

Sample Midterm Exam 2

Simulation and Modeling (CSCI 3010U)

Practice Exam (80 min.)

Family name: _____

Given names: _____

Student number: _____

Question	Marks		
1	_____	/	15
2	_____	/	15
3	_____	/	15
4	_____	/	15
5	_____	/	10
Total	_____	/	70

Instructions

- Write in pen.
- No need to use a calculator. Express your answers as fractions.
- Be happy.
- Read widely.
- Space for rough work is provided at the end of this booklet.
- Always pack sunscreen.
- Total pages: 11.

Question 1 [15 Marks]: Collision Response

Two disks are moving on a frictionless 2D surface. Disk A has mass $m_A = 2 \text{ kg}$ and is moving with velocity $\mathbf{v}_A = (3, 0) \text{ m/s}$. Disk B has mass $m_B = 1 \text{ kg}$ and is moving with velocity $\mathbf{v}_B = (-1, 0) \text{ m/s}$. The disks collide head-on along the x-axis. The coefficient of restitution is $e = 0.5$.

[Part A] (20%)

Write down the formula for the coefficient of restitution e in terms of the velocities before and after collision.

[Part B] (30%)

Using conservation of momentum, write down the equation relating the velocities before and after collision.

[Part C] (50%)

Calculate the final velocities \mathbf{v}'_A and \mathbf{v}'_B of both disks after the collision.

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Question 2 [15 Marks]: Random Number Generation

[Part A] (40%)

A Linear Congruential Generator (LCG) is defined by the recurrence relation:

$$X_{n+1} = (a \cdot X_n + c) \pmod{m}$$

Given $a = 5$, $c = 3$, $m = 8$, and seed $X_0 = 0$, compute the first 8 values generated by this LCG (i.e., X_1, X_2, \dots, X_8).

[Part B] (30%)

What is the period of this generator? Explain your answer.

[Part C] (30%)

Explain one limitation of using a Linear Congruential Generator for Monte Carlo simulations.

Question 3 [15 Marks]: Monte Carlo Integration

You wish to estimate the value of the integral:

$$I = \int_0^1 x^2 dx$$

using Monte Carlo integration.

[Part A] (30%)

Write down the Monte Carlo estimator for this integral using N random samples uniformly distributed in $[0, 1]$.

[Part B] (40%)

Suppose you draw the following 5 random samples from a uniform distribution on $[0, 1]$:

$$x_1 = 0.2, \quad x_2 = 0.4, \quad x_3 = 0.6, \quad x_4 = 0.8, \quad x_5 = 1.0$$

Use these samples to estimate the value of the integral I . Show your work.

[Part C] (30%)

The true value of $\int_0^1 x^2 dx = \frac{1}{3}$. How does your estimate compare? If you wanted to improve the accuracy of your Monte Carlo estimate, what could you do?

Question 4 [15 Marks]: Discrete Event Simulation

A car wash has a single washing bay. Cars arrive at the following times: Car A arrives at time $t = 0$ minutes, Car B arrives at time $t = 3$ minutes, and Car C arrives at time $t = 5$ minutes. Each car takes exactly 4 minutes to wash.

[Part A] (30%)

Define the following terms in the context of Discrete Event Simulation:

(i) **Entity:**

(ii) **Resource:**

(iii) **Queue:**

[Part B] (50%)

Complete the following table showing the state of the car wash over time:

Time (min)	Event	Washing Bay Status	Queue Length
0			
3			
4			
5			
8			
12			

[Part C] (20%)

Calculate the total time each car spends in the system (waiting time + service time):

- Car A: _____
- Car B: _____
- Car C: _____

Question 5 [10 Marks]: Random Walks

Consider a 1D random walk where a particle starts at position $x = 0$. At each time step, the particle moves either $+1$ (right) or -1 (left) with equal probability.

[Part A] (50%)

A particle performs the following random walk starting at $x = 0$:

Step 1: $+1$, Step 2: $+1$, Step 3: -1 , Step 4: $+1$, Step 5: -1 , Step 6: -1

(i) What is the final position of the particle after 6 steps?

(ii) What is the total distance traveled by the particle?

[Part B] (50%)

If you simulate 1000 random walks, each consisting of 100 steps, and record the final position of each walk:

(i) What do you expect the average final position to be? Explain your reasoning.

(ii) Would all particles end up at the same position? Why or why not?

Rough work

Rough work

End of exam.