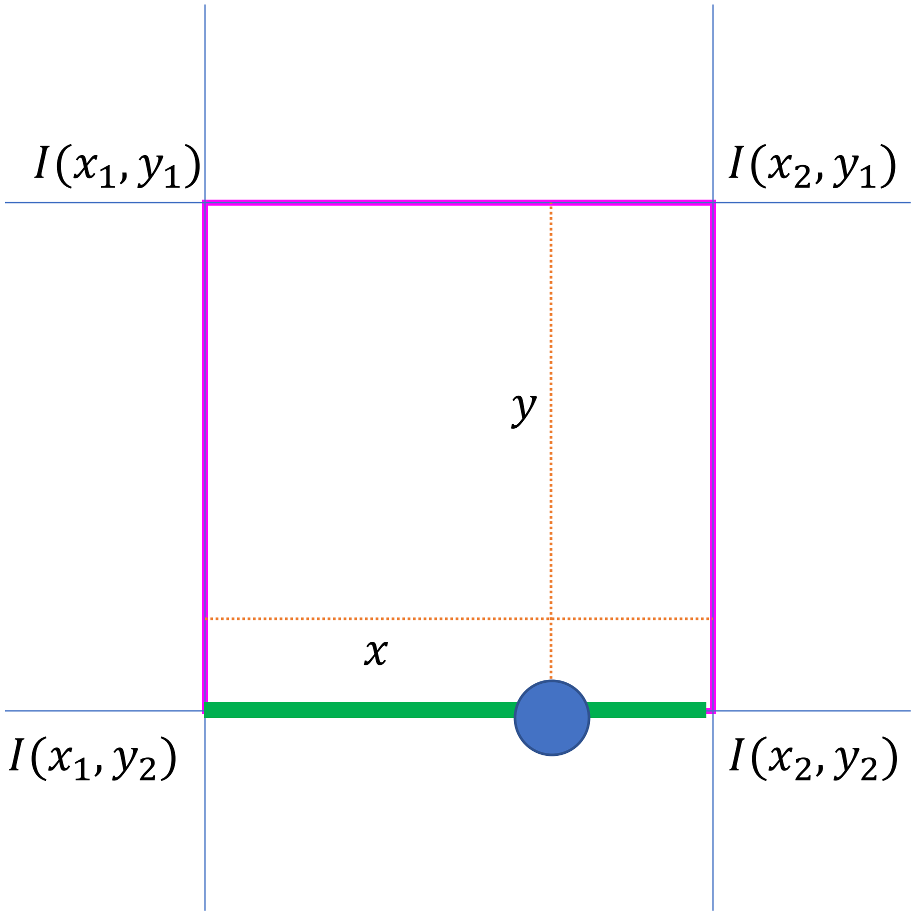

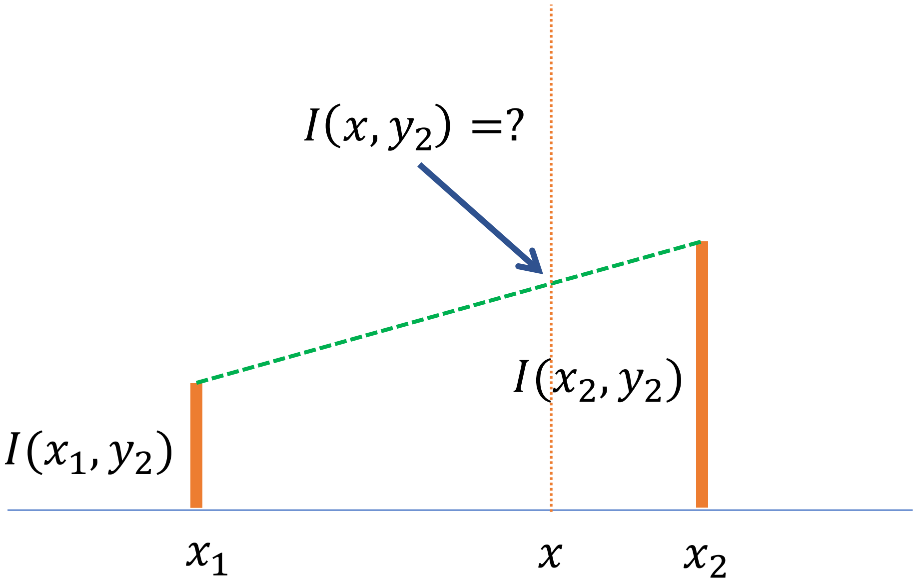

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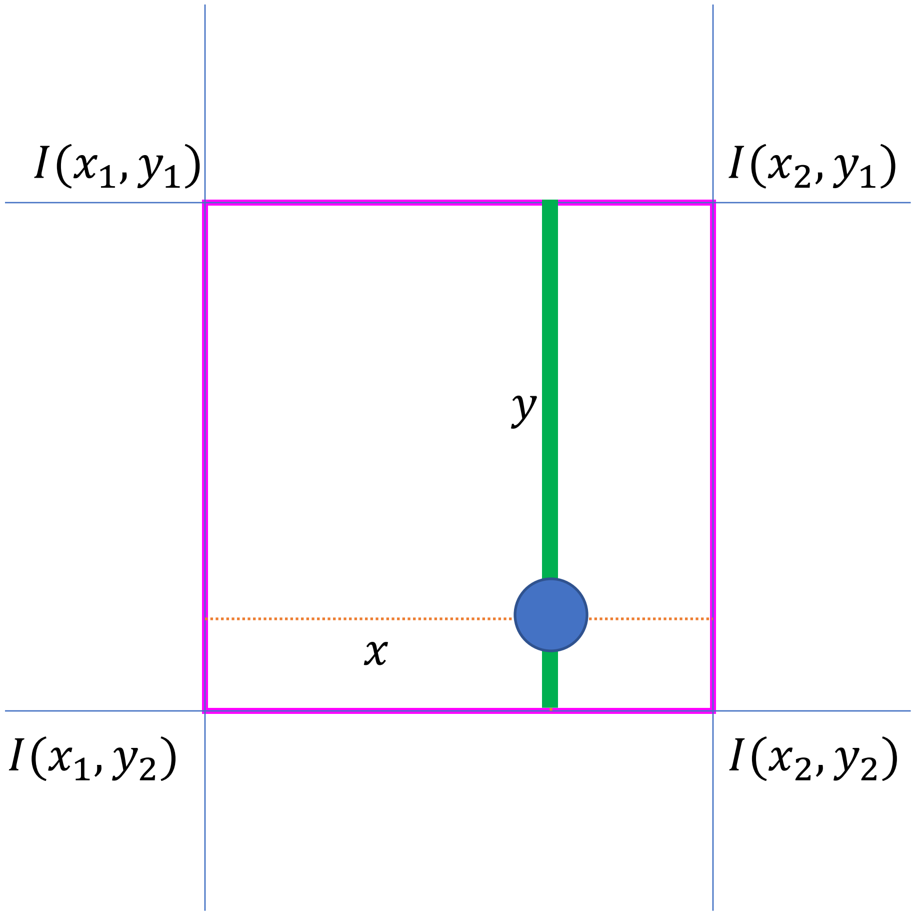

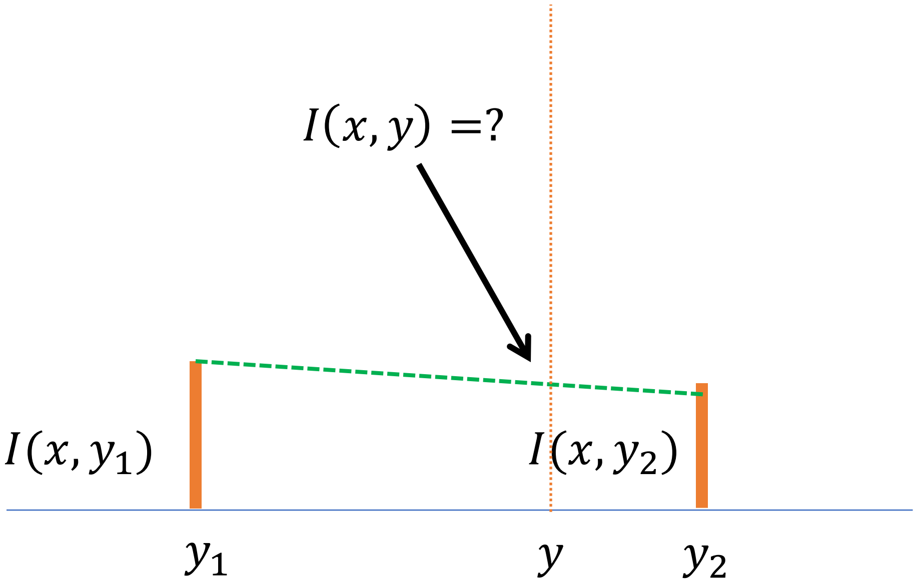

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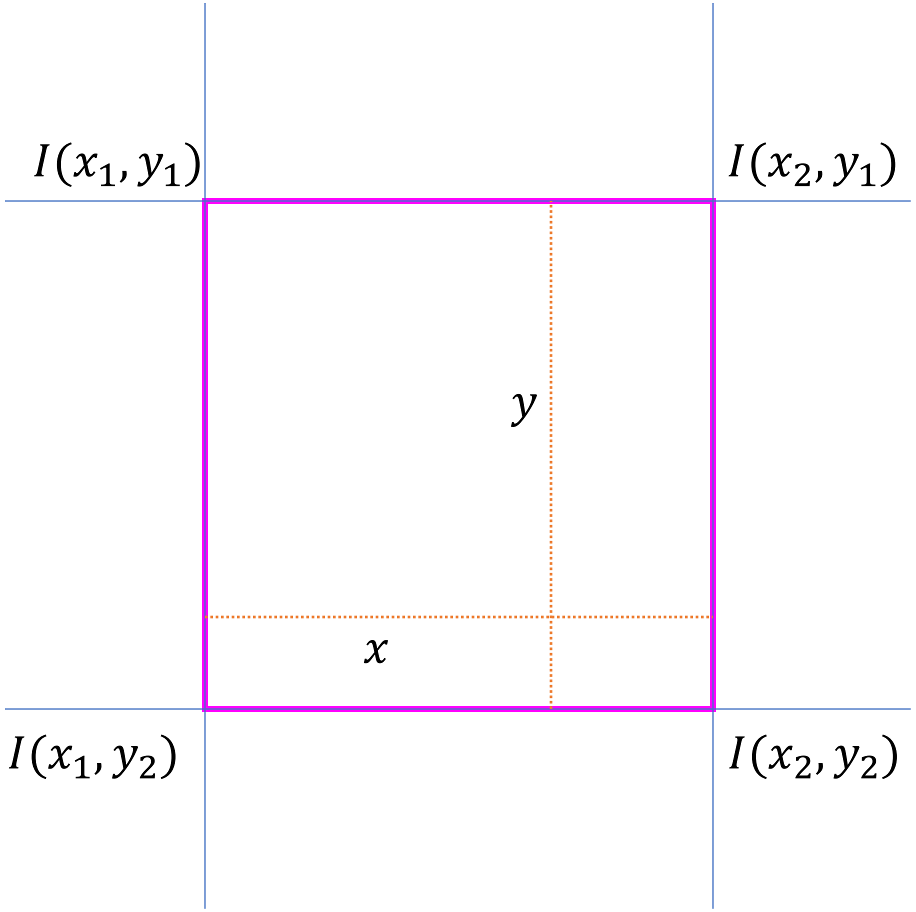


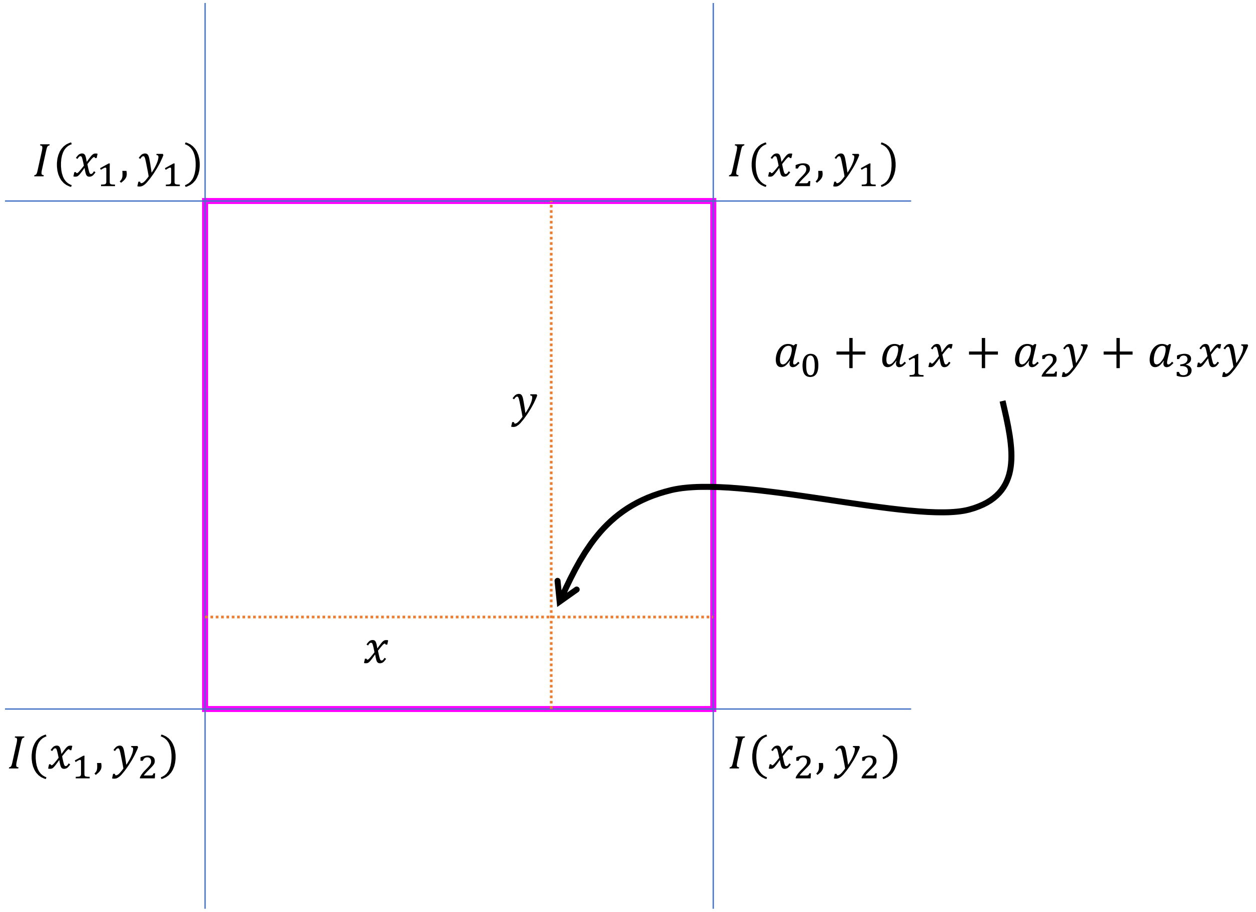
Multi-linear polynomial

$$f(x, y) = a_0 + a_1 x + a_2 y + a_3 xy$$

Then for $i, j \in [1,2]$

$$I(x_i, y_i) = a_0 + a_1 x_i + a_2 y_j + a_3 x_i\, y_j$$

Bi-Linear interpolation



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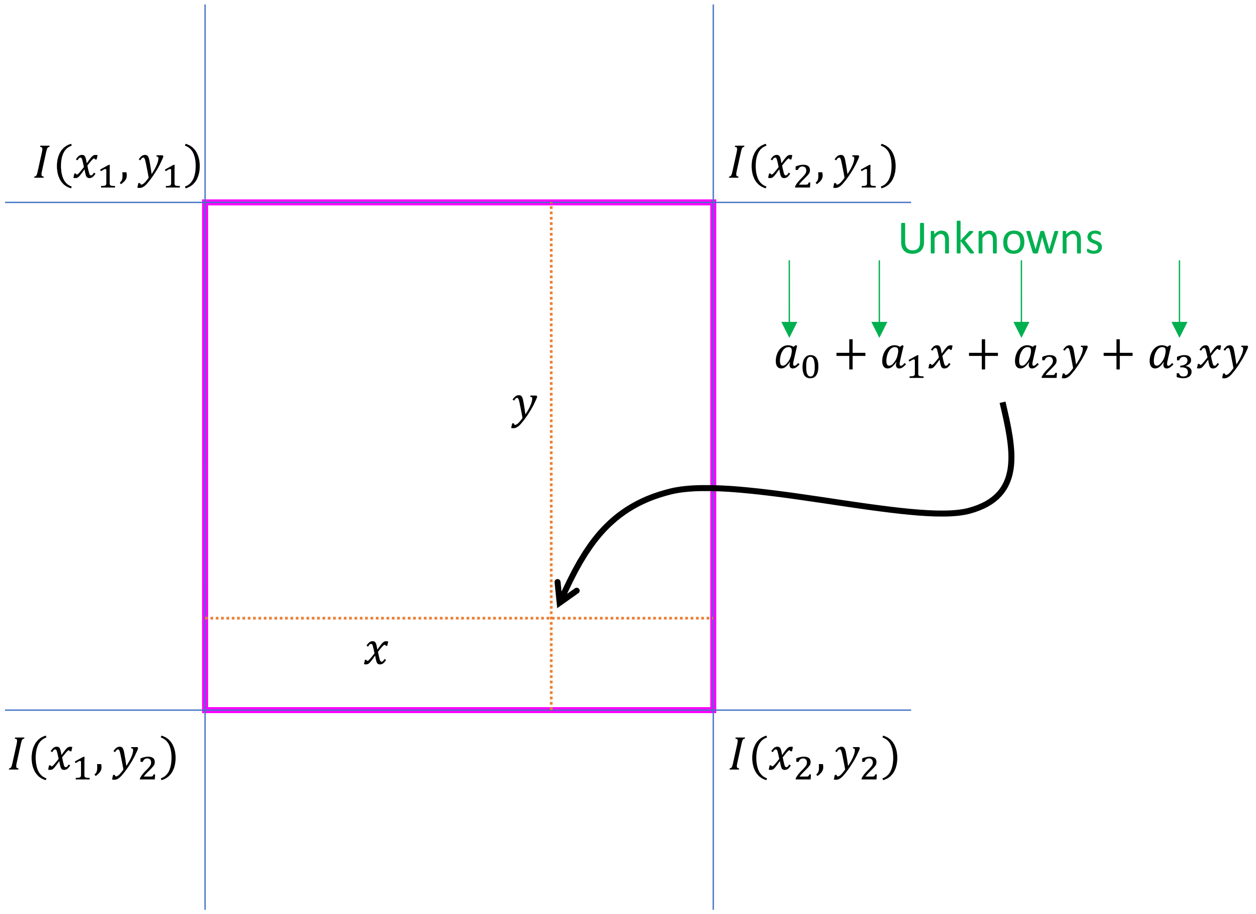

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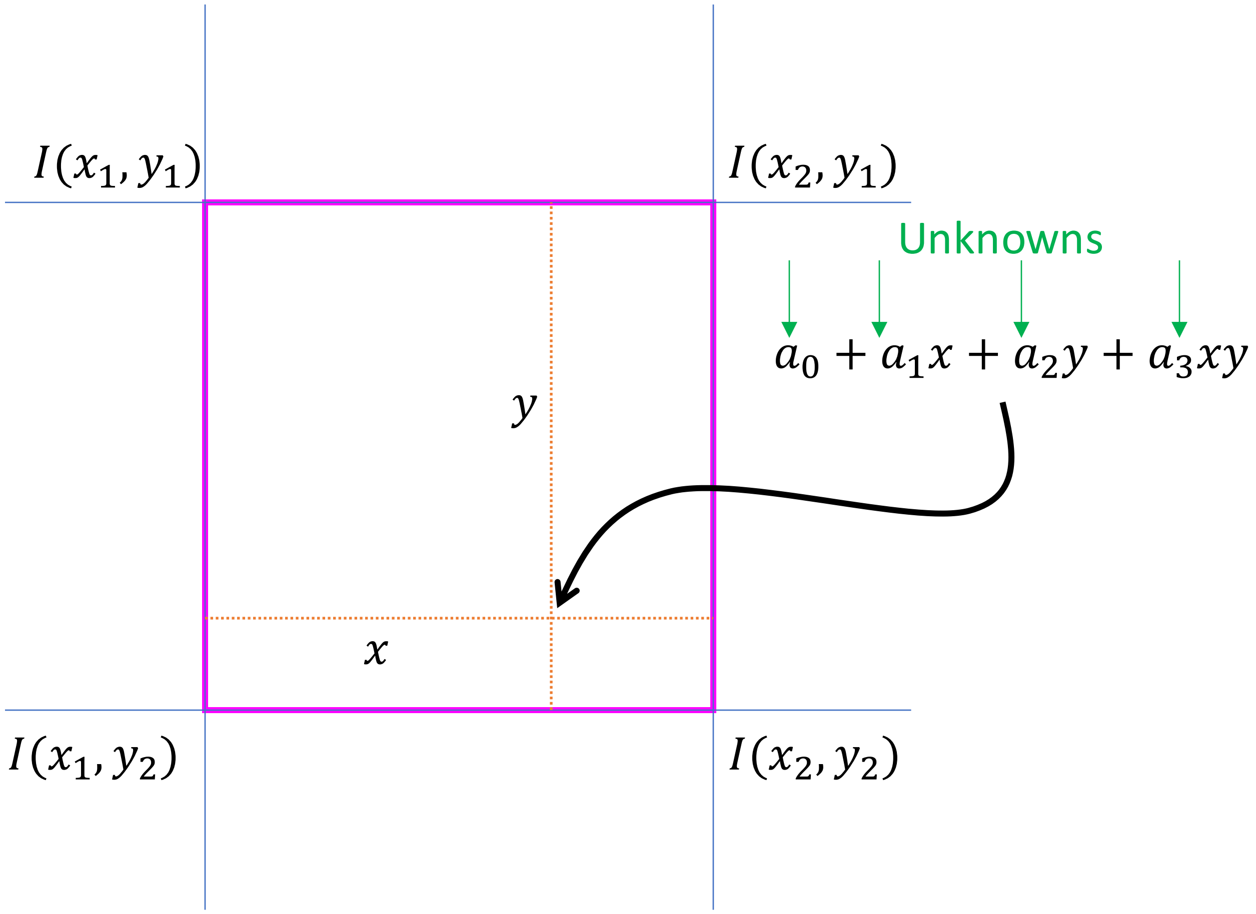

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Solve for the unknowns using the following system of equations

$$\begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_1 & x_2 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ 1 & x_2 & y_2 & x_2 y_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} I(x_1, y_1) \\ I(x_2, y_1) \\ I(x_1, y_2) \\ I(x_2, y_2) \end{bmatrix}$$

Bilinear interpolation: Pros

- Smoothing Effect, which helps reduce jagged edges and pixelation.
- Simple to Implement, requires fewer calculation and computational inexpensive as compared to other mathods
- Maintains linearity between the known data points, which can be desirable in certain applications, such as computer graphics.

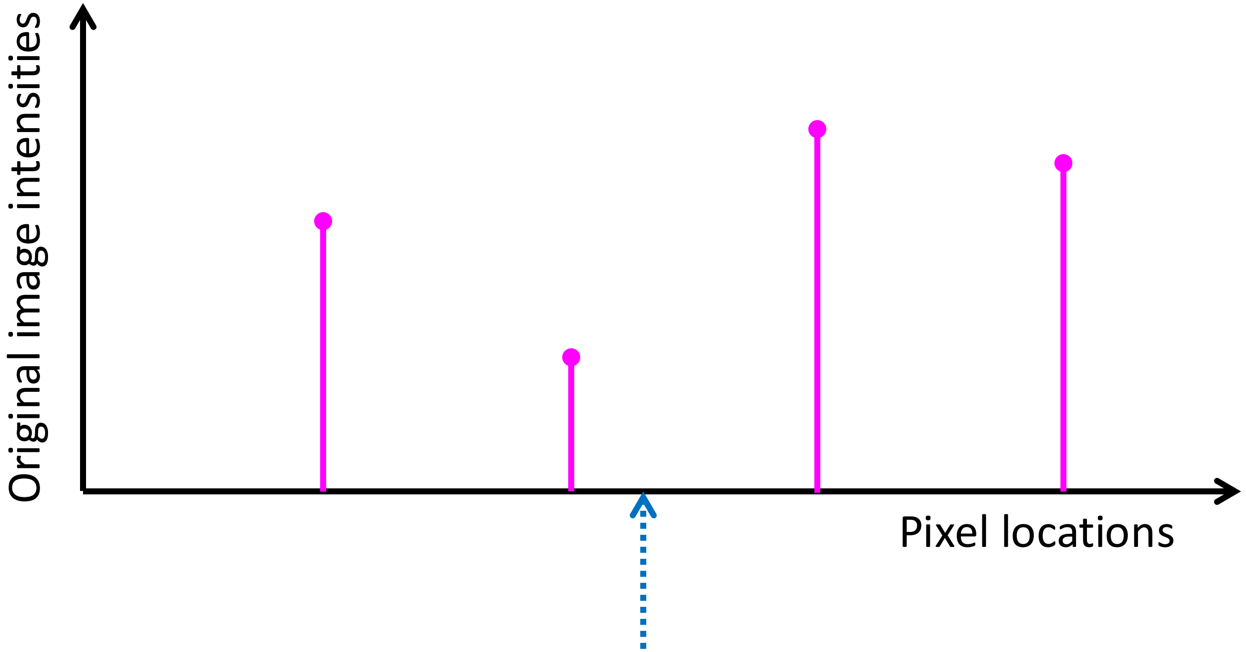
Bilinear interpolation: Cons

- Loss of sharpness and fine details
- Color artifacts
- No consideration for high-frequency components
  - Not suitable for images with intricate patterns or textures
- Not ideal for large scaling
- Limited accuracy and it may not be suitable for photometric applications

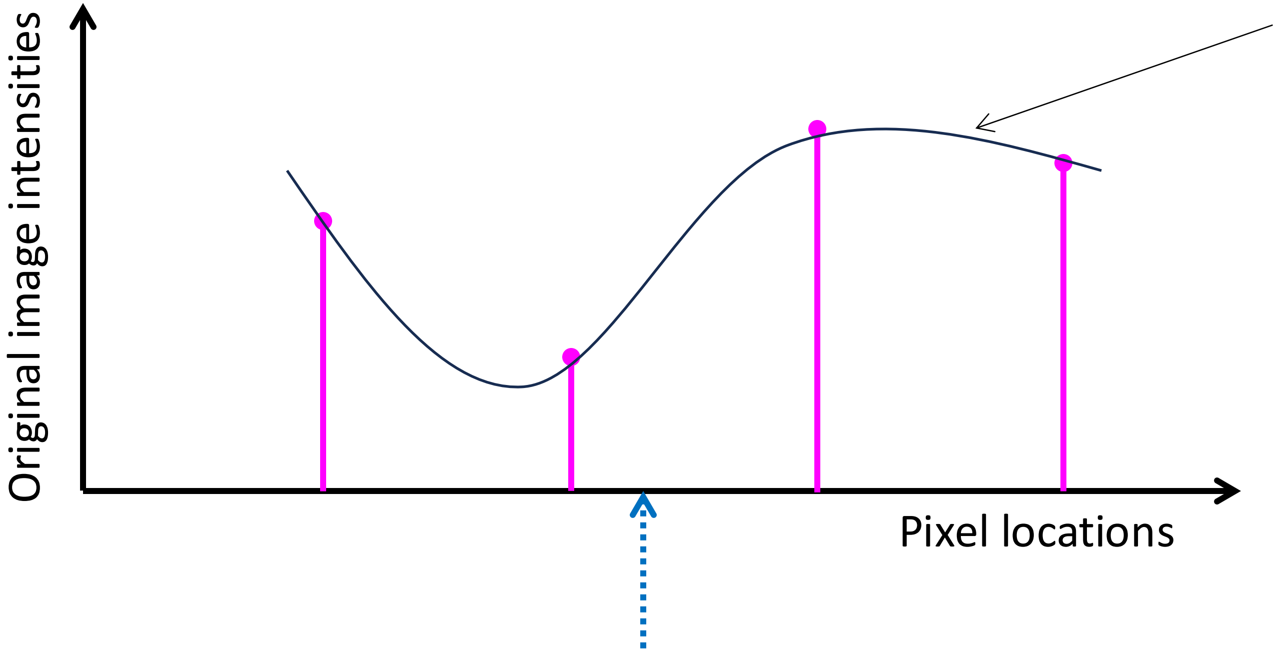
Bicubic interpolation

- Bicubic interpolation is a method for image resizing that calculates new pixel values using the nearest 16 pixels (a 4x4 grid).
- It produces smoother and higher-quality results compared to simpler methods like nearest-neighbor and bilinear interpolation.

Bicubic Interpolation (in 1D)



Cubic Interpolation (in 1D)



Approximate local structure using a cubic polynomial

$$f(x) = ax^3 + bx^2 + cx + d$$

This equation has four unknowns, so we need at least four points to fit this model (to the available image intensities)

Bicubic interpolation: Pros

- Better Image Quality:
  - Reduces jagged edges and pixelation, handling edges and gradients effectively.
- Improved Detail Preservation:
  - Retains fine details, ideal for upscaling images.
- Smooth Transitions:
  - Minimizes artifacts like sudden intensity changes, providing a natural look.
- Widely Used:
  - Implemented in many image processing tools, making it a well-established method.

Bicubic interpolation: Cons

- Slower Performance:
  - More computationally expensive than simpler methods, making it slower on large images.
- Blurring:
  - Can introduce blurriness, especially when scaling down.
- Halo Artifacts:
  - Sometimes causes halo effects around edges in high-contrast areas.
- Over-Smoothing:
  - May smooth out fine details too much during upscaling, leading to a soft image.

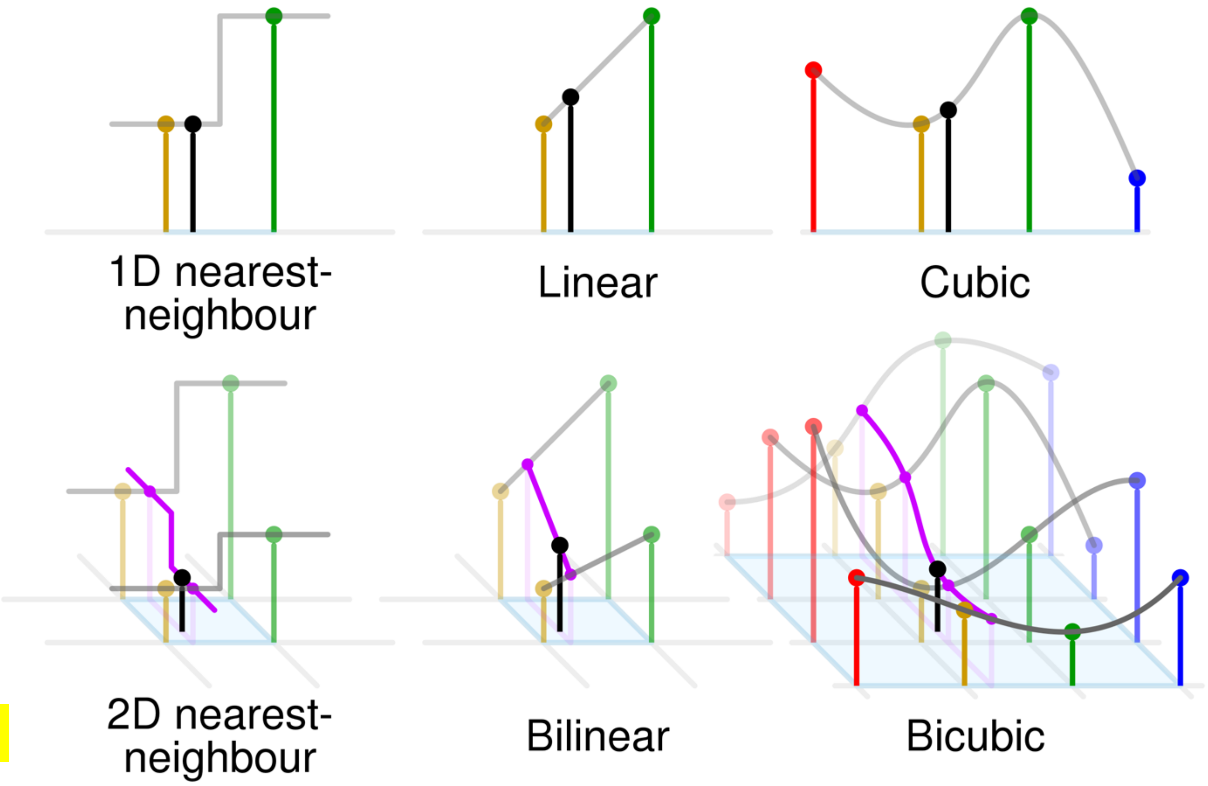
Bicubic interpolation: Best use cases

- Moderate upscaling where image sharpness is not the highest priority but smoothness is.
- General-purpose resizing for photographs and images with a balance of speed and quality.

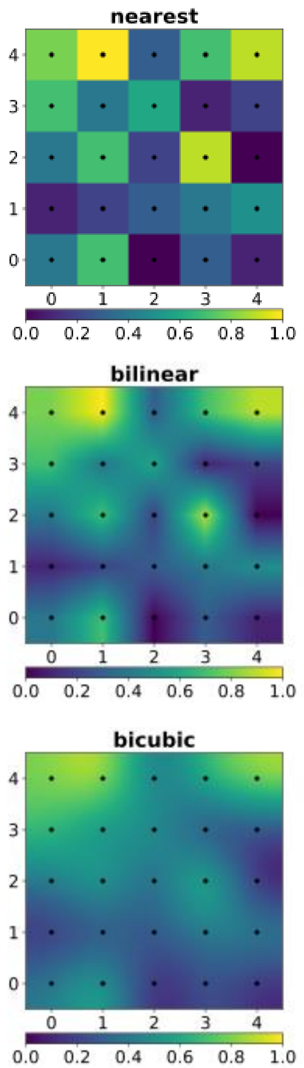
Summary

- Image interpolation methods
- Nearest neighbor interpolation
- Bilinear interpolation

Black dot denotes the sampled pixel value



(CMG Le. Wikipedia)



On image interpolation

https://www.menti.com/bltyg9abucso