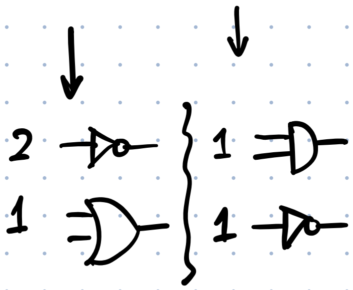
February 11, 2026

Focus: Digital logic optimization

a	b	$a'+b'$	$(ab)'$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



* Boolean Algebra - Axioms

(i)

AND Form	OR Form	
$1x = x$	$0 + x = x$	Identity law
$0x = \emptyset$	$1 + x = 1$	Null law
$xx = x$	$x + x = x$	Idempotent law
$xx' = \emptyset$	$x + x' = 1$	Inverse law

(ii) $x'' = x$ Double compliment law

(iii)

AND Form	OR Form	
$xy = yx$	$x+y = y+x$	commutative law
$(xy)z = x(yz)$	$(x+y)+z = x+(y+z)$	associative law
$x+yz = (x+y)(x+z)$	$x(y+z) = xy + xz$	distributive law

(iv)

$x(x+y) = x$	$x + xy = x$	Absorption law
$(xy)' = x'+y'$	$(x+y)' = x'y'$	DeMorgan's law

* Example: $f(x,y,z) = xy + x'z + yz$ → 5 gates

$= xy + x'z + yz1$ identity law

$= xy + x'z + yz(x+x')$ inverse law

$= xy + x'z + yzx + yzx'$ distributive law

$= xy + x'z + xyz + x'yz$ commutative law ×2

$= xy + x'z + (xy)z + (x'z)y$ associative law ×2

$= xy + (xy)z + x'z + (x'z)y$ commutative

$= xy(1+z) + x'z(1+y)$ distributive

$= xy + x'z$ → 4 gates null ×2

* Example: $f(x,y,z) = (x+y)(x+x)$

$= (x+y)x$

$= x + xy$

$= x$

* $f(x,y,z) = xy'z' + xy'z + xyz' + xyz$

$= x(y'z' + y'z + yz' + yz)$

$= x(y'\underbrace{(z'+z)}_{=1} + y\underbrace{(z'+z)}_{=1})$

$= x\underbrace{(y'+y)}_{=1}$

$= x1$

$= x$

* Algebraic approach → not obvious what to do next.
  → systematic approach

Boolean expression simplification has been known to be NP-complete.