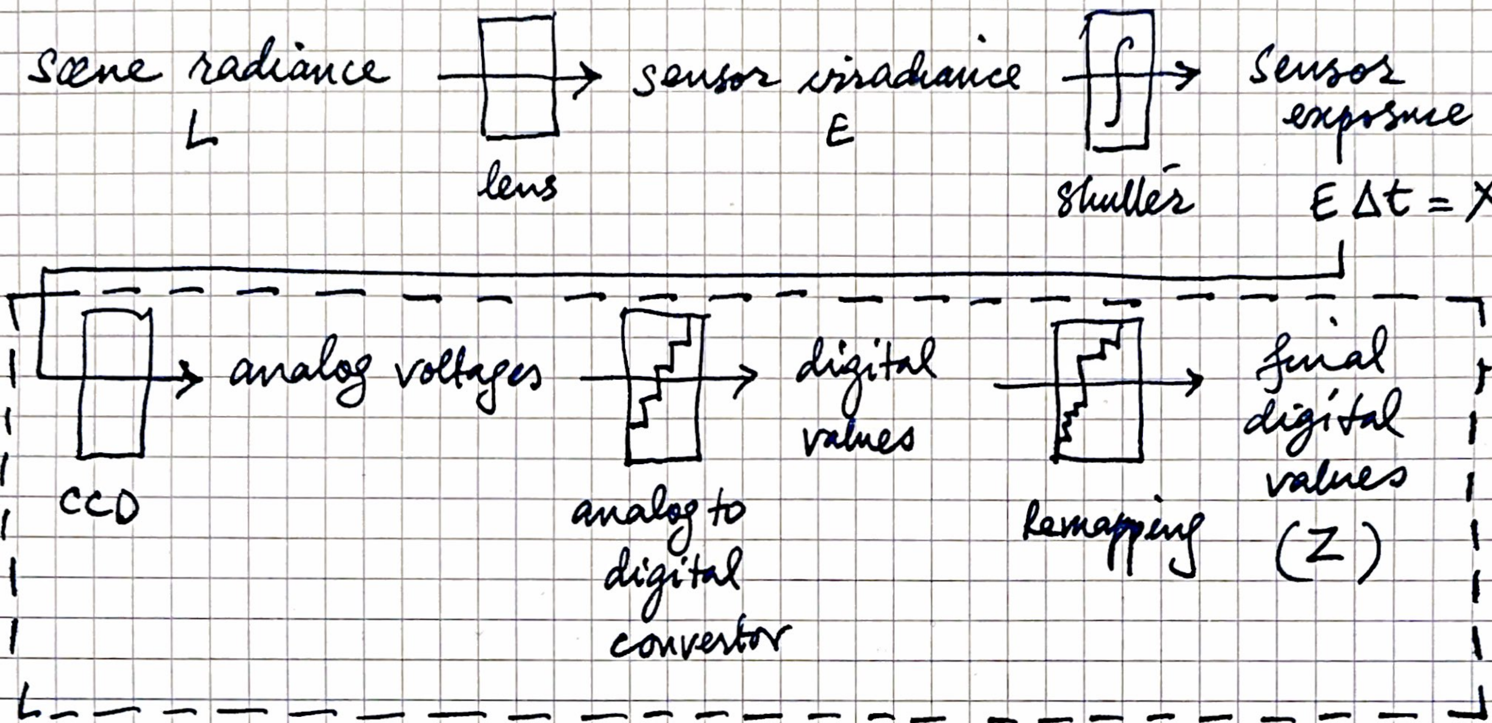
High Dynamic Range (HDR) Photography



$$Z = f(E\Delta t)$$

observed pixels

some unknown function.
(camera response function)

Q. How to compute the camera response function?

Idea 1: Invert f and use logs.

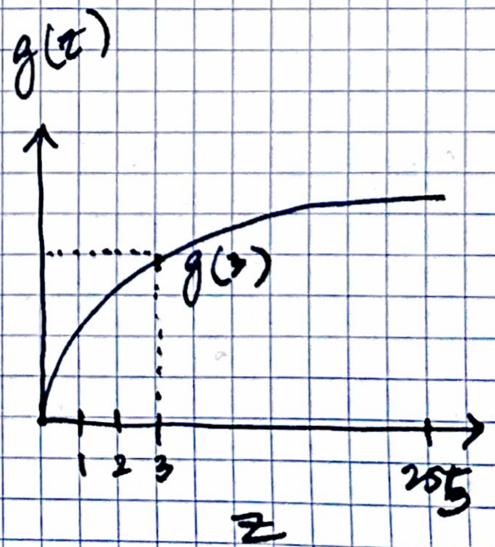
$$Z = f(E\Delta t)$$

$$f^{-1}(Z) = E\Delta t$$

$$\log f^{-1}(Z) = \log E + \log \Delta t$$

$$g(Z) = \log E + \log \Delta t$$

Need to compute these values.

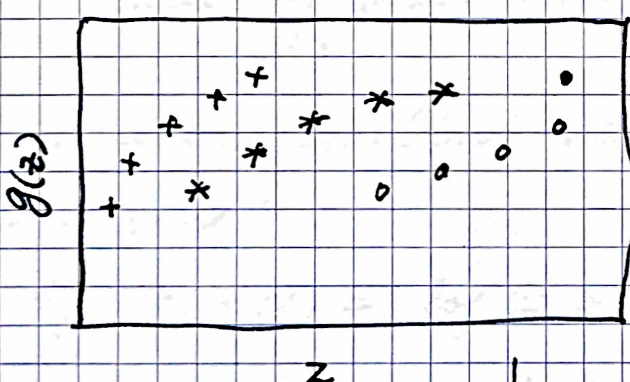


Q. How many unknowns?

$g(0), g(1), \ldots, g(255) \longrightarrow$ 256 unknown values.

Estimating $g(z)$

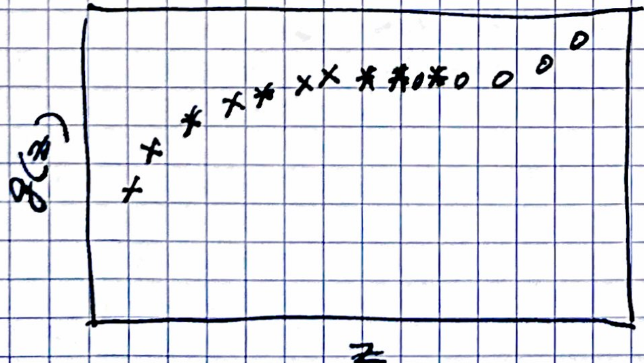
* Approach 1.
  - One pixel and many images.
  - Finely adjust Δt in range $[0, \infty]$
  - Plot $\log \Delta t$ as a function of the pixel observed intensity.
  - What about $\log E$, It is unknown. Does it matter?
    - $\log E$ is a constant.
* Approach 2
  - Few pixels and few images.



3 pixels and 5 images

plot of $g(z)$ from three pixels

↓ computed function.



Plot of normalized $g(z)$ after determining pixel exposures.

Q. What are the benefits of using Approach 2 over Approach 1?

A. We need to take a lot more images for Approach 1 to work.

* Approach 2: N pixels with P images.

$$g(Z_{ij}) = \log E_i + \log \Delta t_j$$

i^{th} pixel

in j^{th} image

irradiance for i^{th} pixel

exposure interval of j^{th} image.

Goal: compute $g(0), g(1), \ldots, g(255)$

compute $\log E_i$

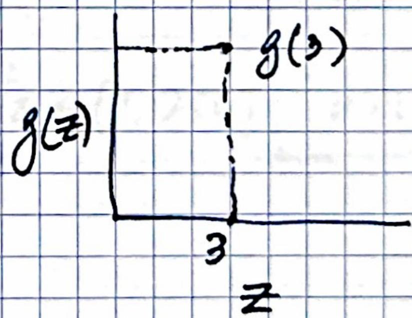
given N pixel intensities in P images
(i) (j)

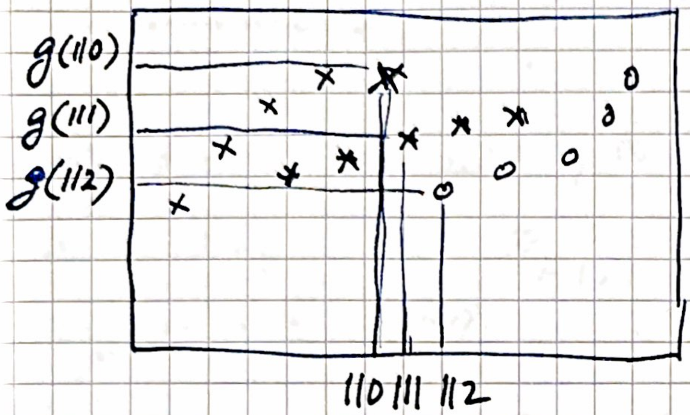
Q. How many equations we have and how many unknowns?

A. $\forall_{ij}$ $g(Z_{ij}) = \log E_i + \log \Delta t_j$ — ①

$(N)(P)$ equations

$N + 256$ unknowns





Observation: we want

$$g(111) - g(110) = g(112) - g(111)$$

$$\Rightarrow \quad 2g(111) - g(110) - g(112) = 0$$

Extend this to all z values.

$$\forall_{z \in [1,254]} \quad 2g(z) - g(z-1) - g(z+1) = 0 \quad \text{— (2)}$$

We can not make this equation for $z = 0$ and for $z = 255$.

For (2), 254 equations with 256 unknowns.

Algorithm:

- Solve a linear system composed of
  - $(N)(P)$ equations (1)
  - 254 equations (2)

  having $(N + 256)$ unknowns.

Lets try to solve this system.

			# Eq.s
(1)	$g(Z_{ij}) = \log E_i + \log \Delta t_j$	$\forall_{ij}$	$(N)(P)$
(2)	$2g(z) - g(z-1) - g(z+1) = 0$	$\forall_{z \in [1,254]}$	254

Simplified notation:

$$g(Z_{ij}) = g_{ij}$$

$$\log E_i = e_i$$

$$\log \Delta t_j = \delta_j$$

① becomes $\quad g_{ij} = e_i - \delta_j$

Say pixel 99 in 7^{th} image has intensity $z_{99,7} = 128$. Then the associated equation is:

$$g_{128} - e_{99} = \delta_7 .$$

Once again the goal is to estimate $g_0, g_1, \cdots, g_{255}$ and $e_0, e_1, \dots, e_N$ using ① and ②. Lets write out the vector of unknowns. Remember that the goal remains to solve $Ax = b$.

$$\underset{\substack{\uparrow \\ \text{vector of unknowns.}}}{x}$$

$$x = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{255} \\ e_0 \\ e_1 \\ \vdots \\ e_N \end{bmatrix}$$

Putting it all together.

$$\left[\begin{array}{c|c} 0 \cdots 0 \; \underset{\substack{\uparrow \\ z_{ij}^{th} \text{ column}}}{1} \; 0 \cdots 0 & 0 \cdots 0 \; \underset{\substack{\uparrow \\ 256+i \\ \text{column}}}{-1} \; 0 \cdots\cdots 0 \end{array}\right] \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{z_{ij}} \\ \vdots \\ g_{255} \\ e_0 \\ e_1 \\ \vdots \\ e_i \\ \vdots \\ e_N \end{bmatrix} = \delta_j$$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} & & & \text{column } z_{ij} & & & & & & & \text{column } 256+i & & & \end{array} \\
ij^{th}\text{ row} & \left[\begin{array}{cccccccccccccc} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \end{array}\right]
\end{array}
\begin{bmatrix} g_0 \\ \vdots \\ g_{255} \\ e_1 \\ \vdots \\ e_N \end{bmatrix} = \begin{bmatrix} \vdots \\ \delta_j \\ \vdots \end{bmatrix}$$

co-efficients of $g_0 \cdots g_{255}$ | co-efficients of $e_1 \cdots e_N$

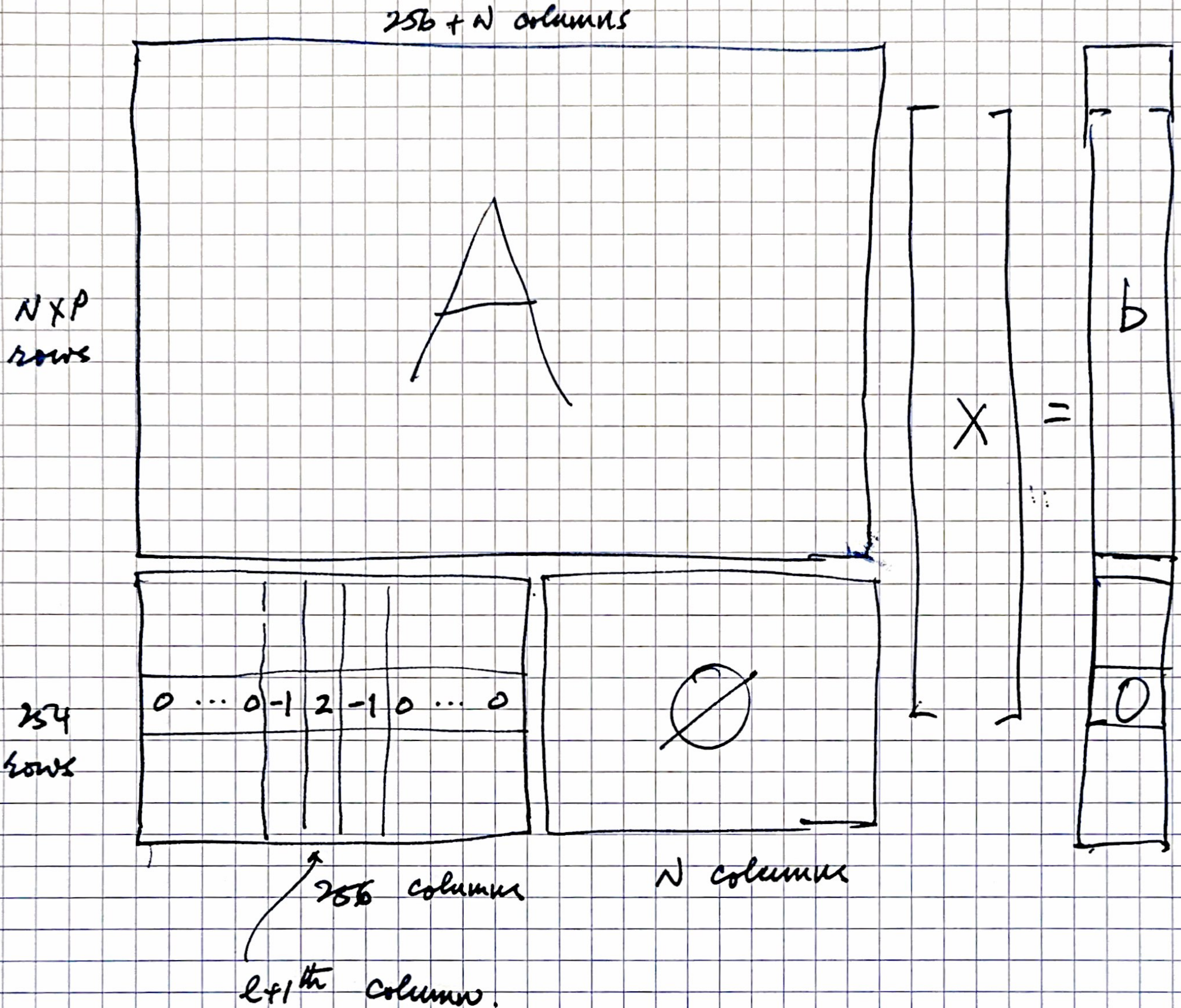
If pixel i has intensity z_{ij} in image j

This is an $Ax = b$ system that we can solve. However we need to ensure that we have more (or the) same equations as the number of unknowns $(256+N)$. ~~However~~ We can achieve this by using an appropriate number of pixels (N) and images (P).
However this is inefficient to solve and we can fix this issue by using additional equations using 2.

Intuition behind 2

- Real camera response function varies smoothly.
- Add more equations to enforce smoothness.
- 2 forces near constant rate of change.

$$2g(z) - g(z-1) - g(z+1) = 0.$$


256 + N columns
N x P rows
A
X
=
b
254 rows
0 ... 0 -1 2 -1 0 ... 0
∅
0
256 columns
N columns
l+1th column.