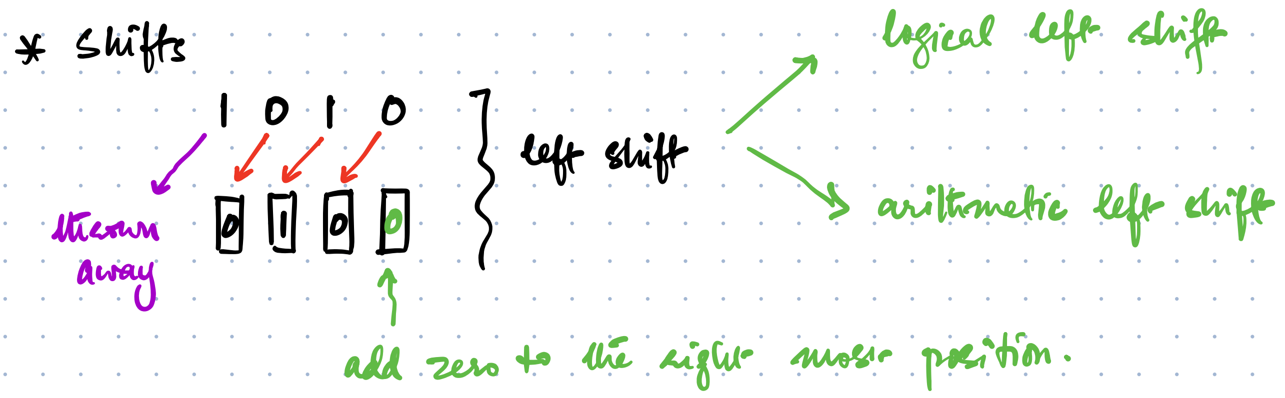
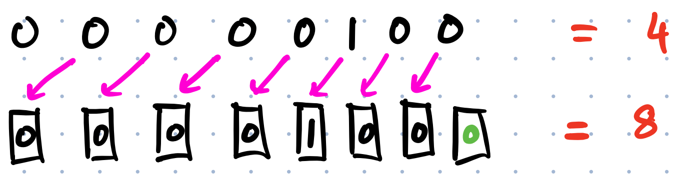
February 4, 2026

* Shifts



Example: what does left-shift do?



left shift by 1-bit is ×2 = 2^1

2-bits ×4 = 2^2

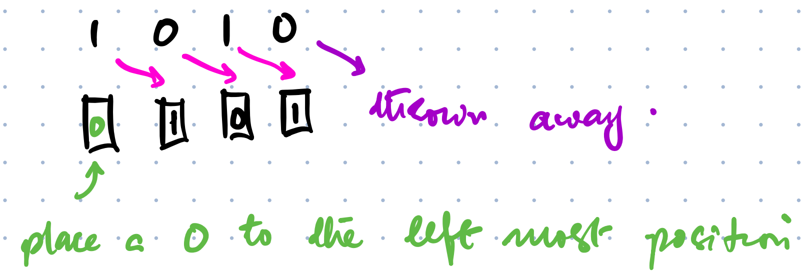
3-bits ×8 = 2^3

⋮

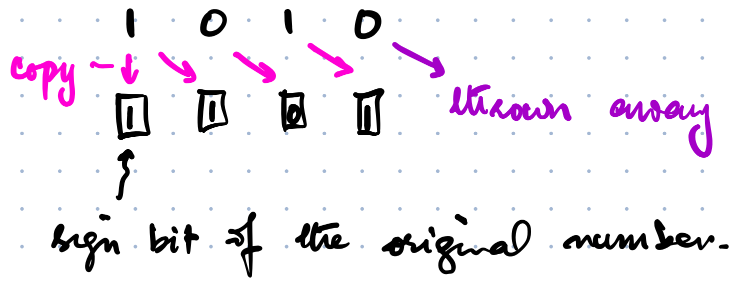
n-bits × 2^n

* Right Shift

(i) logical right shift



(ii) Arithmetic right shift



Takeaways:

(i) Arithmetic left shift by n bits = $\times 2^{n}$

(ii) Arithmetic right shift by n bits = $\times 2^{-n}$

¥ Multiply Binary Numbers

Example: compute $11 \times (-2)$

(task 1): Express -2 as two's compliment. (use 8-bits)

(i) 2: 0 0 0 0 0 0 1 0

(ii) 1's compliment: 1 1 1 1 1 1 0 1

(iii) +1: $\boxed{1\ 1\ 1\ 1\ 1\ 1\ 1\ 0}$ -2

(task 2): Break 11 as powers of two's.

$$11 = 8 + 2 + 1$$
$$= 2^3 + 2^1 + 2^0$$

(task 3): lets multiply.

$$11 \times (-2) = (2^3 + 2^1 + 2^0) \times (-2)$$
$$= \underbrace{2^3 \times (-2)}_{\substack{\text{left shift by}\\ \text{3-bits}\\ \text{(a)}}} + \underbrace{2^1 \times (-2)}_{\substack{\text{left shift by}\\ \text{1-bit}\\ \text{(b)}}} + \underbrace{2^0 \times (-2)}_{\substack{\text{do nothing}\\ \text{(c)}}}$$

-2: 1 1 1 1 1 1 1 0

(a) $\boxed{1\ 1\ 1\ 1\ 0\ 0\ 0\ 0}$

(b) 1 1 1 1 1 1 0 0

(c) 1 1 1 1 1 1 1 0

① 1 1 1 1 1

1 1 1 1 1 1 0 0

1 1 1 1 1 1 1 0

---

① 1 1 1

1 1 1 1 1 0 1 0

1 1 1 1 0 0 0 0

---

negative number ← ① 1 1 0 1 0 1 0

① 1 1 0 1 0 1 0

Flip 0 0 0 1 0 1 0 1

Add 1 0 0 0 1 0 1 1 0

$$\cdots\ \underset{\underset{16}{\downarrow}}{2^4}\ 2^3\ \underset{\underset{4}{\downarrow}}{2^2}\ \underset{\underset{2}{\downarrow}}{2^1}\ 2^0 \quad = -22 .$$

Answer.

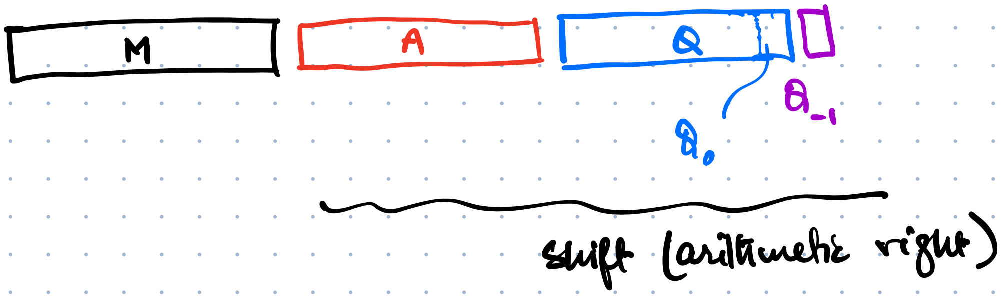
Booth's algorithm

- Speeds up the multiplication of two signed numbers in 2's compliment form.
- Reduce the numbers of additions and subtractions by looking for runs of 1.
- Compute $\underset{\text{Multiplicand}}{\underset{\uparrow}{M}} \times \underset{\text{Multiplier}}{\underset{\uparrow}{Q}}$

Accumulator : A, initialized to 0, same bit length as M.

Q_{-1} bit, initialized to 0, set to the right of Q.

Count, set to the number of bits in M.



Decision logic

Q_0	Q_{-1}	Actions	why
0	0	shift only	
0	1	A = A + M, then shift	
1	0	A = A − M, then shift	
1	1	Shift only.	

Shift Step

- Arithmetic right shift on the combined block of [A] [Q] [Q_{-1}]

Example: Compute 2 × (−3)

(i) M = 0 0 1 0

(ii) −M = 1 1 1 0 (−2)

(iii) Q = 1 1 0 1 (−3)

(iv) Q_{-1} = 0

(v) A = 0 0 0 0

0 0 1 0
→ 1 1 0 1 −2
1 1 1 0

0 0 1 1
1 1 0 0 −3
1 1 0 1

	Q_0 Q_{-1}	Q_0	Q_{-1}	
	0 0 0 0 | 1 1 0 1 | 0	1	0	A = A − M, shift
①	A−M 1 1 1 0 1 1 0 1 0 → 1 1 1 1 | 0 1 1 0 | 1	0	1	A = A + M, shift
②	0 0 0 1 0 1 1 0 1 → 0 0 0 0 | 1 0 1 1 | 0	1	0	A = A − M, shift
③	1 1 1 0 1 0 1 1 0 → 1 1 1 1 | 0 1 0 1 | 1	1	1	shift
④	1 1 1 1 | 1 0 1 0 | 1			

0 0 0 0
1 1 1 0
1 1 1 0

① 1 1
1 1 1 1
0 0 1 0
0 0 0 1

0 0 0 0
1 1 1 0
1 1 1 0

Final result : [A][Q].

$$\text{-ve}\ \boxed{\textcircled{1}\ 1\ 1\ 1\ 1\ 0\ 1\ 0} \longrightarrow \underline{\underline{-6}} = 2\times(-3)$$

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1 \\ 1\ + \\ \hline 0\ 0\ 0\ 0\ 0\ \underbrace{1\ 1\ 0}_{6} \end{array}$$