Image Gradients

Computational Photography (CSCI 3240U)

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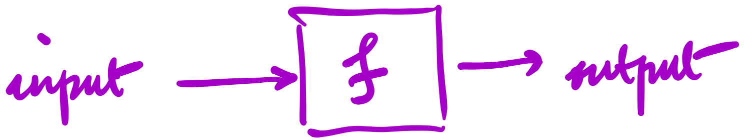
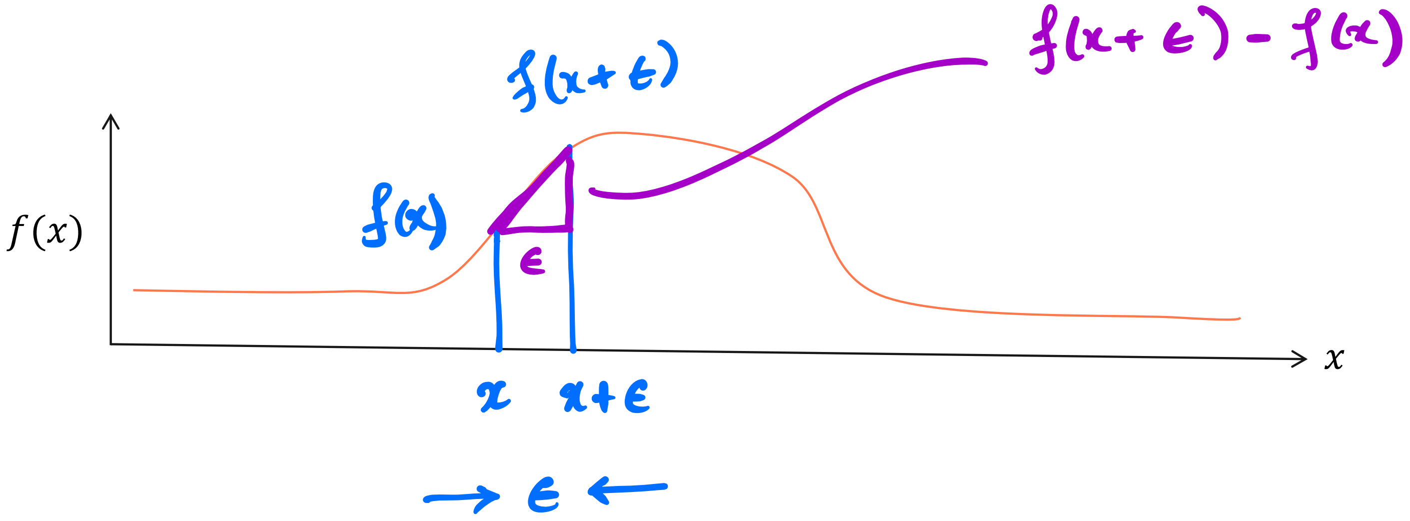
Today’s lecture

- Why do we care about image gradients?
- Computing image gradients
- Sobel filters
- Gradient magnitude and directions
- Visualizing image gradients

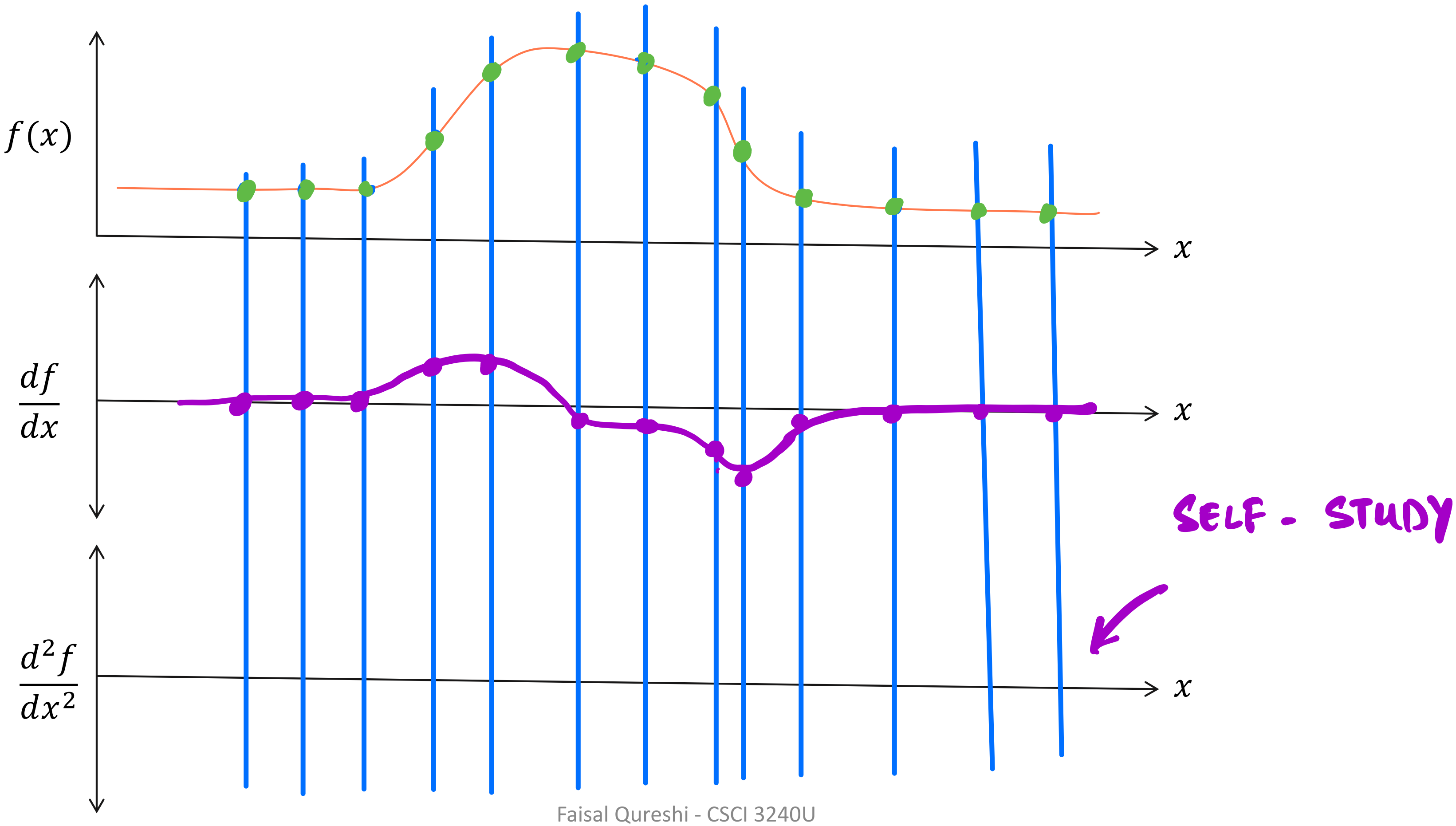
Derivative



$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$



Derivative



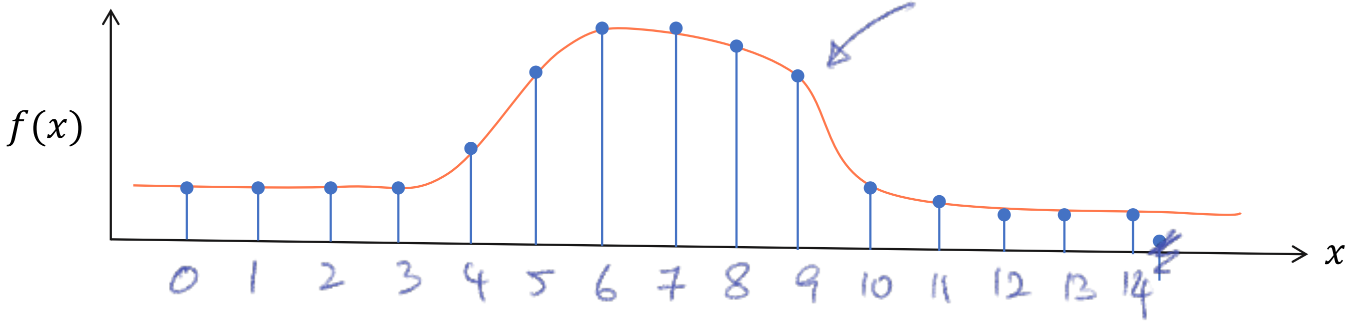
Derivative

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

$f(x) = x^2 \sin(x)$

$f'(x) = 2x \sin(x) + x^2 \cos(x)$

we have discrete fn.



$\epsilon = 1$

Finite-difference approximation

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

Use finite difference approximation to compute image derivatives

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

edge edge

$I =$

	0	1	2	3	4	5	6
I	1	1	9	8	6	0	0
I'	∅	8	-1	-2	-6	0	
I''	8	-9	-1	-4	6		

Observation: (1) derivative magnitude correspond to edges.
(2) sign tells going up or down

Use finite difference approximation to compute image derivatives

$I * [1,1,1] \rightsquigarrow$ Image blur

$I * G_\sigma \rightsquigarrow$ Gaussian blur

POWER OF LINEAR FILTERING

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

$I =$

0	1	2	3	4	5	6
1	**1**	**9**	**8**	**6**	**0**	**0**

$I' =$

0	**8**	**-1**	**-2**	**-6**	**0**	**?**

signal Filter

$I * [1, -1] =$

0	8	-1	-2	-6	0	//////

convolution

Partial derivatives

Grayscale: $I(x,y)$

Color image (RGB): $I(x,y,c)$

Hyperspectral image: $I(x,y,\lambda_1,\lambda_2,\cdots,\lambda_n)$

ASIDE: $f(x,y;\theta)$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

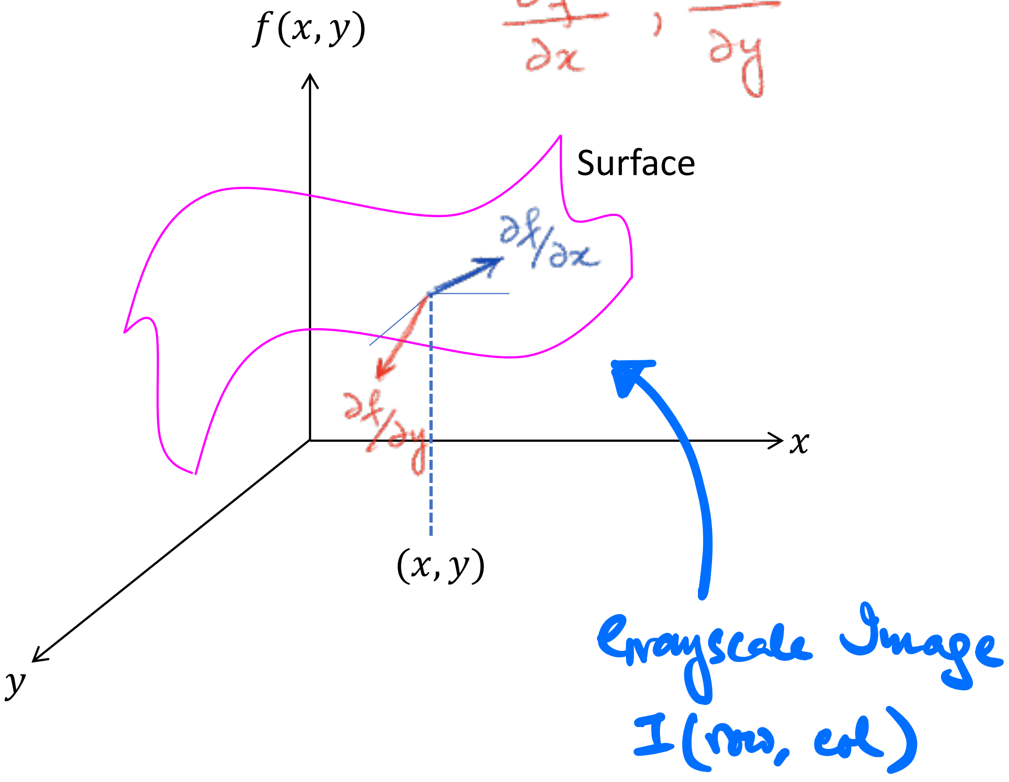


Image derivatives in x and y directions

$I =$

1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

This is a 2D fn.

$\frac{\partial I}{\partial x}$

$I_x = I * [1, -1] =$

0	8	-1	-7	
0	0	0	0	
2	2	3	-7	
-2	-1	6	-2	

$I_y = I * [1, -1]^T =$

$\frac{\partial I}{\partial y}$

Image gradient ∇I

$$\nabla I = \left[\frac{\partial I(x,y)}{\partial x}, \frac{\partial I(x,y)}{\partial y}\right]$$

$I_x =$

0	8	-1	-7	
0	0	0	0	
2	2	3	-7	
-2	-1	6	-2	

$I_y =$

7	7	-1	0	7
-7	-5	-3	0	7
4	0	-3	0	5

$\nabla I_{0,0} = [0, 7]^T$

$I =$

1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

x

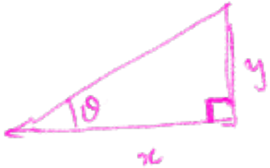
y

$\nabla I_{2,2} = [3, -3]^T$

Gradient direction and magnitude

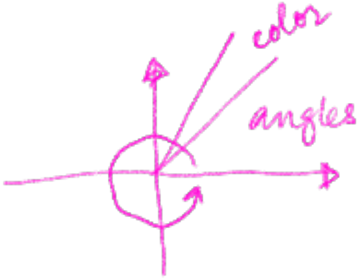
$$\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

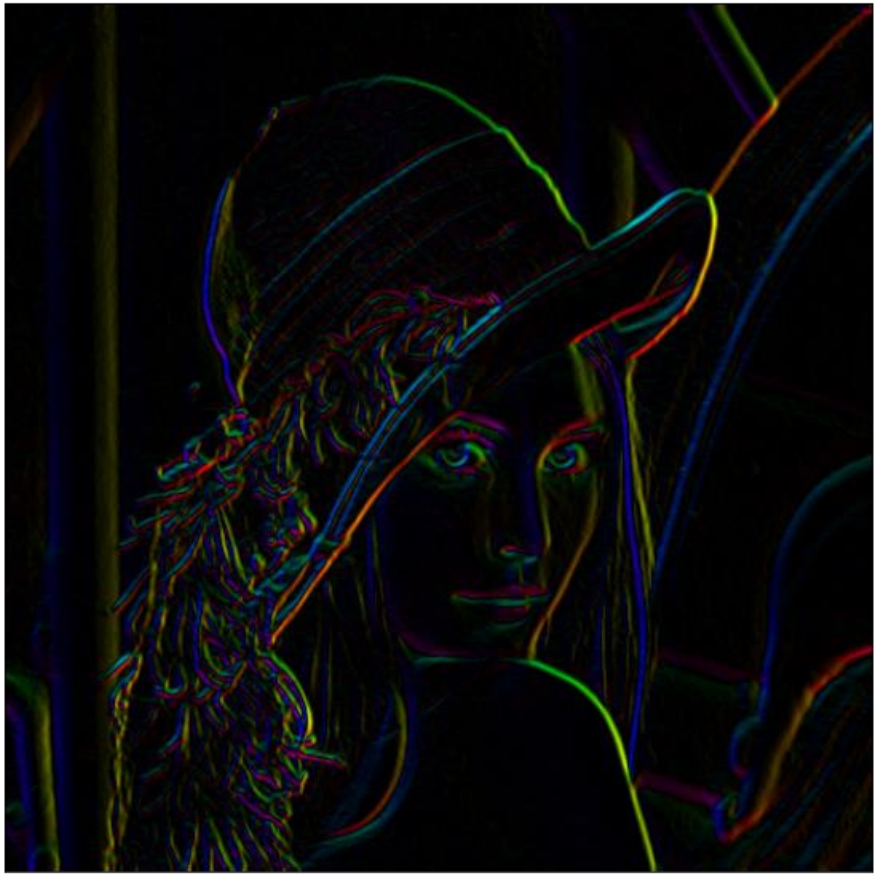
$$\theta = \tan^{-1}\left(\frac{\partial I}{\partial y} \Big/ \frac{\partial I}{\partial x}\right)$$



$$\nabla I_{2,2} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$\|\nabla I_{2,2}\| = \sqrt{9+9} = \sqrt{18}$$





Filters for computing image derivatives

Sobel

$$H_x = \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline\end{array} \qquad H_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline\end{array}$$

Prewire

$$H_x = \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline 1 & 0 & -1 \\ \hline\end{array} \qquad H_y = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline\end{array}$$

Roberts

$$H_x = \begin{array}{|c|c|}\hline 0 & -1 \\ \hline 1 & 0 \\ \hline\end{array} \qquad H_y = \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & -1 \\ \hline\end{array}$$

Image noise and gradients

I Goal: I'

$\nabla * I$ poor result

some filter for computing derivatives

$\nabla * (\underbrace{G_\sigma * I}_{\text{blur it first}})$ better results

re-write

$(\nabla * G_\sigma) * I$

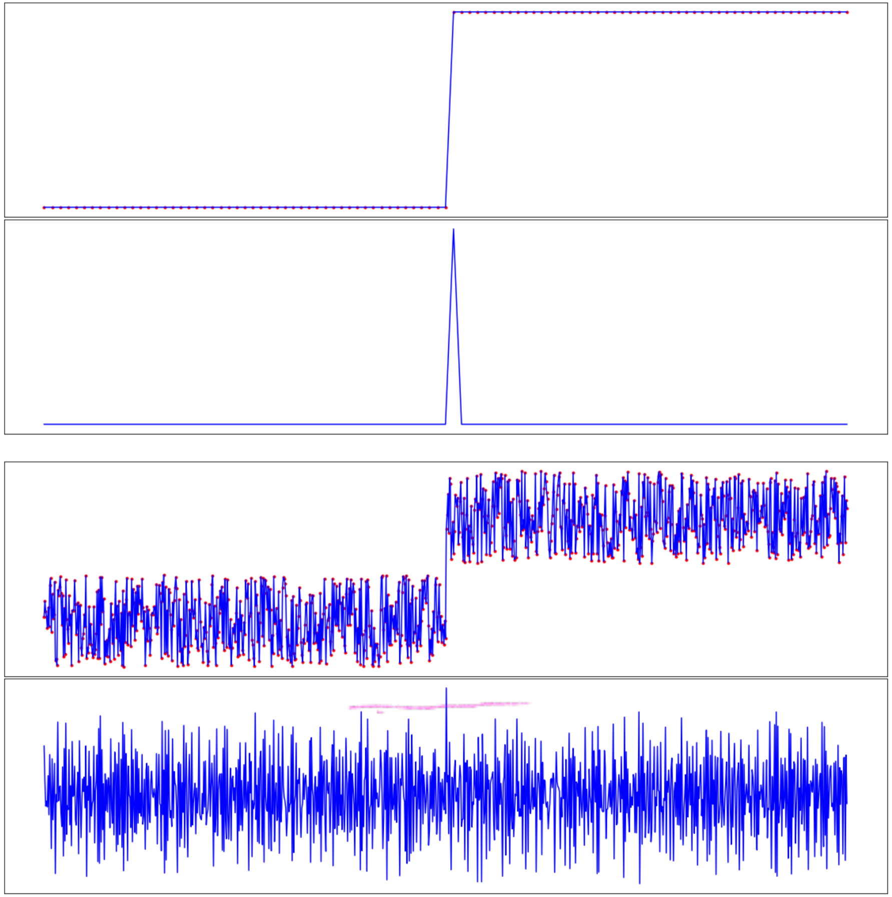
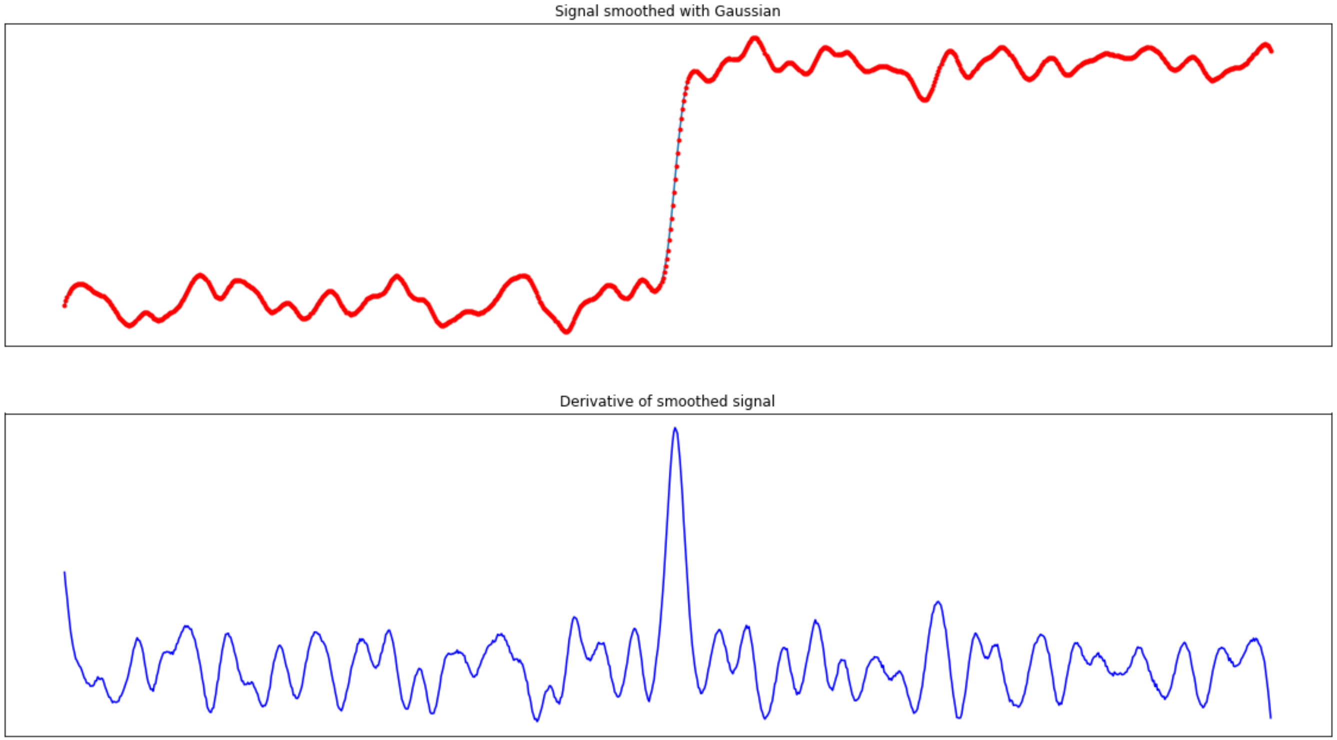


Image noise and gradients



Summary

- Image gradients
- Finite-difference approximation filters
- Gradient magnitude and direction
- Image noise and gradients