Survey

Spatial Processes – Part 2

Computational Photography (CSCI 3240U)

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Special thanks to Ioannis Gkioulekas

- Many of the slides are taken with his permission from the computational photography course that he has developed at CMU

Image Enhancement

- Make an image more suitable for a particular application than the original image
- Types of techniques
  - Point processing
  - Spatial processing (pixel neighbourhoods) ← Today's Focus
  - Frequency domain processing

Spatial Filtering

- Two main types
  - Linear filtering
  - Non-linear filtering
- Linear filters
  - Remove, isolate, modify frequencies in the image
  - Foundation based upon the convolution theorem
- Non-linear filters
  - Based upon image statistics

Linear Filtering in 1D

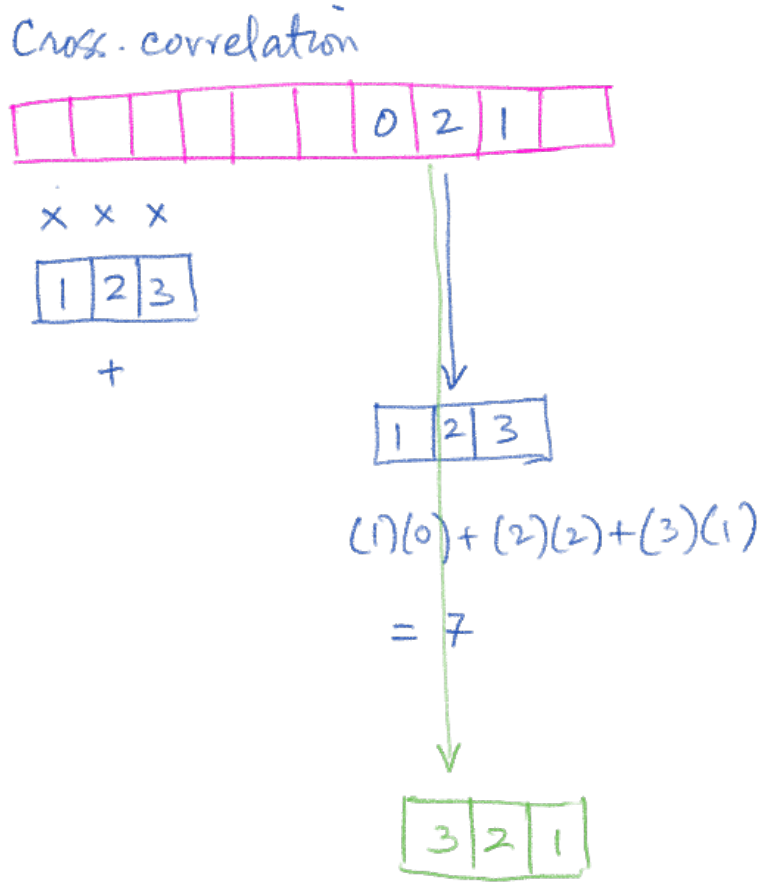
Cross-correlation

$$CC(i) = \sum_{k \in [-w,w]} \mathbf{f}(i+k)\mathbf{h}(k)$$

f

Convolution

$$(\mathbf{f} * \mathbf{k})_i = \sum_{k \in [-w,w]} \mathbf{f}(i-k)\mathbf{h}(k)$$



Linear Filtering in 1D

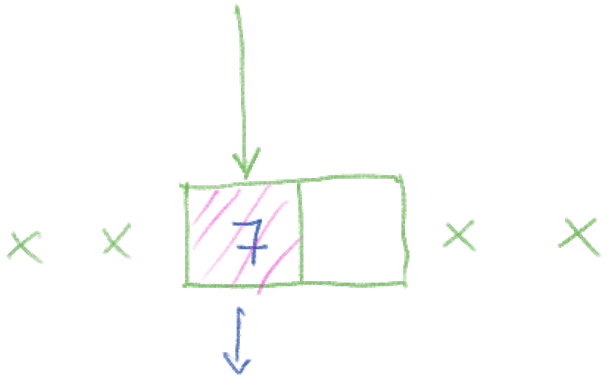
half-width = $w = 2$

total length = $2w+1$

1	2	4	-3	2	0

$*$

2	0	0	1	1
-2	-1	0	1	2



∵ convolution

1	1	0	0	2

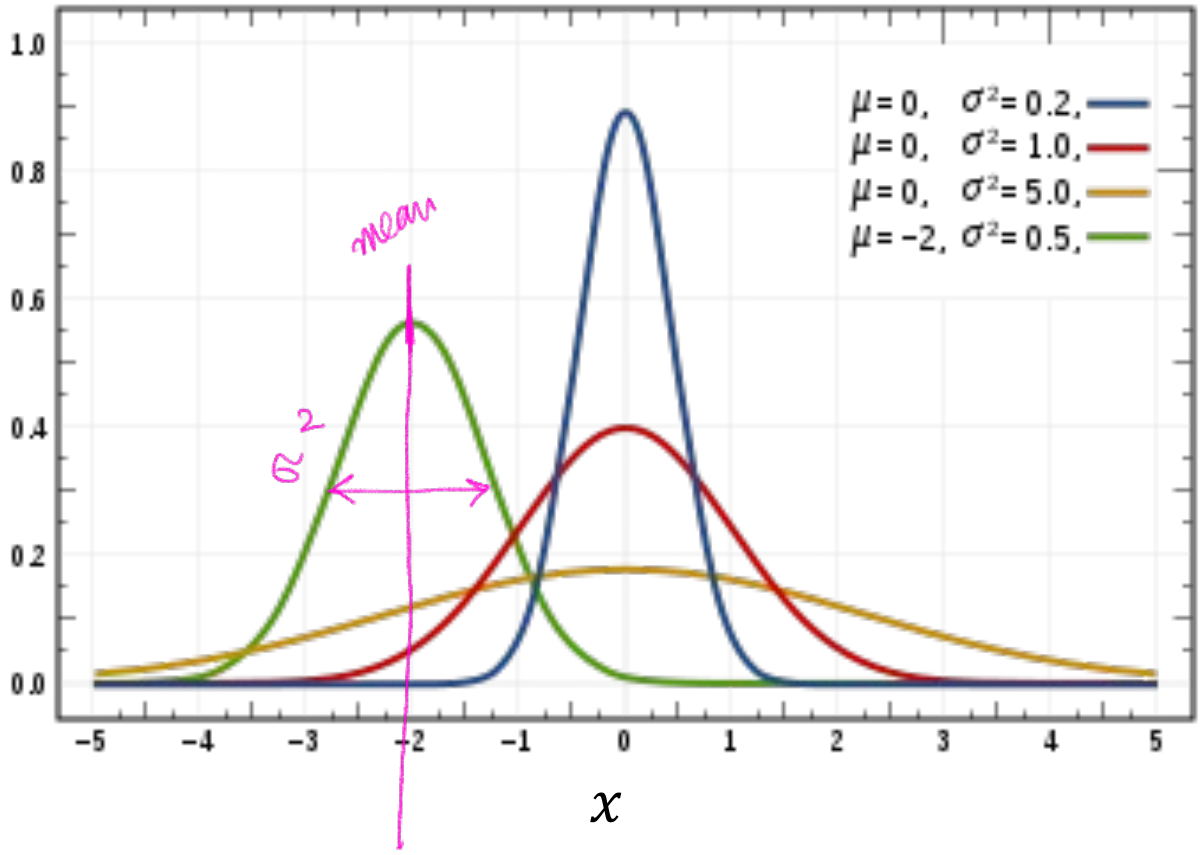
$(1)(1) + (1)(2) + (0)(4) + (0)(-3) + (2)(2)$

$= 1 + 2 + 4$

$= 7$

Gaussian in 1D

$G(x; \mu, \sigma^2)$



From Wikipedia

Gaussian in 1D

$$G(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\exp^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Here μ and σ refer to the mean and standard deviation of this Gaussian.

Aside: Mean and Standard Deviation

Given n data points $\{x_1, x_2, \cdots, x_n\}$

$$\mu = \mathrm{E}[x] = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\sigma^2 = \mathrm{E}[(x-\mu)^2] = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

Gaussian in 1D

Fit Gaussian to the following data:

1	**2**	**2**	**-3**	**2**	**0**

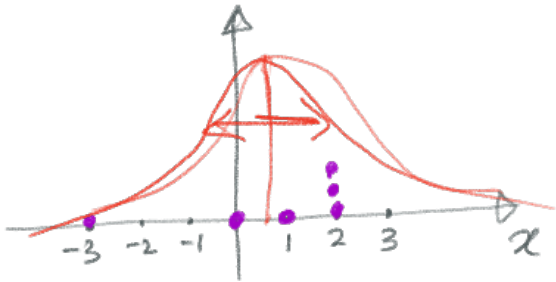
6 data points.

Mean of data $= \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1+2+2-3+2+0}{6} = \frac{4}{6} = \frac{2}{3}$.

$\sigma^2 = \left[(1-\tfrac{2}{3})^2 + (2-\tfrac{2}{3})^2 + (2-\tfrac{2}{3})^2 + (-3-\tfrac{2}{3})^2 + (2-\tfrac{2}{3})^2 + (0-\tfrac{2}{3})^2\right] / 6$

$= \ldots$



$g(x; \mu, \sigma^2)$

↑ Fn. of x

↑ ↑ parametrized by μ and σ^2

Gaussian in 1D

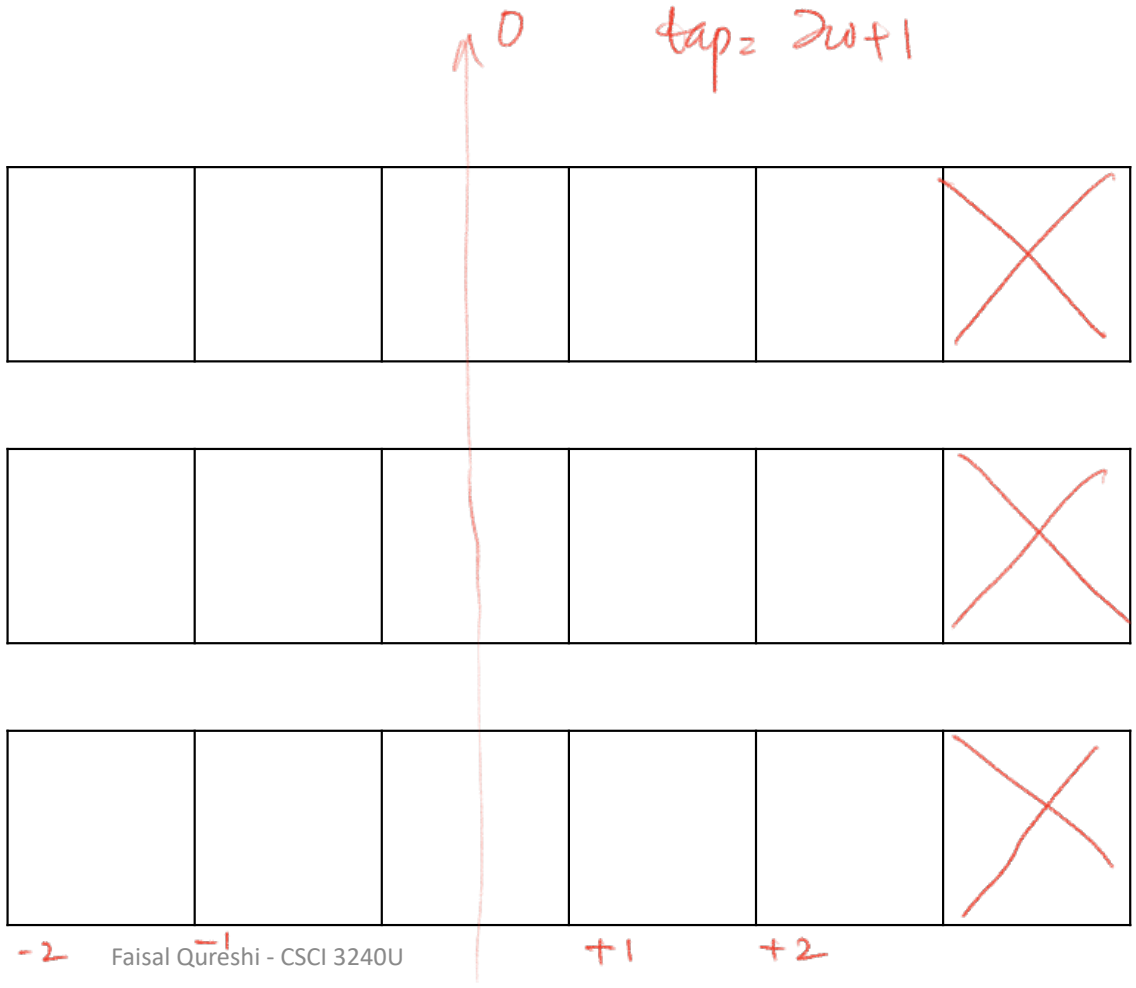
Compute 5-tap Gaussian filter with $\sigma = 2$

$$G(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\exp^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{2\sqrt{2\pi}}\exp^{-x^2/8}$$

$x \in \{-2, -1, 0, +1, +2\}$

$-w, \quad , 0, \quad , +w$



Linear Filtering in 2D

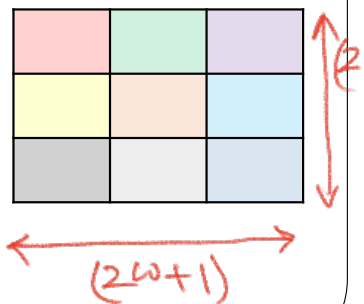
$h = w = 1$

CC

1	3	5	0	1	1
2	1	-1	3	0	4
0	1	2	2	3	4
1	3	4	9	10	3
3	7	8	2	1	4
1	5	6	4	2	1

Image

,



Kernel

=

?			

Linear Filtering in 2D

CC

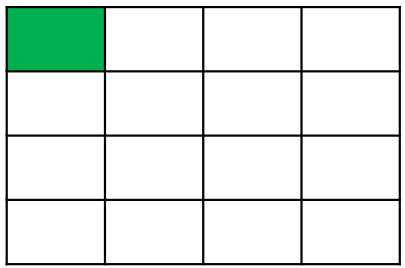
1	3	5	0	1	1
2	1	-1	3	0	4
0	1	2	2	3	4
1	3	4	9	10	3
3	7	8	2	1	4
1	5	6	4	2	1

Image

,

Kernel

=



Linear Filtering in 2D

CC

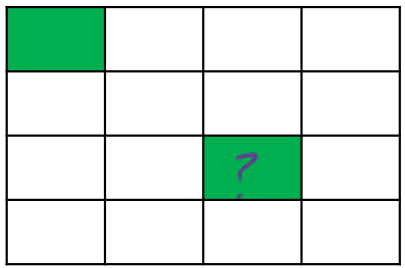
1	3	5	0	1	1
2	1	-1	3	0	4
0	1	2	2	3	4
1	3	4	9	10	3
3	7	8	2	1	4
1	5	6	4	2	1

Image

,

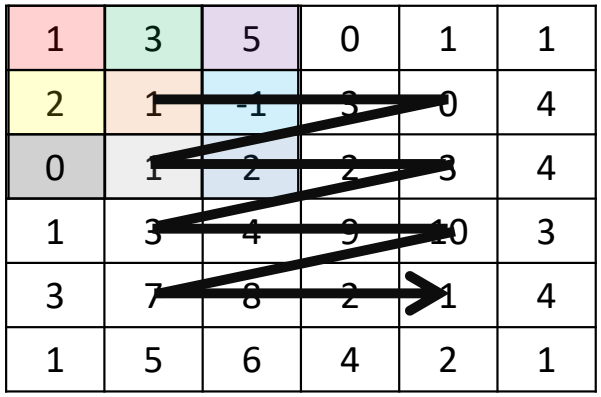
Kernel

=



Linear Filtering in 2D

CC



Image

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Kernel

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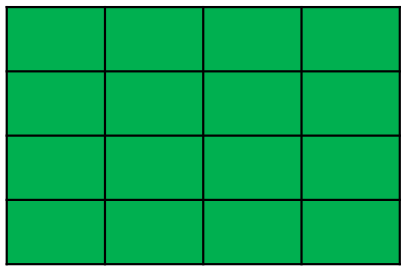


cross-correlation

Linear Filtering in 2D

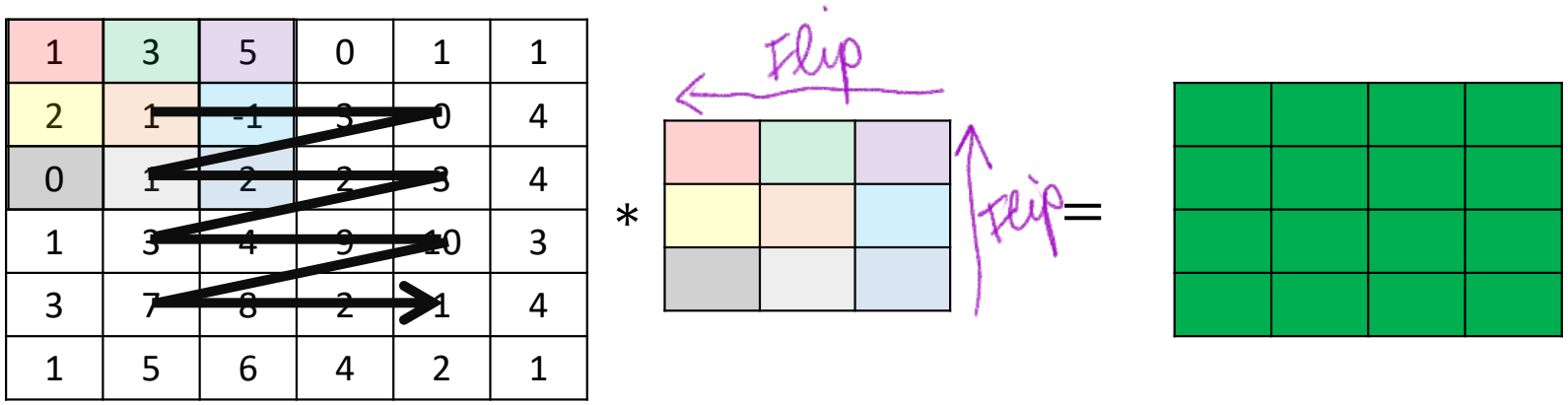
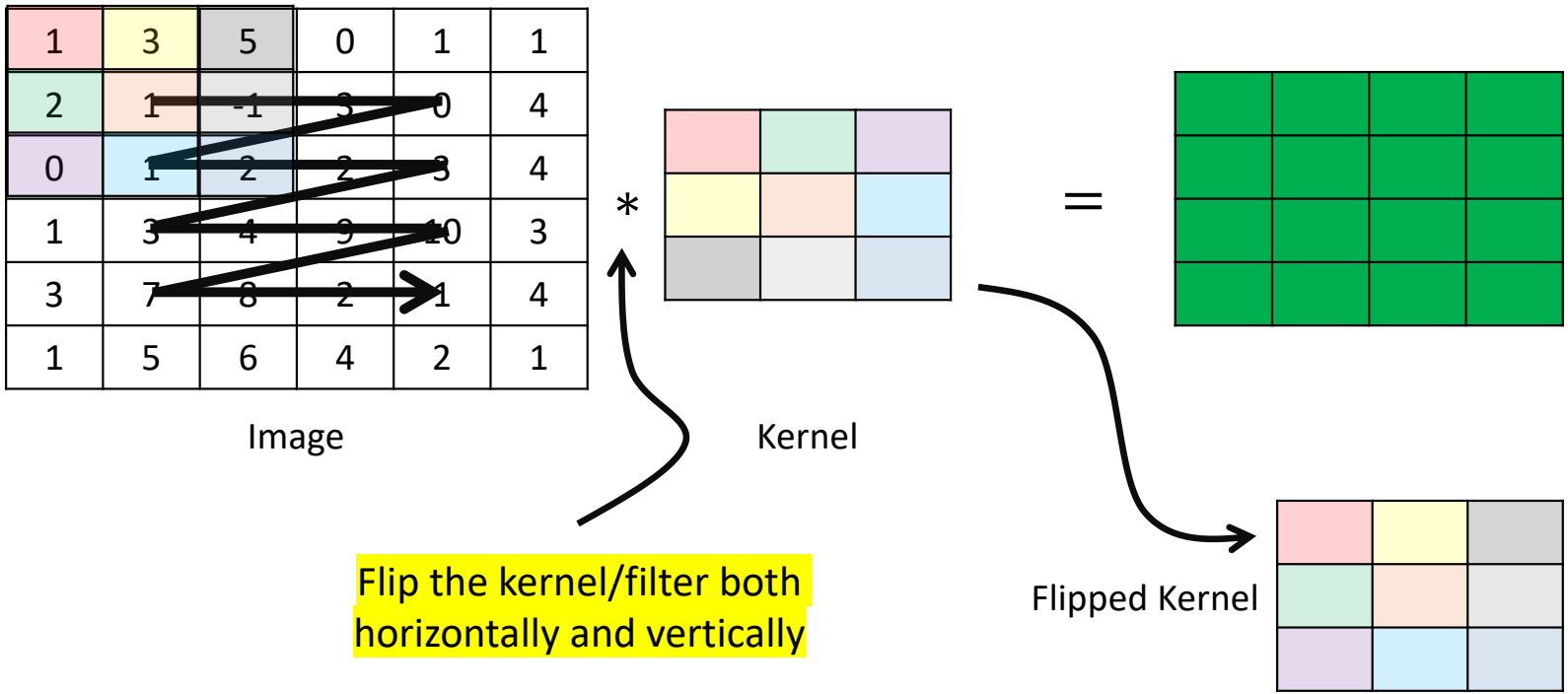


Image Kernel

Linear Filtering in 2D



Linear Filtering in 2D

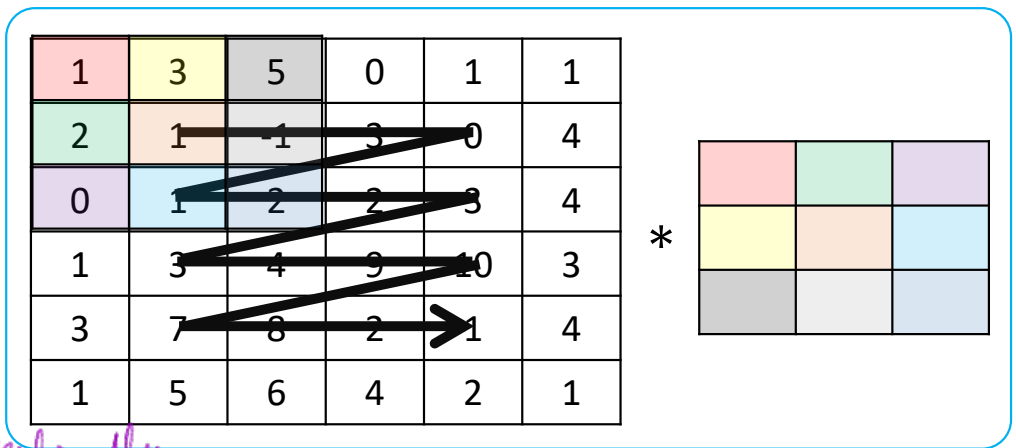
Cross-correlation

$$CC(i,j) = \sum_{\substack{k\in[-w,w]\\ l\in[-h,h]}} \mathbf{f}(i+k, j+l)\mathbf{h}(k,l)$$

Convolution

$$(\mathbf{f} * \mathbf{k})_{i,j} == \sum_{\substack{k\in[-w,w]\\ l\in[-h,h]}} \mathbf{f}(i-k, j-l)\mathbf{h}(k,l)$$

Confirm that this is akin to flipping the kernel horizontally & vertically.



Linear Filtering in 2D: Number of multiplications and additions

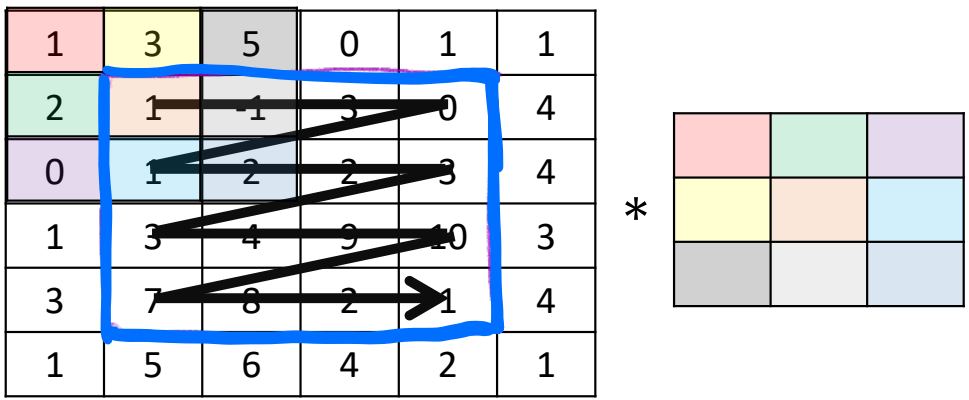


Image Kernel

Image: 1000 x 1000

Filter: 15 x 15

Size of the output.

places = (4)(4) = 16

multiplications at each location
= (3)(3) = 9

additions at each location
= 8

Putting every thing together
= 16 x (9 MUL + 8 ADD)

Multivariate Guassian (in k-dimensions)

$$G(\boldsymbol{x};\boldsymbol{\mu},\Sigma) = \frac{1}{\sqrt{(2\pi)^k \det(\Sigma)}} exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right)$$

where

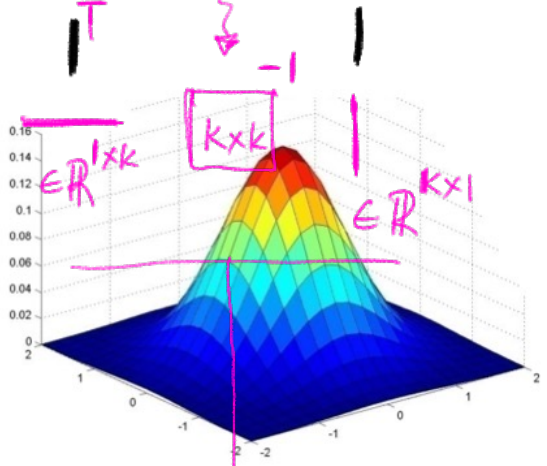
$\boldsymbol{x} \in R^k$

$\boldsymbol{\mu} \in R^k$

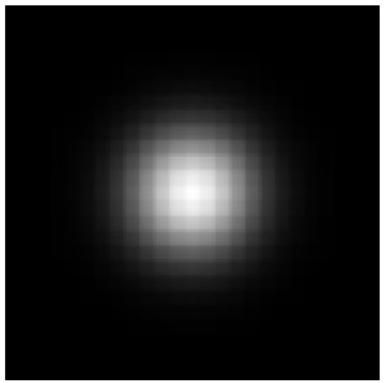
$\Sigma \in R^{k \times k}$

covariance matrix

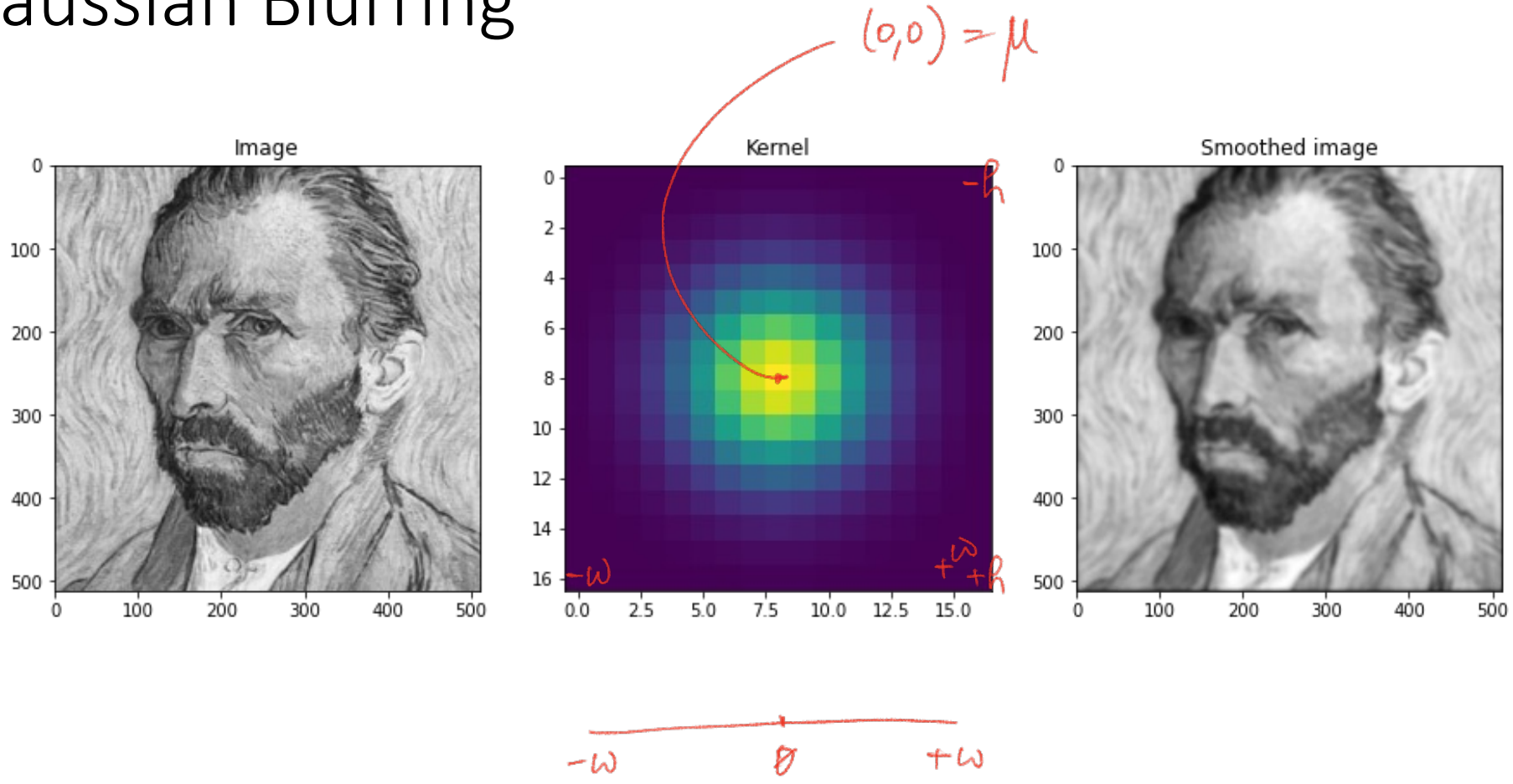
determinant



Gaussian in 2D



Gaussian Blurring



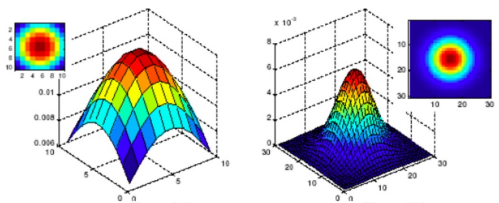
Gaussian Blurring

- We often use the following approximation of a Gaussian function

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

3×3

- Gaussian functions have infinite support, but discrete Gaussian kernels are finite

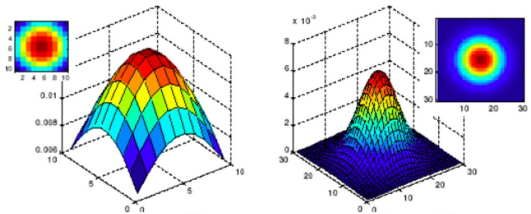


σ = 5 with 10 x 10 kernel

σ = 5 with 30 x 30 kernel

Gaussian Blurring

- Variance controls how broad or peaky the filter is



σ = 5 with 10 x 10 kernel

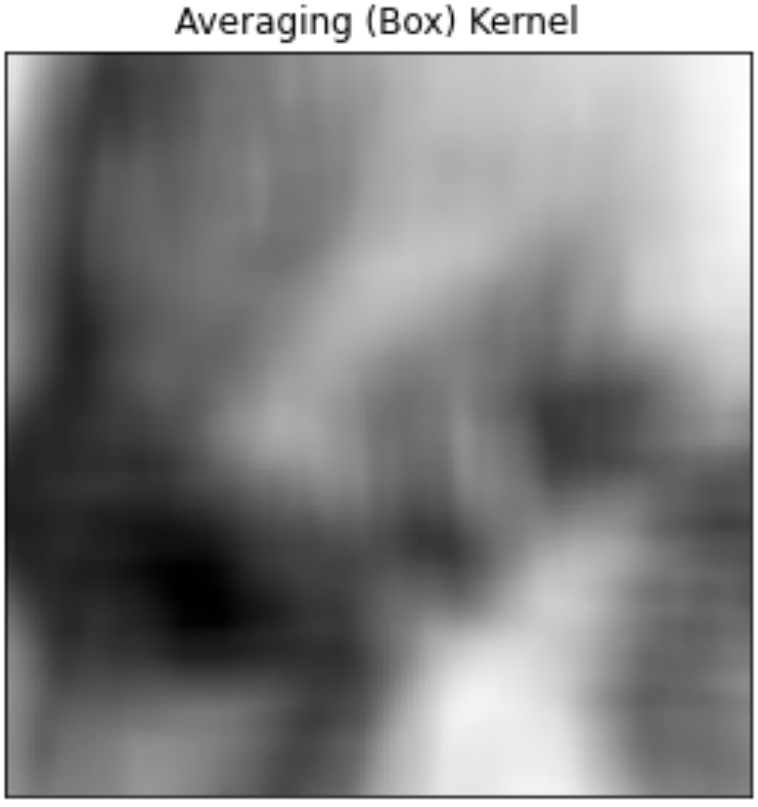
σ = 5 with 30 x 30 kernel

- Removes high-frequency components from the image
  - Blurs the image
  - Acts as a low-pass filter

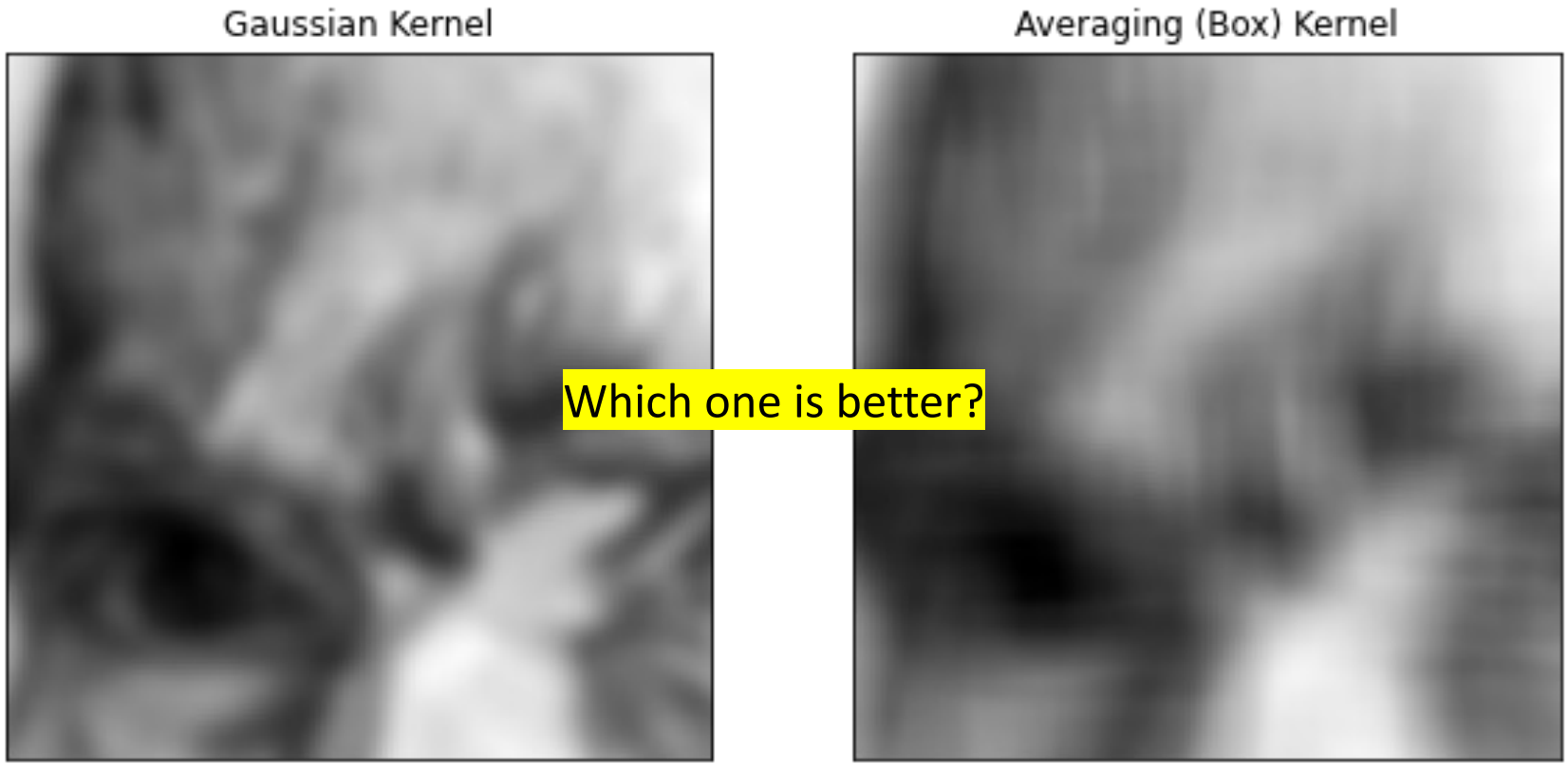
Gaussian Blurring

- Convolving twice with Gaussian kernel of width σ^2 is the same as convolving once with kernel of width $\sigma\sqrt{2}$
- Applying a Gaussian filter with variance σ_1^2, followed by applying a Gaussian filter with variance σ_2^2 is the same as applying once with Gaussian filter with variance $\sqrt{\sigma_1^2 + \sigma_2^2}$
- All values are positive
- Values sum to 1?
  - Why is this relevant?
- This size of the filter, plus its variance, determines the extent of smoothing

Gaussian Blurring vs. Average (Box) Filtering



Gaussian Blurring vs. Average (Box) Filtering



Summary

- Linear filtering 1D recap
  - Cross-correlation
  - Convolution
  - Gaussian filtering
- Linear filtering in 2D
- Multivariate Gaussians
- Gaussian Blurring

Check out Linear Filtering notes here.