Spatial Processes

Computational Photography (CSCI 3240U)

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- Many of the slides are taken with his permission from the computational photography course that he has developed at CMU

Image Enhancement

- Make an image more suitable for a particular application than the original image
- Types of techniques
  - Point processing
  - Spatial processing
  - Frequency domain processing

E.g., Human perception

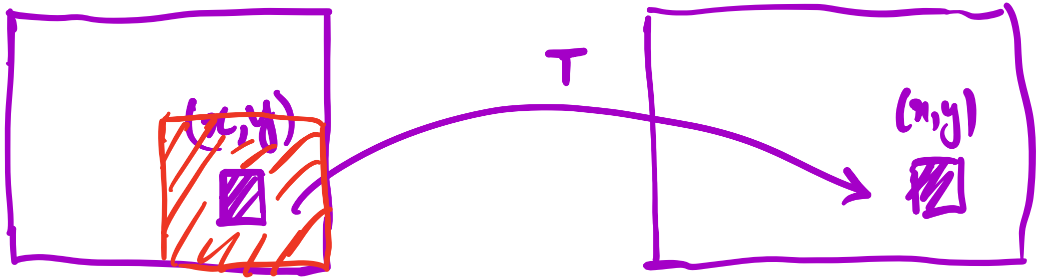


Image Enhancement

- Make an image more suitable for a particular application than the original image
- Types of techniques
  - Point processing
  - Spatial processing (pixel neighbourhoods)
  - Frequency domain processing

E.g., Human perception

Image Enhancement

- Make an image more suitable for a particular application than the original image
- Types of techniques
  - Point processing
  - Spatial processing (pixel neighbourhoods) ← Today's Focus
  - Frequency domain processing

Spatial Processing

- Input image: $f(x, y)$
- Output image: $g(x, y)$
- T is an operator on f or a set of f
  - T is defined over some neighbourhood N of (x, y)
  - T can operate over a set of images

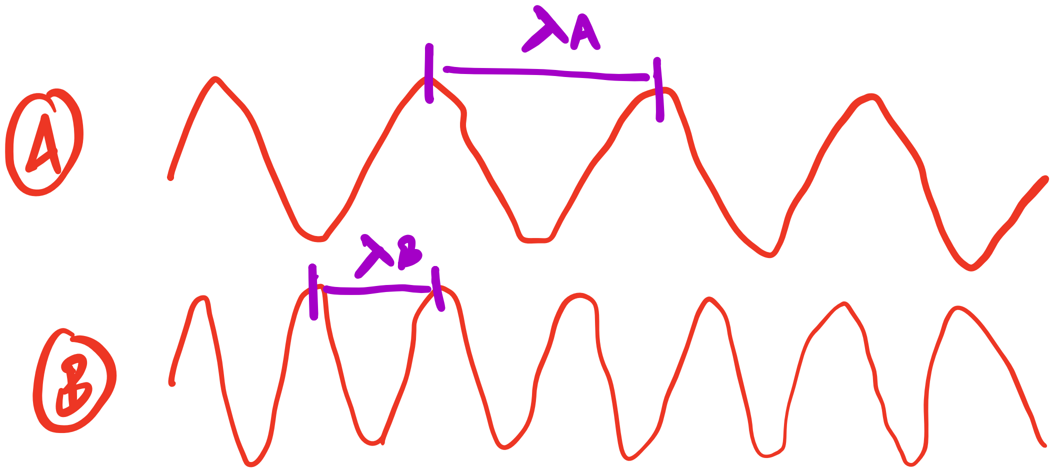
Under what conditions a pixel at (x,y) may be different from its neighbours.

① Object edges

② Shading

③ Texture

Spatial Filtering



- Two main types
  - Linear filtering
  - Non-linear filtering
- Linear filters
  - Remove, isolate, modify frequencies in the image
  - Foundation based upon the convolution theorem

$\lambda_B < \lambda_A$

$f_B > f_A$

CNNs

- Non-linear filters
  - Based upon image statistics

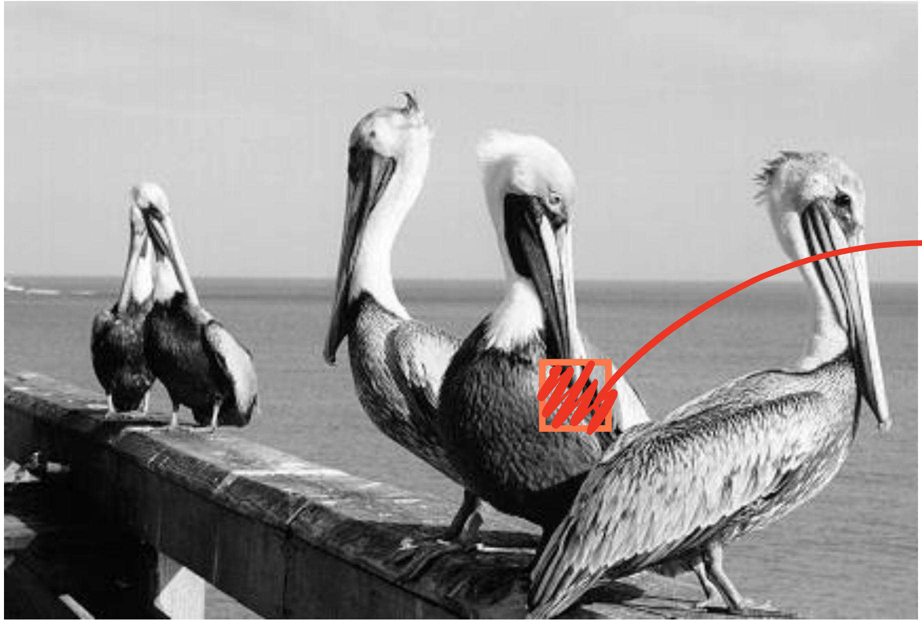
An Example of Spatial Filtering

$f(x, y)$

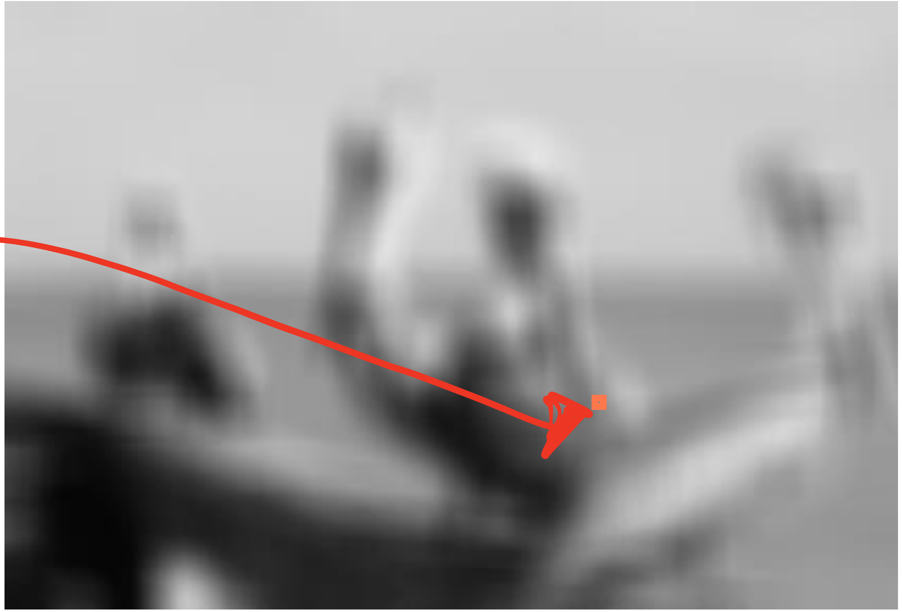
$g(x, y)$

5 x 5 neighbourhood

An Example of Spatial Filtering



$f(x, y)$



$g(x, y)$

5 x 5 neighbourhood

Linear Filtering in 1D

1	2	4	1	0	9	1	3	5	2

0 1 2

7

10

20

$\mathcal{N}: -1, 0, 1$

$1-1,\ 1+0,\ 1+1$

$\mathcal{N}: -2, -1, 0, 1, 2$

Kernel/Filter: $\boxed{1}\boxed{1}\boxed{1}$ Summing kernel.

1	2	4	1	0	9	1	3	5	2

0 1 2 ?

Stride: 1 7 10 20

1	1	1

?

Boundary Condition.

$$\begin{bmatrix}1\\2\\4\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\end{bmatrix} = (1)(1)+(2)(1)+(4)(1) = 7$$

1	2	4	1	0

1/3	1/3	1/3

Average / Mean

?

7/3	7/3	5/3

?

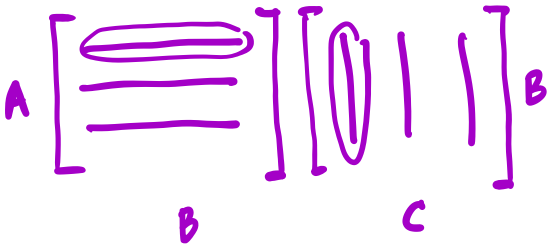
$$(1)(1/3)+(2)(1/3)+(4)(1/3)$$

$$= \frac{1+2+4}{3} = 7/3$$

0	1	2	3	4
1	2	4	1	0

for $i = 1$ to 3

dot-product



$$\begin{array}{cc} 3 & \begin{bmatrix} 7/3 \\ 7/3 \\ 5/3 \end{bmatrix} \\ & 1 \end{array} = \begin{array}{cc} & 5 \\ 3 & \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \end{bmatrix} \end{array} \begin{array}{cc} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 0 \end{bmatrix} & 5 \\ 1 & \end{array}$$

Linear Filtering in 1 d

- Signal: f
- Kernel (sometimes called mask or filter): h
- Half-width of kernel: w → $2w+1$ → Size of the kernel.

-1	0	+1
2	0	-2

$w = 1$

0	1	2	3	4
1	2	4	1	0

Cross-correlation

$$CC(i) = \sum_{k \in [-w,w]} \mathbf{f}(i+k)\mathbf{h}(k)$$

$$\begin{aligned} CC(2) &= f(2-1)\,h(-1) + f(2-0)\,h(0) + f(2+1)\,h(1) \\ &= (2)(2) + 0 + (1)(-2) = 4-2 = 2 \end{aligned}$$

Convolution

$$(\mathbf{f} * \mathbf{h})_i = \sum_{k \in [-w,w]} \mathbf{f}(i-k)\mathbf{h}(k)$$

$$\begin{aligned} (f*h)_2 &= f(3)h(-1) + f(2)h(0) + f(1)h(1) \\ &= (1)(2) + 0 + (2)(-2) = -2 \end{aligned}$$

Compute $CC(i)$

	0	1	2	3	4	5	6	7
$f =$	**1**	**3**	**4**	**1**	**10**	**3**	**0**	**1**
$h =$	**1**	**0**	**-1**					

$$CC(i) = \sum_{k \in [-w,w]} \mathbf{f}(i+k)\mathbf{h}(k)$$

$CC(4) =$

-2

Compute $f * h$

	0	1	2	3	4	5	6	7
$f =$	**1**	**3**	**4**	**1**	**10**	**3**	**0**	**1**

$h =$	**1**	**0**	**-1**

$$(\mathbf{f} * \mathbf{k})_i = \sum_{k \in [-w,w]} \mathbf{f}(i-k)\mathbf{h}(k)$$

$(f * h)_4$

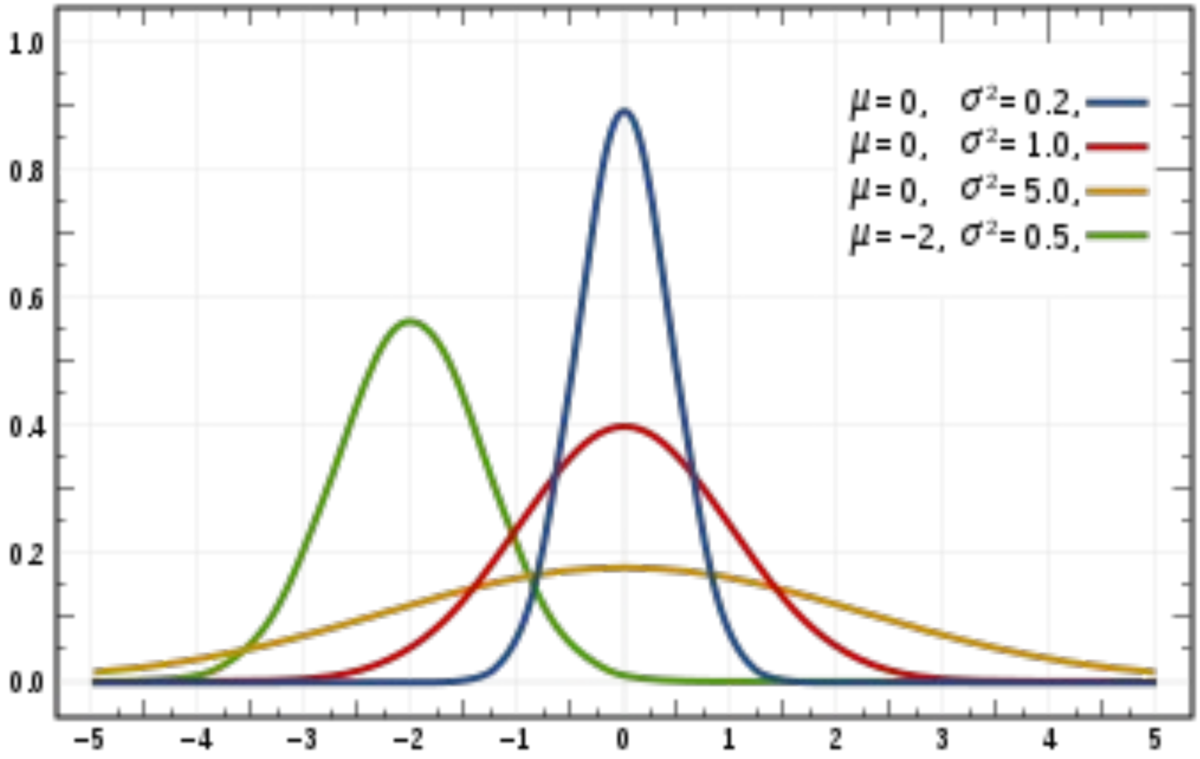
Flip

-1	0	1

+2

Gaussian in 1D

$G(x; \mu, \sigma^2)$



x

From Wikipedia

Gaussian in 1D

$$G(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Here μ and σ refer to the mean and standard deviation of this Gaussian.

Find a Gaussian that best describes the data

Aside: Mean and Standard Deviation

Given n data points $\{x_1, x_2, \cdots, x_n\}$

$$\mu = \mathrm{E}[x] = \frac{1}{n}\sum_{i=1}^{n} x_i$$

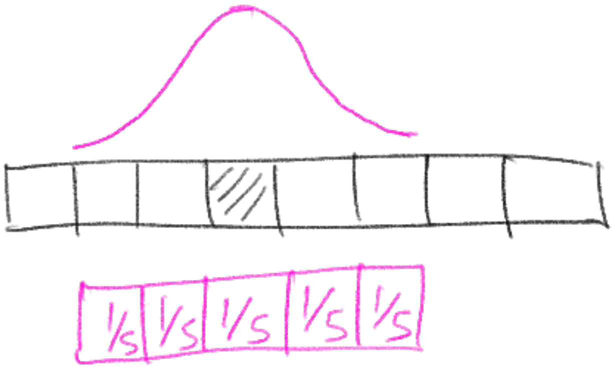
$$\sigma^2 = \mathrm{E}[(x-\mu)^2] = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Gaussian in 1D

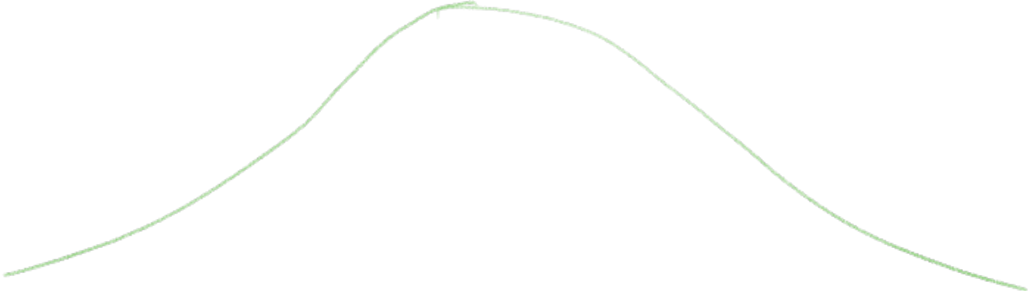
scaling

$$G(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

e^{-1/σ^2}	$e^{-1/2\sigma^2}$	$e^0 = 1$	$e^{-1/2\sigma^2}$	e^{-1/σ^2}
0	1	2	3	4



$\mu = 2$



Linear Filtering in 2D

1	3	5	0	1	1
2	1	-1	3	0	4
0	1	2	2	3	4
1	3	4	9	10	3
3	7	8	2	1	4
1	5	6	4	2	1

Image

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Kernel

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ 1 \\ 2 \\ 2 \\ 3 \\ 4 \\ 9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\} 1/9$$

Summary

- Linear filtering 1D
  - Cross-correlation
  - Convolution
- Gaussian filtering in 1D
- Linear filtering in 2D

Check out Linear Filtering notes [here](#).