Image Stitching

Computational Photography (CSCI 3240U)

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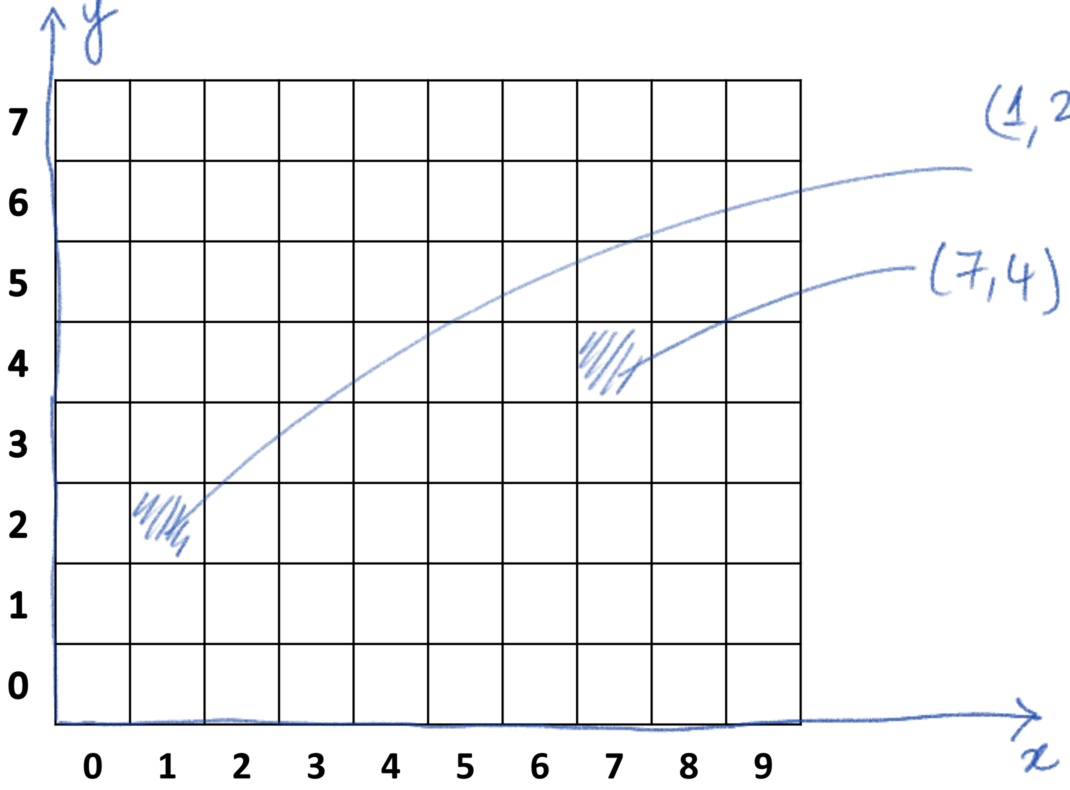
Today

- Image stitching

Euclidean vs. Homogeneous Coordinates



Standard Euclidean representation of position $\vec{p} = (x, y)$.

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = (1)\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (2)\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

basis vectors

Coordinates.

$$\begin{bmatrix} x \\ y \end{bmatrix} = x \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Homogeneous coordinates (also called Projective representation of point $\vec{p}$).

$$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} \quad \text{for any } \lambda \neq 0.$$

point $\vec{p}$ — Homogeneous coordinates of point $\vec{p}$

Examples:

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \\ 4 \end{bmatrix} = \cdots$$

Cartesian — Homogeneous.

Q. Convert homogeneous coordinate $\begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$ to the cartesian point?

A. $\begin{bmatrix} 3/7 \\ 2/7 \end{bmatrix}$

RECIPE

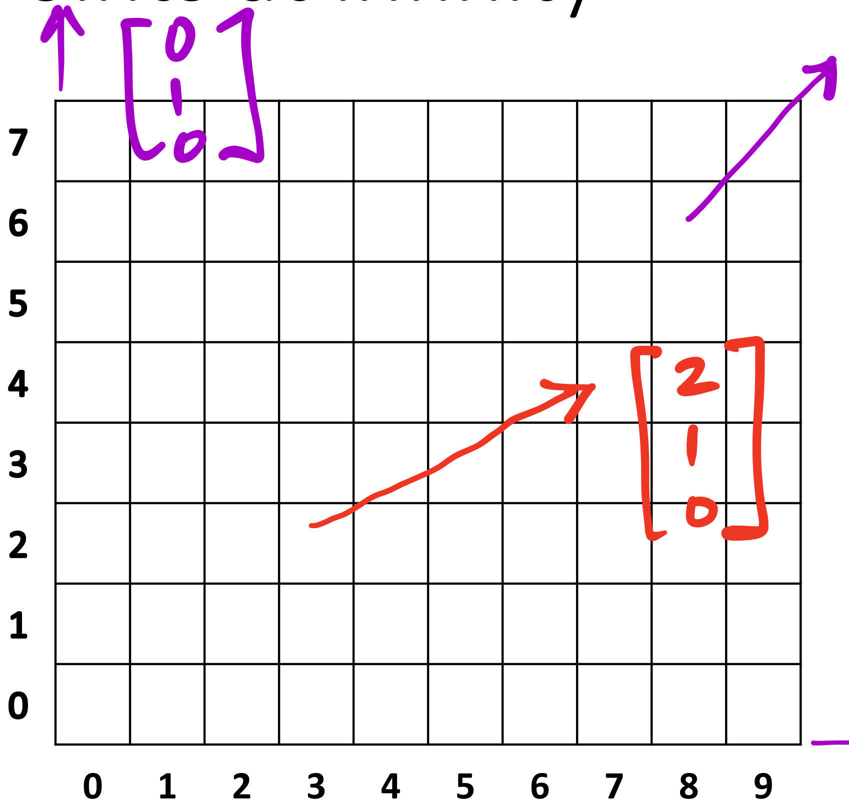
$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} \rightarrow \begin{bmatrix} a/c \\ b/c \end{bmatrix}$$

$$\begin{bmatrix}x\\y\\1\end{bmatrix} \simeq \lambda\begin{bmatrix}x\\y\\1\end{bmatrix} = \begin{bmatrix}\lambda x\\\lambda y\\\lambda\end{bmatrix}$$

Equality

$$\xrightarrow{\text{Cartesian}} \begin{bmatrix}x\\y\end{bmatrix}$$

Points at infinity



Homogeneous Number.

$$\begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} \xrightarrow{\text{convert to cartesian}} \begin{bmatrix} 3/0 \\ 3/0 \end{bmatrix}$$

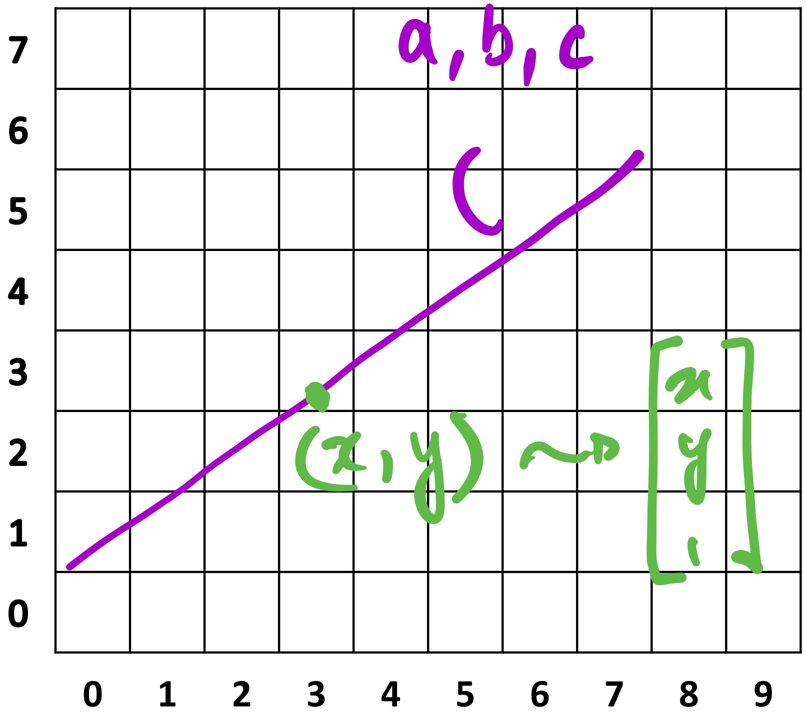
$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 0 \\ 0.000001 \end{bmatrix} ?$$

Very far in the x direction

1. We are able to describe points at infinity
2. Representing directions.

Line equations in homogeneous coordinates



$y = mx + b$ ✓

$$\boxed{ax + by + c = 0}$$

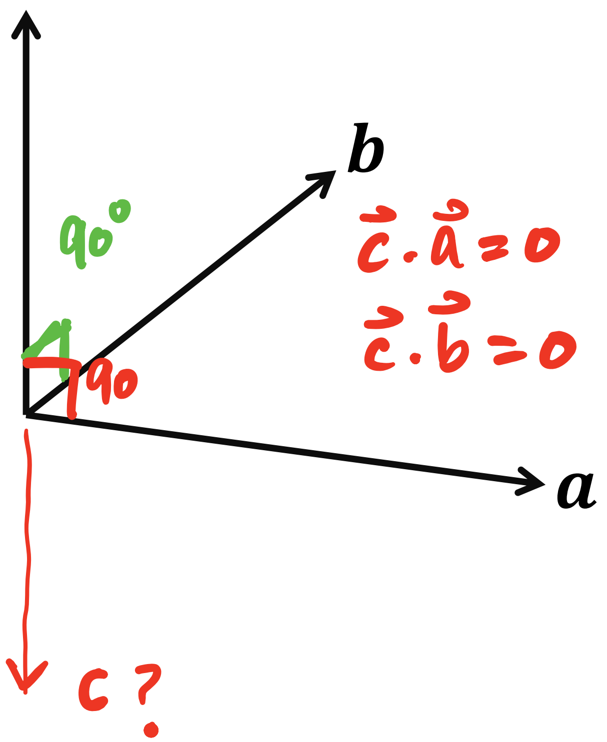
line parameters

$$\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Homogeneous Pt.

Cross-product of two vectors

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$$



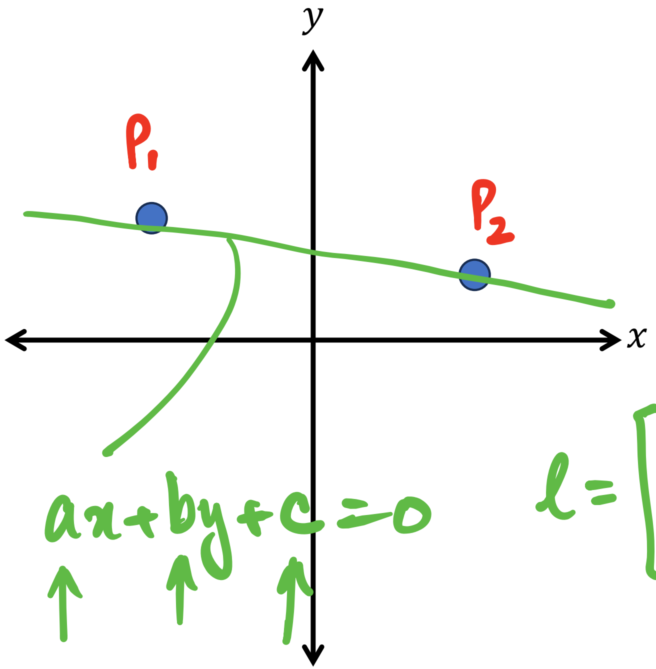
$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \boldsymbol{a_x} & \boldsymbol{a_y} & \boldsymbol{a_z} \\ \boldsymbol{b_x} & \boldsymbol{b_y} & \boldsymbol{b_z} \end{vmatrix}$$

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{a_z} & \boldsymbol{a_y} \\ \boldsymbol{a_z} & \boldsymbol{0} & -\boldsymbol{a_x} \\ -\boldsymbol{a_y} & \boldsymbol{a_x} & \boldsymbol{0} \end{bmatrix} \boldsymbol{b}$$

The line passing through two points

$P_1 = (x_1, y_1)$

$P_2 = (x_2, y_2)$



$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$ax+by+c=0$

a, b, c: unknown.

$$l = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

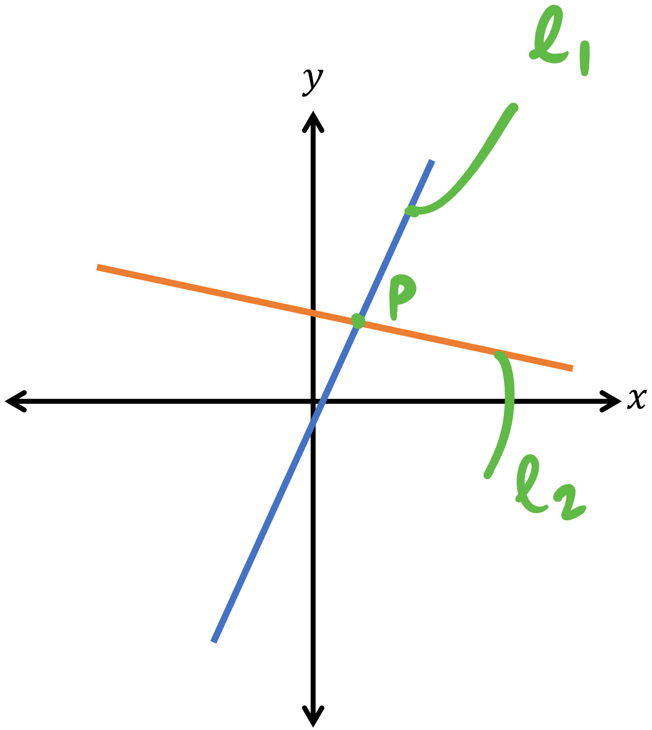
$$P_1 = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

$$l \cdot P_1 = 0 \qquad l \cdot P_2 = 0$$

$$\ell = p_1 \times p_2$$

Use cross-product to compute ℓ that is orthogonal to p_1 and p_2.

The point of intersection of two lines



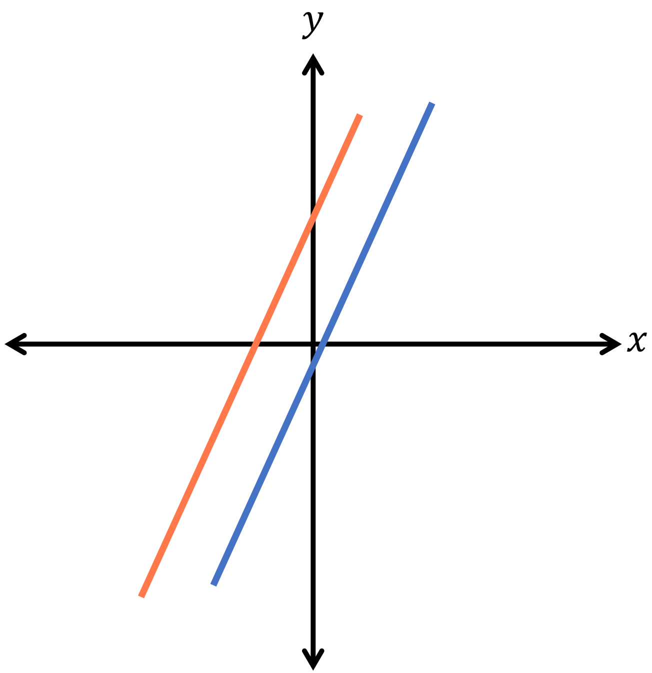
Observation:

$l_1 \perp p \quad \because \quad l_1 \cdot p = 0$

$l_2 \perp p \quad \because \quad l_2 \cdot p = 0$

$l_1 \times l_2 = p$

Intersecting two parallel lines



$x + 2y + 1 = 0$

$x + 3y + 0 = 0$

Find the intersection pt.

$x + 2y + 1 = 0$

$6x + 12y + 6 = 0$

Find the intersection point.

Q. $x + 2y + 1 = 0$

$x + 3y = 0$

Approach 1:

$$\cancel{x} + 3y = 0$$
$$\underset{-}{\cancel{x}} \underset{-}{+} 2y \underset{-}{+ 1} = 0$$

$$y - 1 = 0$$
$$y = 1$$

$\therefore x = -3$

Intersection Pt. $= \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

Approach 2.

$$l_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad l_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

$$l_1 \times l_2 = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$\downarrow$

Cartesian $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

Q. $x + 2y + 1 = 0$

$6x + 12y + 6 = 0$

Approach 1:

No can't solve. These equations are linearly dependent.

$\rightarrow$ lines do not intersect.

These are parallel lines.

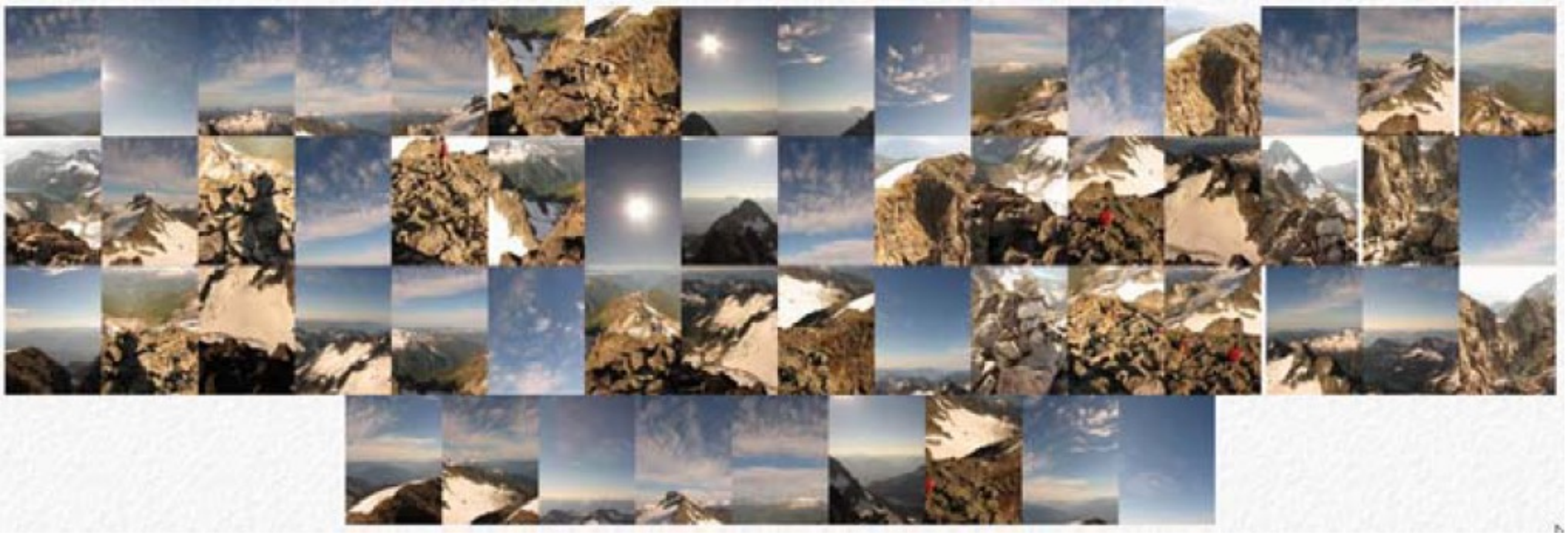
$$l_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad l_2 = \begin{bmatrix} 6 \\ 12 \\ 6 \end{bmatrix}$$

$$l_2 \times l_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Convert to Cartesian

$$\begin{bmatrix} 0/0 \\ 0/0 \end{bmatrix}$$

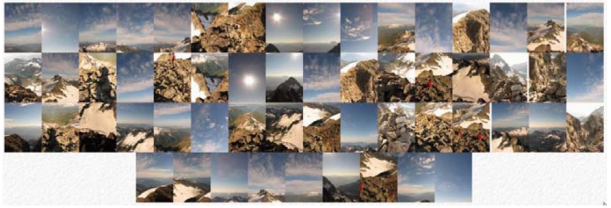
Image stitching



57 images

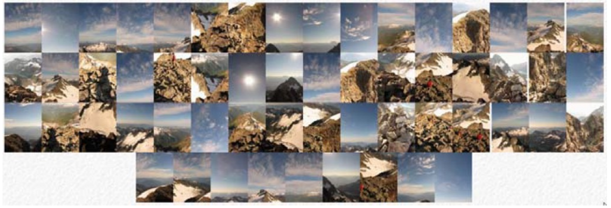
Camera should change orientation only, not position.
Keep camera settings (gain, focus, speed, aperture) fixed, if possible.

Image stitching



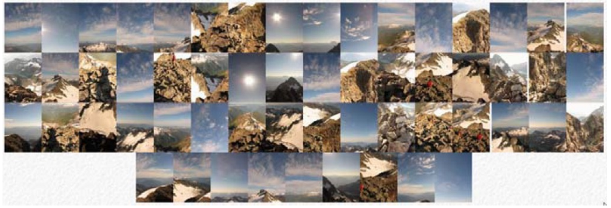
Using 28 out of 57 images

Image stitching



Using all 57 images

Image stitching (Autostitch)



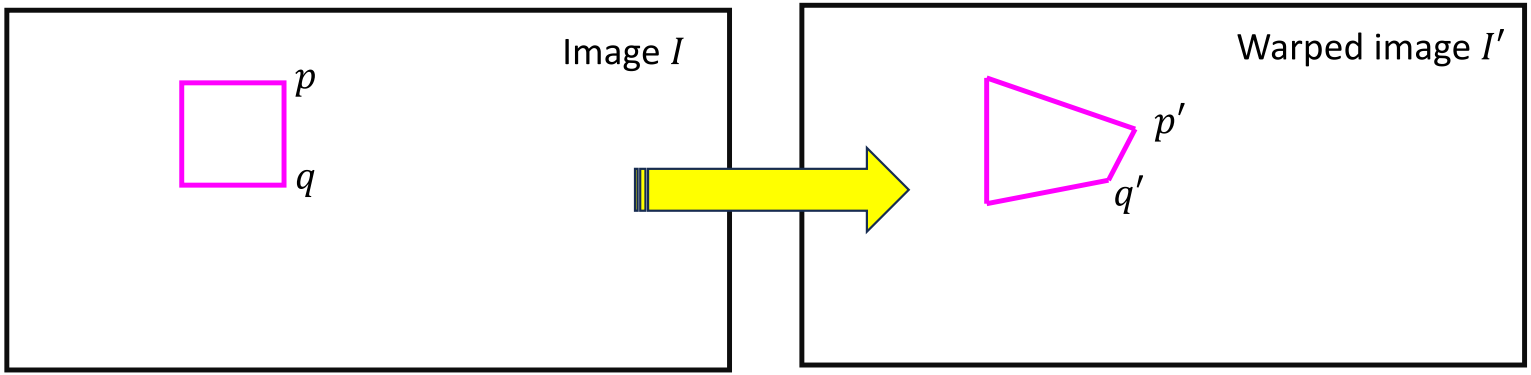
Seams are not visible

Using all 57 images. Laplacian blending.

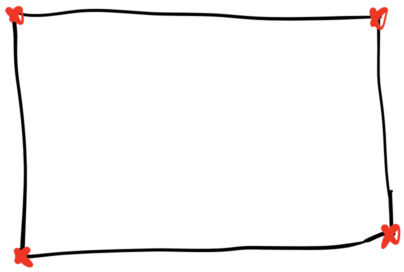
Brown & Lowe; ICCV 2003

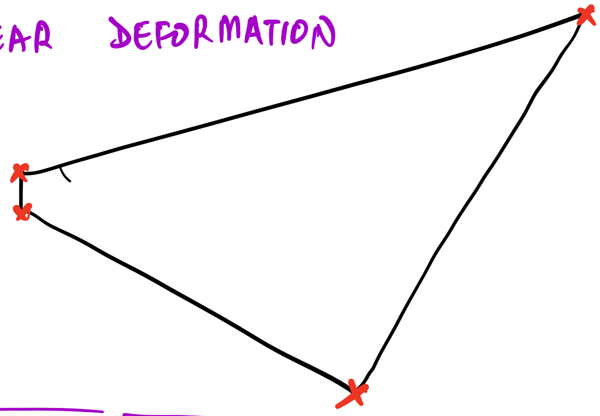
Linear image wraps

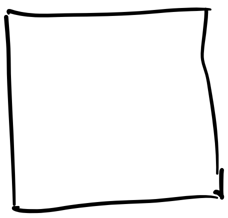
- To align multiple photos for image stitching, we must warp these images in such a way that all lines are preserved.
  - Lines before warping remain lines after warping
- Linear image wraps and *homographies*

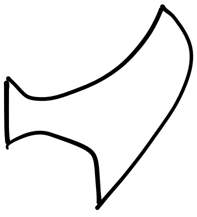


LINEAR DEFORMATION





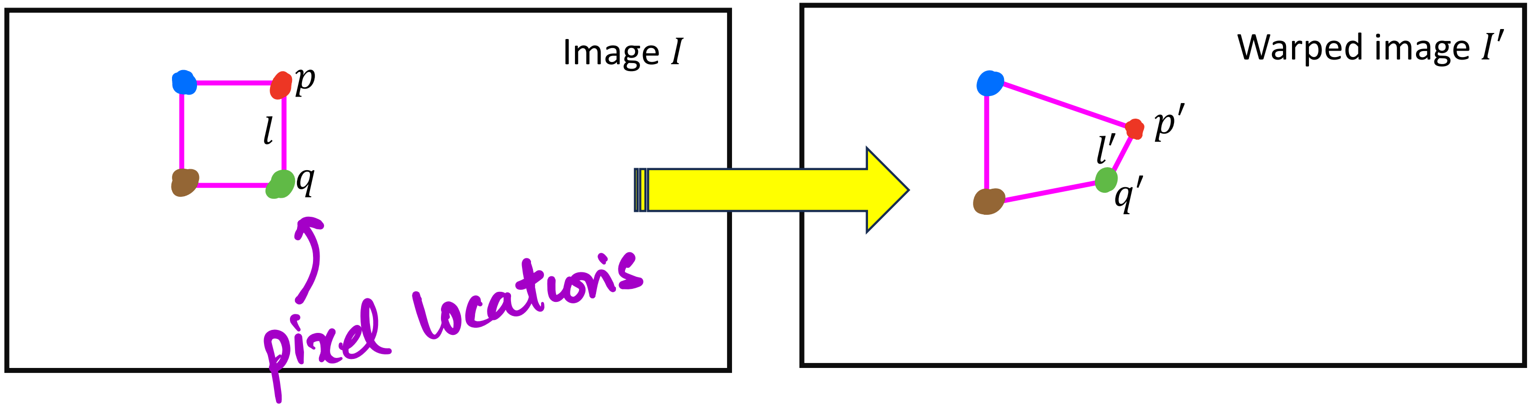




Non-linear deformation

Linear image wraps

- Definition: an image warp is linear if every 2D line l in the original image is transformed into a line l' in the warped image
- Property: Every linear warp can be expressed as a 3×3 matrix H that transforms homogeneous image coordinates (we won't discuss the proof here)



$$\underset{H}{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}} \underset{P}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}} \longrightarrow \begin{bmatrix} x' \\ y' \\ \lambda \end{bmatrix} = \begin{bmatrix} x'/\lambda \\ y'/\lambda \end{bmatrix}$$

Homogeneous Coordinate of point (x, y)

Warping images using homography

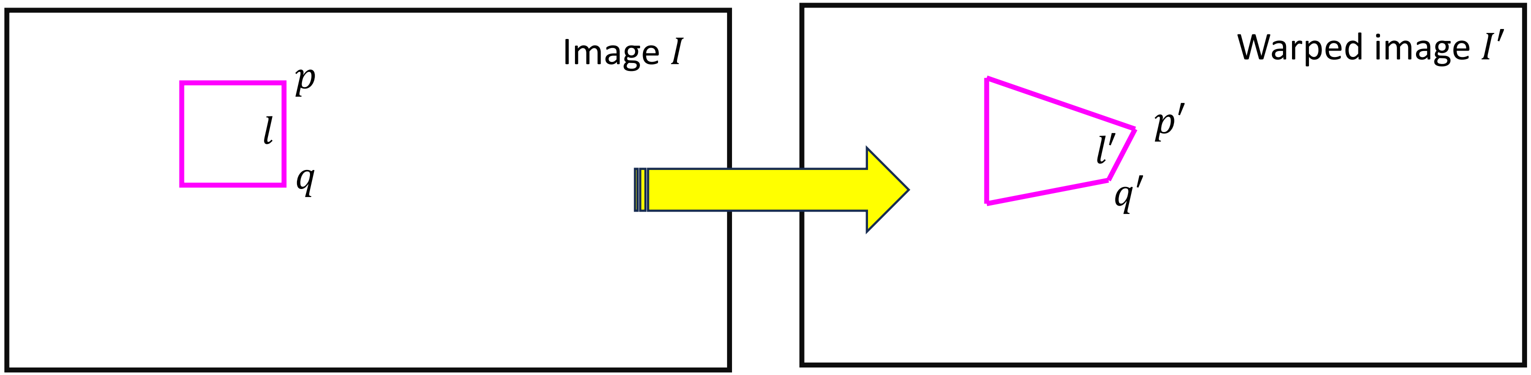
Linear warping equation: $I(\boldsymbol{p}) = I'(H\boldsymbol{p})$

Intensity at pixel in the source image I with homogeneous coordinates $\boldsymbol{p}$

Intensity at pixel in the warped image I' with homogeneous coordinates $H\boldsymbol{p}$

Matrix H is called homography

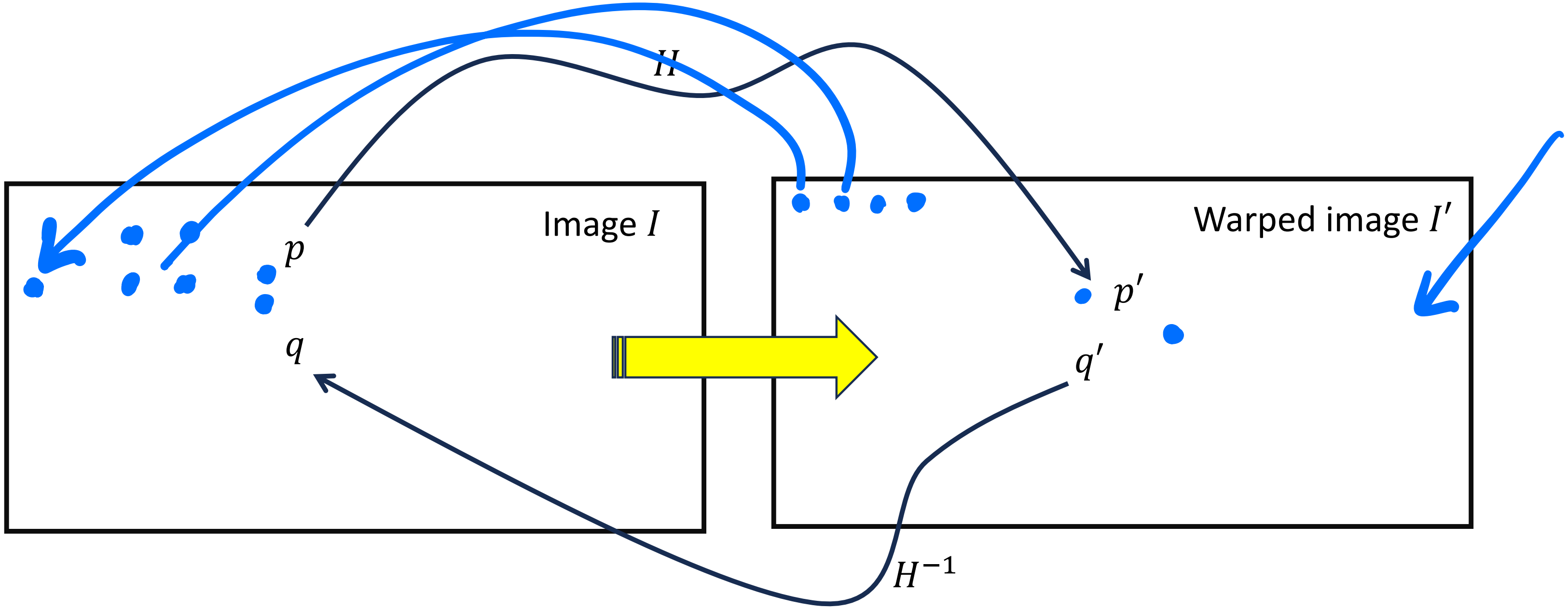
Scaling H by a factor $\lambda \neq 0$ does not change homography



Warping images using homography

Linear warping equation:

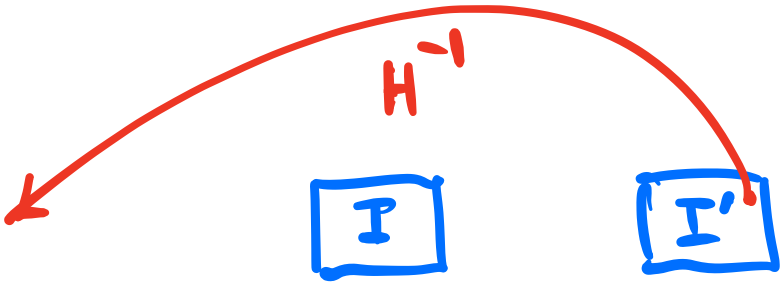
$$I(\boldsymbol{p}) = I'(H\boldsymbol{p}) \text{ and also } I'(\boldsymbol{q}') = I(H^{-1}\boldsymbol{q}')$$



Computing warp I' from I and H

- Compute H^{-1}
- To compute the color of pixel (u, v) in the warped image
  - Compute $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = H^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$
  - Copy color from $I\left(\frac{a}{c}, \frac{b}{c}\right)$

What if location $\left(\frac{a}{c}, \frac{b}{c}\right)$ is not valid pixel locations?



Homography & image mosaicing

- Every photo taken from a tripod-mounted camera is related by a homography
- Assumptions
  - No lens distortion
  - Camera's center of projection does not move while camera is mounted on the tripod
- Problem
  - These homographys that relate photos taken from a tripod-mounted camera are *unknown*
    - We need to estimate them