Layered architectures

Machine Learning (CSCI 5770G)

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Logistic regression and layers

- Lets look at how we can specify logistic regression as layers.
- The ability to specify such models as layers is key to designing neural networks.
- We will also discuss *backpropagation*.

2-class softmax classifier

2-class softmax classifier

- Define cost $C(\theta)$ that needs to be minimized to train this classifier.
- We set this cost equal to the negative log-likelihood for this 2-class softmax classifier

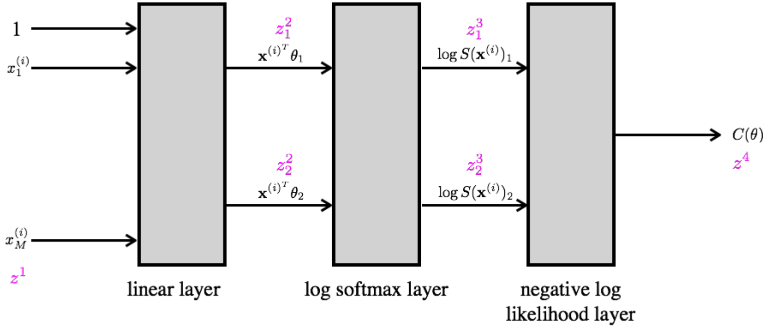
Negative log-likelihood

$$C(\theta) = l(\theta)$$
$$= -\sum_{i=1}^{N} \mathbb{I}_0(y^{(i)}) \log \frac{e^{\mathbf{x}^{(i)^T}\theta_1}}{e^{\mathbf{x}^{(i)^T}\theta_1} + e^{\mathbf{x}^{(i)^T}\theta_2}} + \mathbb{I}_1(y^{(i)}) \log \frac{e^{\mathbf{x}^{(i)^T}\theta_2}}{e^{\mathbf{x}^{(i)^T}\theta_1} + e^{\mathbf{x}^{(i)^T}\theta_2}}$$

Layer representation

$$C(\theta) = -\sum_{i=1}^{N} \mathbb{I}_0(y^{(i)}) \log \frac{e^{\mathbf{x}^{(i)T}\theta_1}}{e^{\mathbf{x}^{(i)T}\theta_1} + e^{\mathbf{x}^{(i)T}\theta_2}} + \mathbb{I}_1(y^{(i)}) \log \frac{e^{\mathbf{x}^{(i)T}\theta_2}}{e^{\mathbf{x}^{(i)T}\theta_1} + e^{\mathbf{x}^{(i)T}\theta_2}}$$

Layers



Chain rule

$$\frac{\partial f(g(u,v),h(u,v))}{\partial u} =$$

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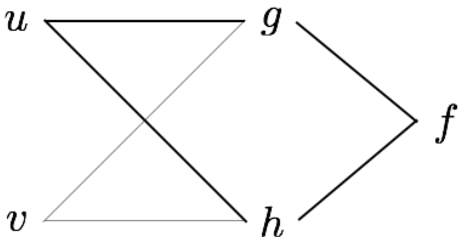
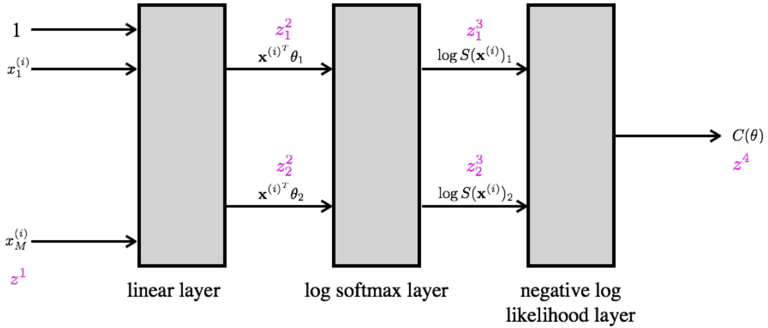
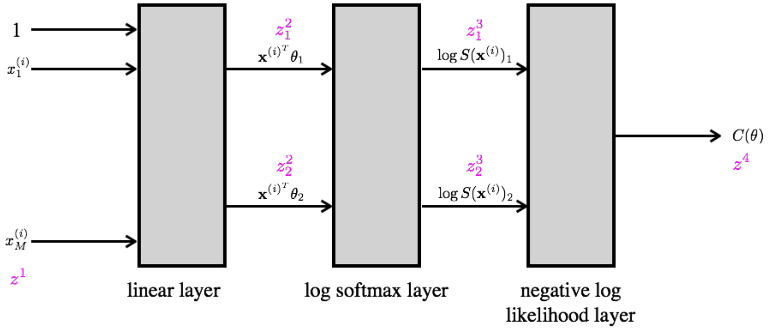


Layers



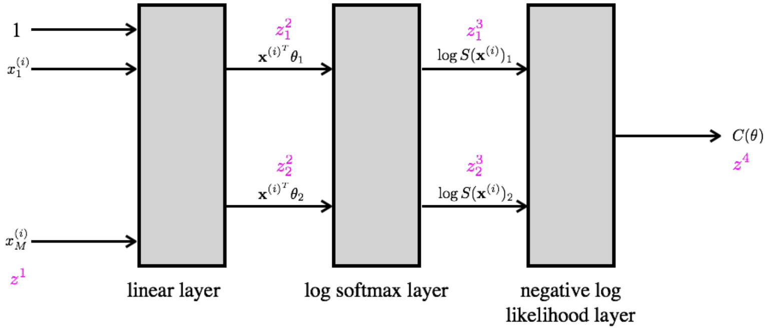
- What are our model paramters?

Layers



- What are our model paramters? (θ_1 and θ_2)
- We are interested in computing $\frac{\partial z^4}{\partial \theta_1}$ and ($\frac{\partial z^4}{\partial \theta_2}$)
  - Why?

Layers



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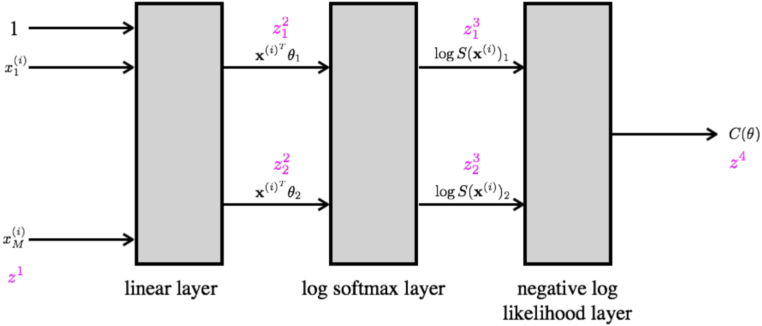
  - Why? (z^4 is the cost that we can minimize using gradient descent using these gradients)

Layers



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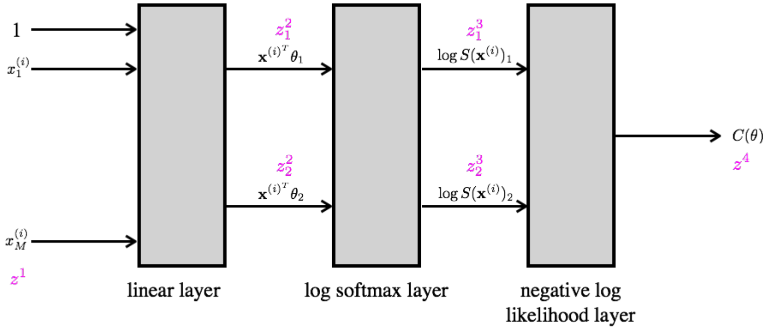
Layers

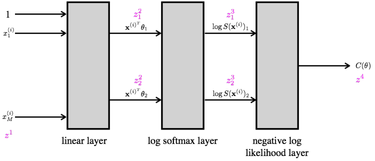


- What are our model paramters? (θ_1 and θ_2)
- We are interested in computing $\frac{\partial z^4}{\partial \theta_1}$ and ($\frac{\partial z^4}{\partial \theta_2}$)
  - Why? (z^4 is the cost that we can minimize using gradient descent using these gradients)
  - How do we even compute $\frac{\partial z^4}{\partial \theta_1}$? (or $\frac{\partial z^4}{\partial \theta_2}$)

Computing $\frac{\partial z^4}{\partial \theta_1}$

We can use the *chain rule* to compute $\frac{\partial z^4}{\partial \theta_1}$.

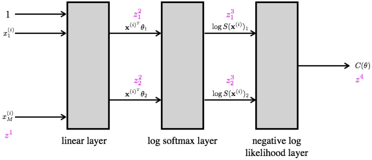
$$\frac{\partial z^4}{\partial \theta_1} =$$



Computing $\frac{\partial z^4}{\partial \theta_1}$

We can use the *chain rule* to compute $\frac{\partial z^4}{\partial \theta_1}$.

$$\frac{\partial z^4}{\partial \theta_1} = \frac{\partial z^4}{\partial z_1^3} \frac{\partial z_1^3}{\partial \theta_1}$$



Computing $\frac{\partial z^4}{\partial \theta_1}$

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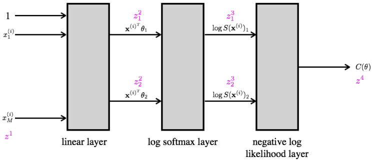
$$
\begin{aligned}
\frac{\partial z^4}{\partial \theta_1} &= \frac{\partial z^4}{\partial z_1^3}\frac{\partial z_1^3}{\partial \theta_1} + \frac{\partial z^4}{\partial z_2^3}\frac{\partial z_2^3}{\partial \theta_1} \\
&= \frac{\partial z^4}{\partial z_1^3}
\end{aligned}
$$



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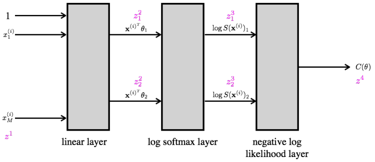
&= \frac{\partial z^4}{\partial z_1^3}\left(\frac{\partial z_1^3}{\partial z_1^2}\frac{\partial z_1^2}{\partial \theta_1} + \frac{\partial z_1^3}{\partial z_2^2}\frac{\partial z_2^2}{\partial \theta_1}\right)
\end{aligned}
$$



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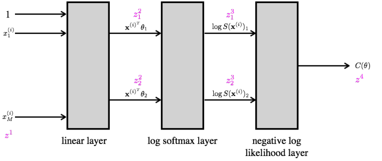
\frac{\partial z^4}{\partial \theta_1} =& \frac{\partial z^4}{\partial z_1^3}\frac{\partial z_1^3}{\partial \theta_1} + \frac{\partial z^4}{\partial z_2^3}\frac{\partial z_2^3}{\partial \theta_1} \\
=& \frac{\partial z^4}{\partial z_1^3}\left(\frac{\partial z_1^3}{\partial z_1^2}\frac{\partial z_1^2}{\partial \theta_1} + \frac{\partial z_1^3}{\partial z_2^2}\frac{\partial z_2^2}{\partial \theta_1}\right) \\
+& \frac{\partial z^4}{\partial z_2^3}
\end{aligned}
$$



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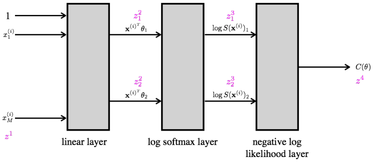
&= \frac{\partial z^4}{\partial z_1^3}\left(\frac{\partial z_1^3}{\partial z_1^2}\frac{\partial z_1^2}{\partial \theta_1} + \frac{\partial z_1^3}{\partial z_2^2}\frac{\partial z_2^2}{\partial \theta_1}\right) \\
&+ \frac{\partial z^4}{\partial z_2^3}\left(\frac{\partial z_2^3}{\partial z_1^2}\frac{\partial z_1^2}{\partial \theta_1} + \frac{\partial z_2^3}{\partial z_2^2}\frac{\partial z_2^2}{\partial \theta_1}\right)
\end{aligned}
$$



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=& \frac{\partial z^4}{\partial z_1^3}\left(\frac{\partial z_1^3}{\partial z_1^2}\frac{\partial z_1^2}{\partial \theta_1} + \frac{\partial z_1^3}{\partial z_2^2}\frac{\partial z_2^2}{\partial \theta_1}\right) \\
&+ \frac{\partial z^4}{\partial z_2^3}\left(\frac{\partial z_2^3}{\partial z_1^2}\frac{\partial z_1^2}{\partial \theta_1} + \frac{\partial z_2^3}{\partial z_2^2}\frac{\partial z_2^2}{\partial \theta_1}\right)
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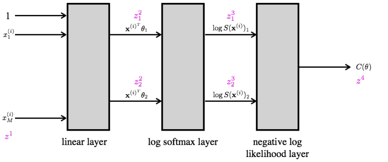
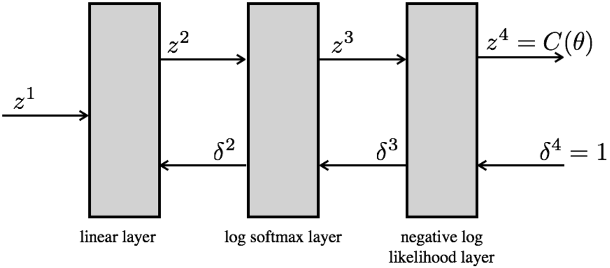


We can similarly compute $\frac{\partial z^4}{\partial \theta_2}$.

Notation

- z^l: input to layer l
- z^{l+1}: output of layer l
- $C(\theta)$: cost (or loss) to be minimized, e.g., negative log-likelihood for the case of the two-class softmax classifier
- Define
$$\delta^l = \frac{\partial C(\theta)}{\partial z^l}$$
- If L denotes the last layer, $\delta^L = 1$, since the output of the last layer is z^L which is equal to the cost $C(\theta)$, i.e.,
$$\delta^L = \frac{\partial C(\theta)}{\partial z^L} = \frac{\partial z^L}{\partial z^L} = 1$$

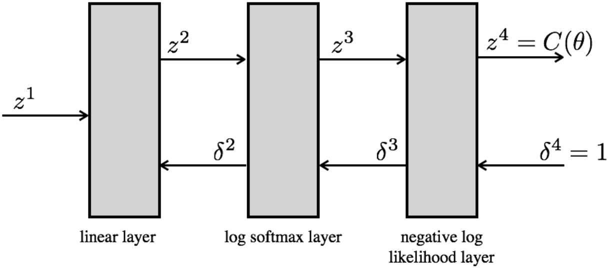
Forward pass and backward pass



Forward pass

Backward pass

Forward pass and backward pass

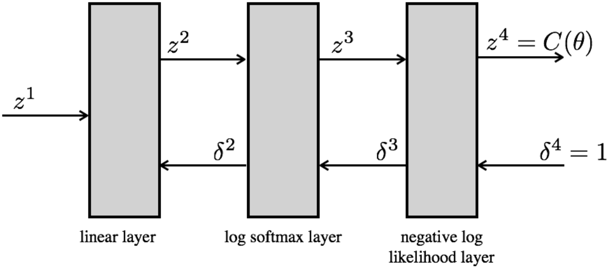


Forward pass

$z^1 = f(\mathbf{x})$ (*input data*)

Backward pass

Forward pass and backward pass



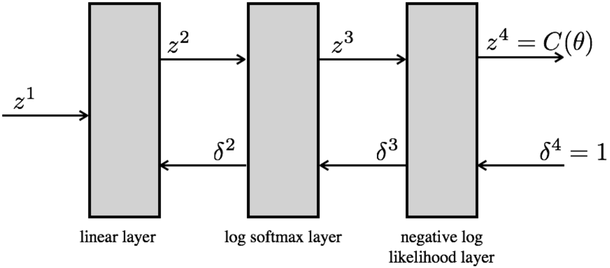
Forward pass

$z^1 = f(\mathbf{x})$ (*input data*)

$z^2 = f(z^1)$ (*linear function*)

Backward pass

Forward pass and backward pass

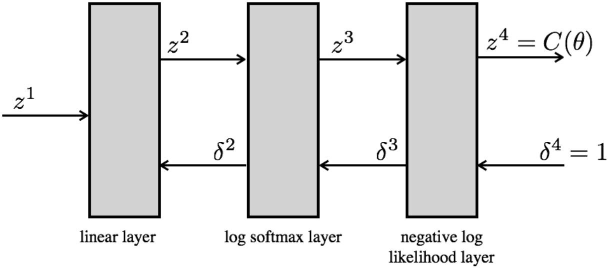


Forward pass

$z^1 = f(\mathbf{x})$ (*input data*)
$z^2 = f(z^1)$ (*linear function*)
$z^3 = f(z^2)$ (*log softmax*)

Backward pass

Forward pass and backward pass

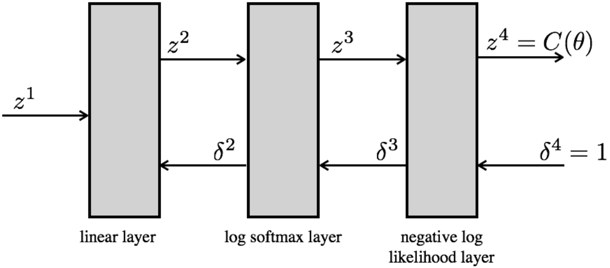


Forward pass

$z^1 = f(\mathbf{x})$ (*input data*)
$z^2 = f(z^1)$ (*linear function*)
$z^3 = f(z^2)$ (*log softmax*)
$z^4 = f(z^3) = C(\theta)$ (*negative log likelihood*, cost)

Backward pass

Forward pass and backward pass



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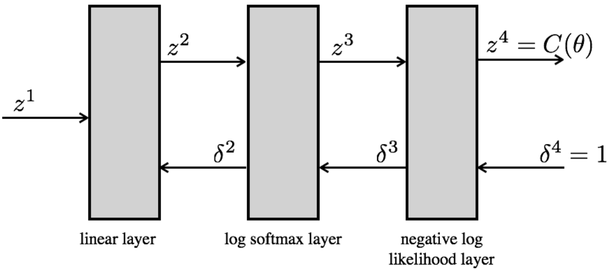
$z^3 = f(z^2)$ (*log softmax*)
$z^4 = f(z^3) = C(\theta)$ (*negative log likelihood,* cost)

Backward pass

$\delta^4 = \frac{\partial C(\theta)}{\partial z^4} = 1$

Forward pass and backward pass



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$z^3 = f(z^2)$ (*log softmax*)
$z^4 = f(z^3) = C(\theta)$ (*negative log likelihood*, cost)

Backward pass

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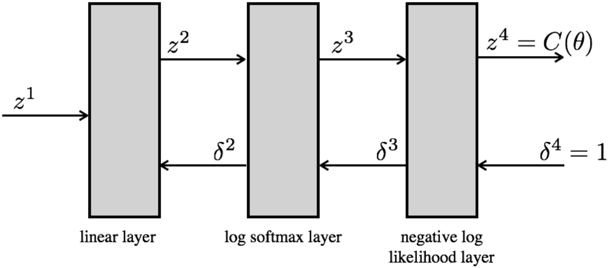
Forward pass and backward pass



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$z^4 = f(z^3) = C(\theta)$ (*negative log likelihood*, cost)

Backward pass

$\delta^4 = \frac{\partial C(\theta)}{\partial z^4} = 1$
$\delta^3 = \frac{\partial C(\theta)}{\partial z^3}$
$\delta^2 = \frac{\partial C(\theta)}{\partial z^2}$

Computing δ^l

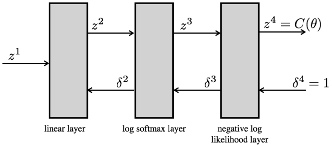
$$\delta^4 =$$

$$\delta^3_1 =$$

$$\delta^3_2 =$$

$$\delta^2_1 =$$

$$\delta^2_2 =$$



Computing δ^l

$$\delta^4 = \frac{\partial C(\theta)}{\partial z^4}$$

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$$\delta^3_2 =$$

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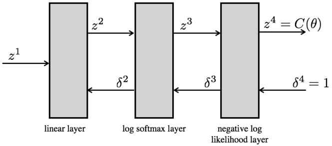

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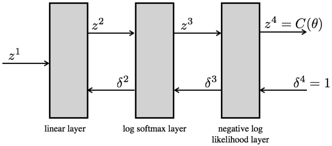

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$$\delta^3_1 = \frac{\partial C(\theta)}{\partial z^3_1} =$$

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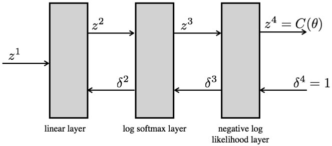

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$$\delta^3_2 =$$

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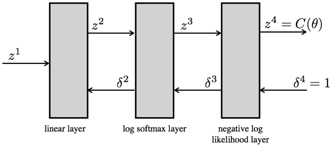


Computing δ^l

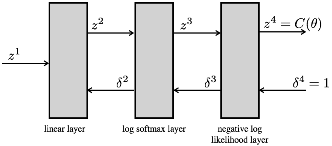
$$\delta^4 = \frac{\partial C(\theta)}{\partial z^4} = \frac{\partial z^4}{\partial z^4} = 1$$

$$\delta_1^3 = \frac{\partial C(\theta)}{\partial z_1^3} = \frac{\partial C(\theta)}{\partial z^4}\frac{\partial z^4}{\partial z_1^3} = \delta^4 \frac{\partial z^4}{\partial z_1^3}$$

$$\delta_2^3 =$$

$$\delta_1^2 =$$

$$\delta_2^2 =$$



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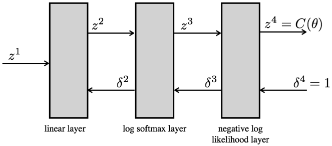
$$\delta_2^3 = \frac{\partial C(\theta)}{\partial z_2^3}$$

$$\delta_1^2 =$$

$$\delta_2^2 =$$



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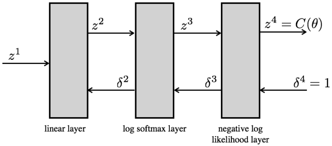
$$\delta^3_2 = \frac{\partial C(\theta)}{\partial z^3_2} = \frac{\partial C(\theta)}{\partial z^4}\frac{\partial z^4}{\partial z^3_2}$$

$$\delta^2_1 =$$

$$\delta^2_2 =$$



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$$\delta^2_1 =$$

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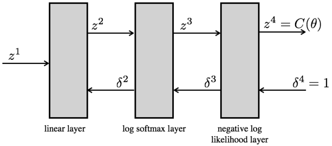

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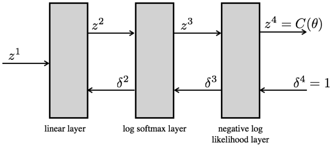
$$\delta^2_1 = \frac{\partial C(\theta)}{\partial z^2_1}$$

$$\delta^2_2 =$$



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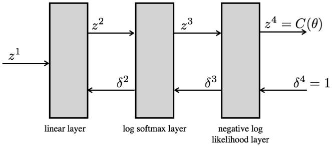

Computing δ^l

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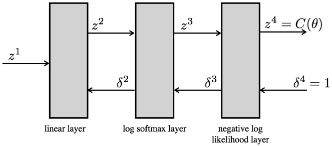

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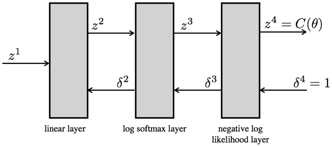

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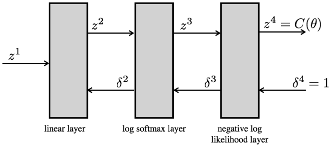

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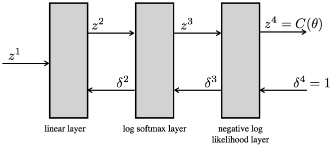
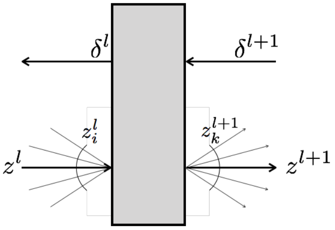


For any differentiable layer l



Layer l

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For a given layer l, with inputs z_i^l and outputs z_k^{l+1}

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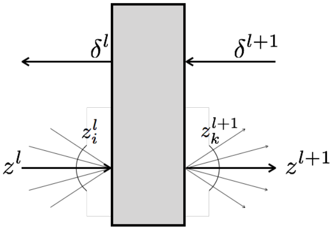

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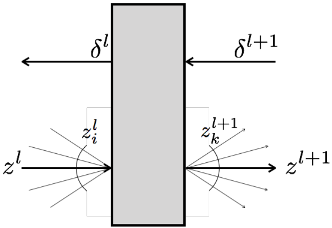

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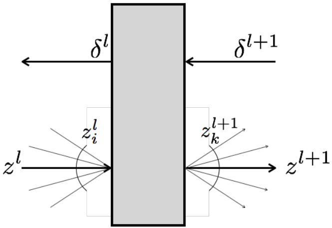

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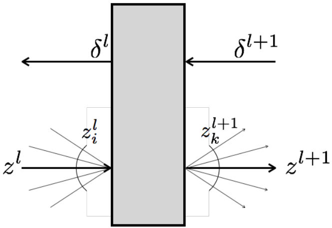

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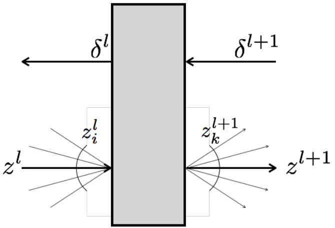

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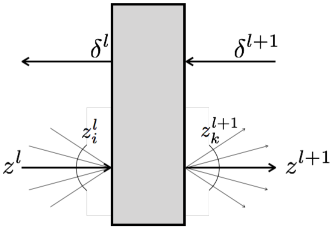

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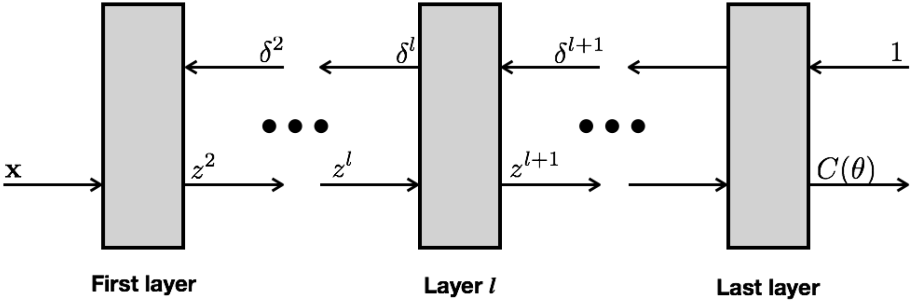
In our 2-class softmax classifier only layer 1 has parameters (θ_0 and θ_1)

Layered architectures

As long as we have differentiable layers, i.e., we can compute $\frac{\partial z_k^{l+1}}{\partial z_i^l}$, we can use *backpropagation* to update the parameters θ to minimize the cost $C(\theta)$.



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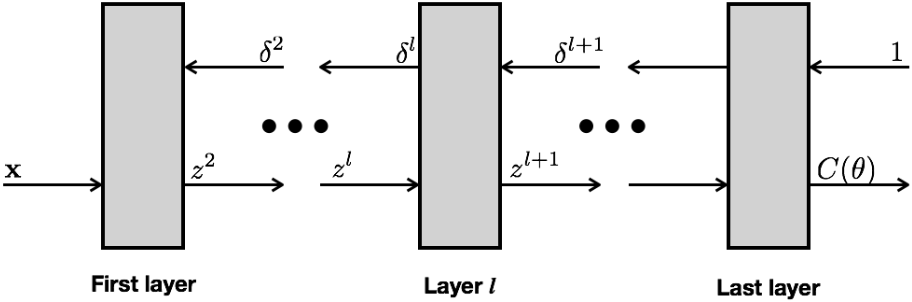

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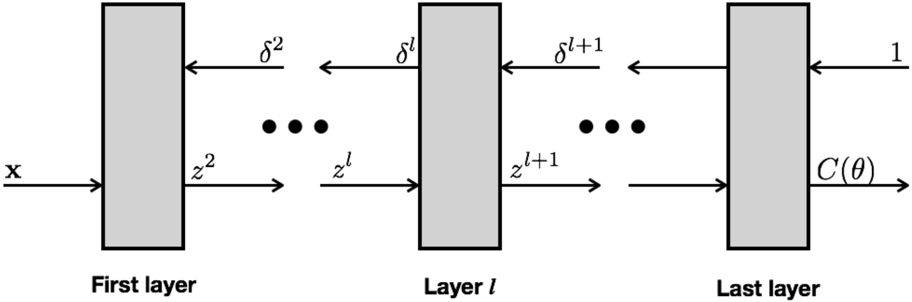

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Layered architecture: consequences

- Compositionality
- Reuse
- Ease of constructing **your own layers**

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A neural network can itself be treated as a layer within another neural network (recursion). This allows us to build new neural networks using exisitng (and sometimes pre-trained) models.

Summary

- Layered view of a 2-class softmax classifier
- Chain rule of differentiation
- Differentiable layers
- Backpropagation

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