Image Stitching

Computational Photography (CSCI 3240U)

Faisal Z. Qureshi
http://vclab.science.ontariotechu.ca
Ontario Tech University

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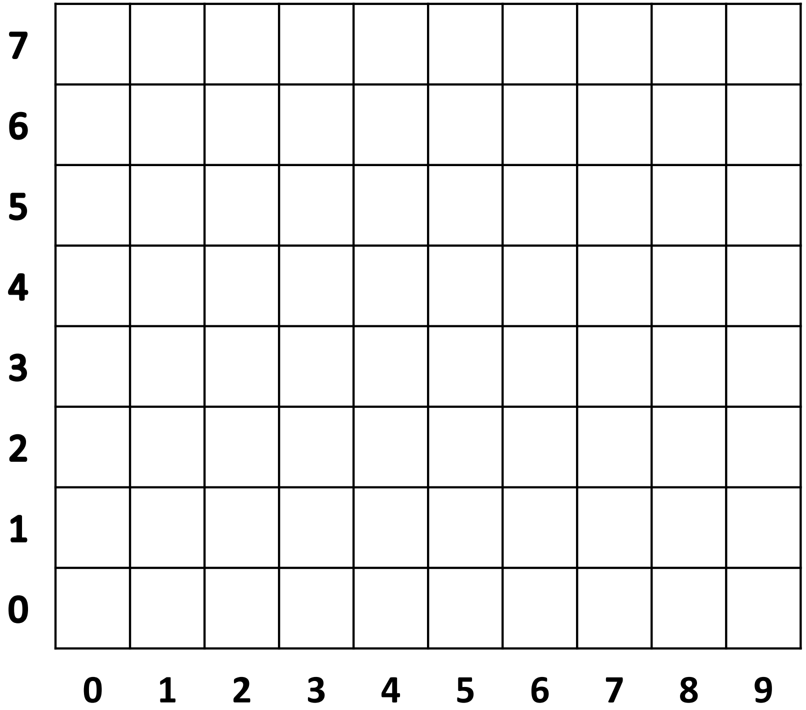
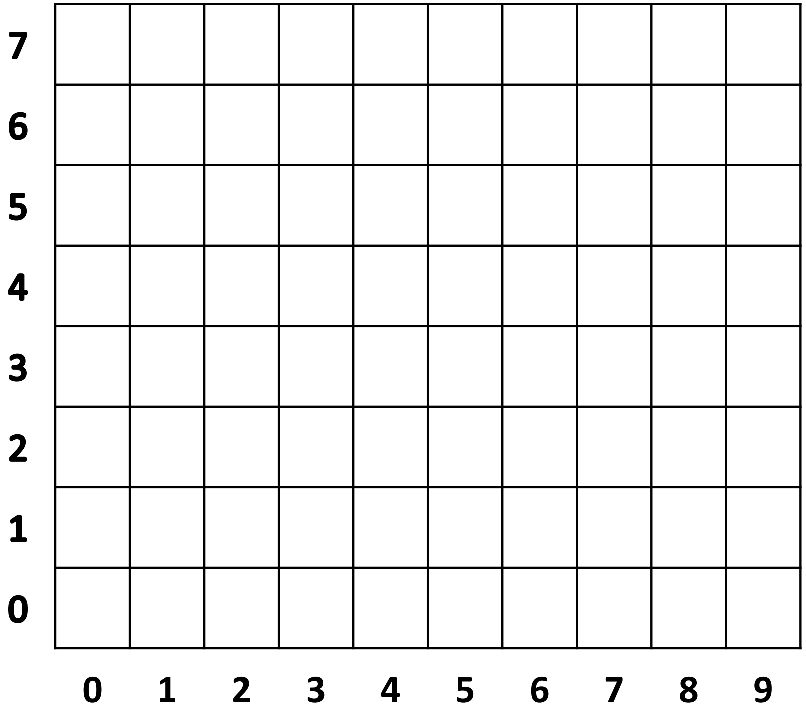
Today

- Image stitching

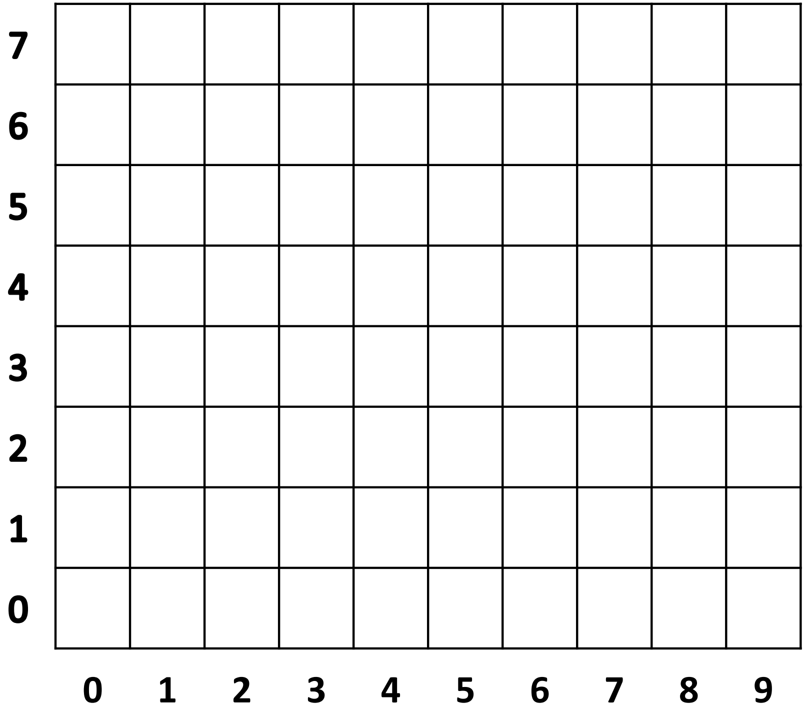
Euclidean vs. Homogeneous Coordinates



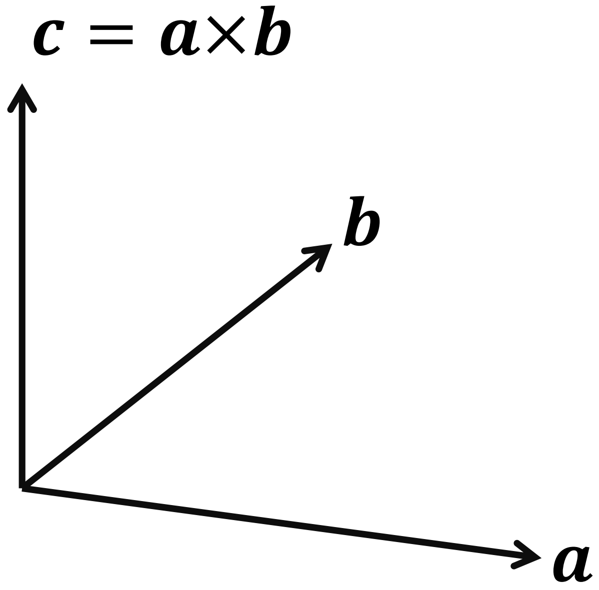
Points at infinity



Line equations in homogeneous coordinates



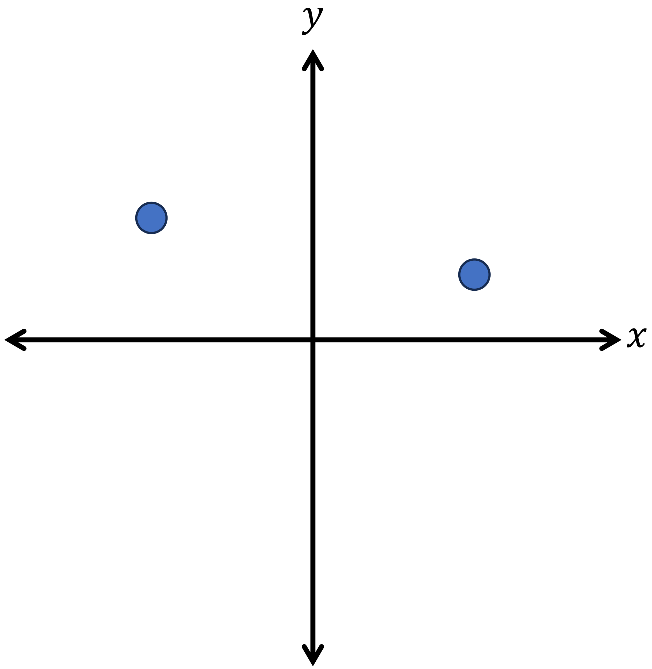
Cross-product of two vectors



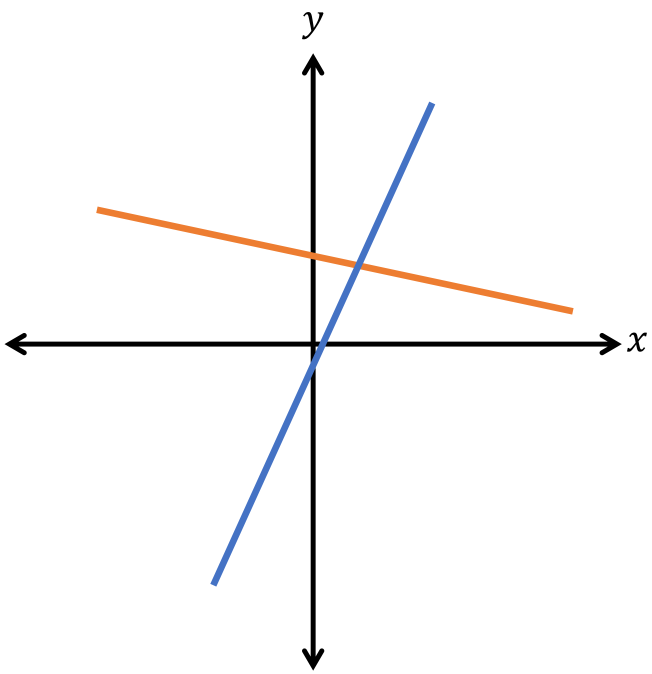
$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \boldsymbol{a_x} & \boldsymbol{a_y} & \boldsymbol{a_z} \\ \boldsymbol{b_x} & \boldsymbol{b_y} & \boldsymbol{b_z} \end{vmatrix}$$

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{a_z} & \boldsymbol{a_y} \\ \boldsymbol{a_z} & \boldsymbol{0} & -\boldsymbol{a_x} \\ -\boldsymbol{a_y} & \boldsymbol{a_x} & \boldsymbol{0} \end{bmatrix} \boldsymbol{b}$$

The line passing through two points



The point of intersection of two lines



Intersecting two parallel lines

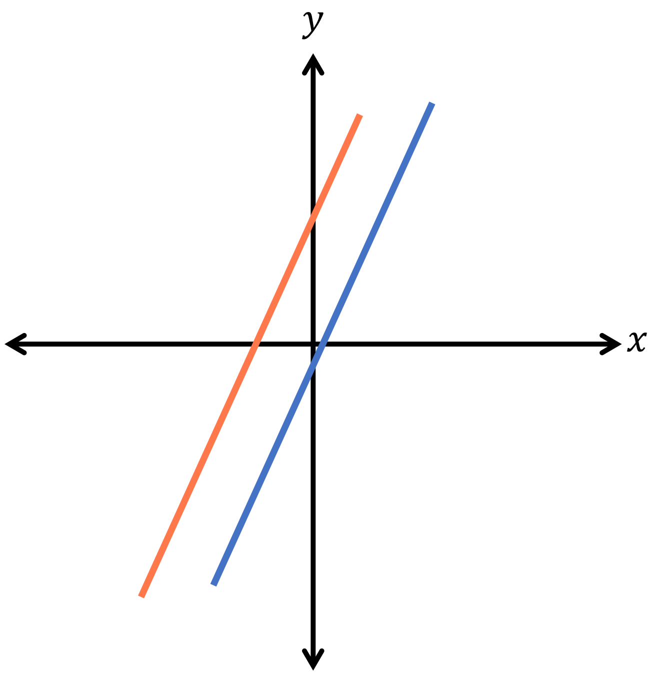
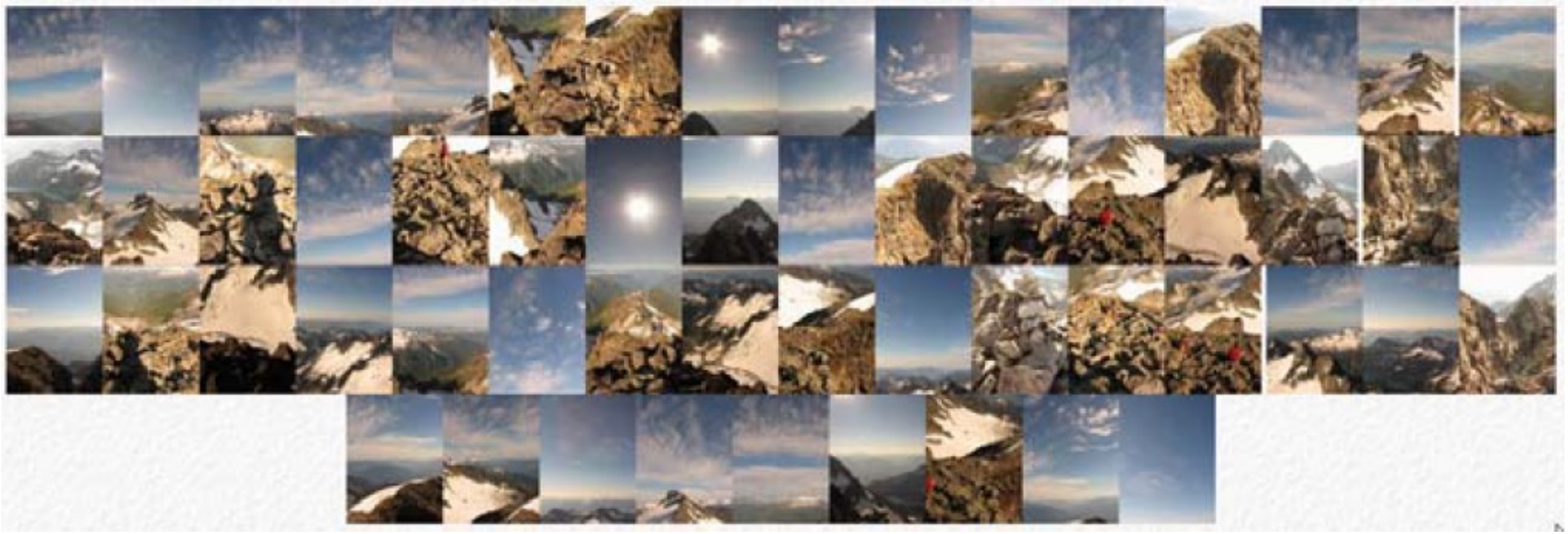


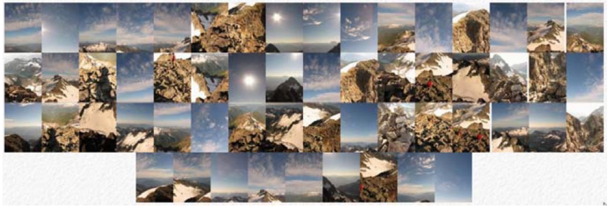
Image stitching



57 images

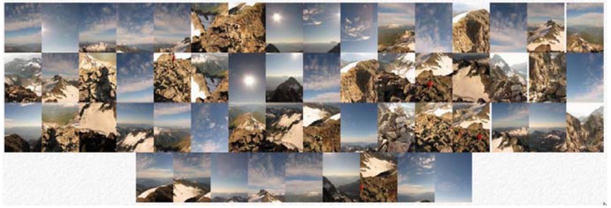
Camera should change orientation only, not position.
Keep camera settings (gain, focus, speed, aperture) fixed, if possible.

Image stitching



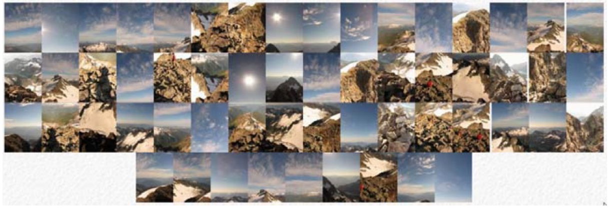
Using 28 out of 57 images

Image stitching



Using all 57 images

Image stitching (Autostitch)



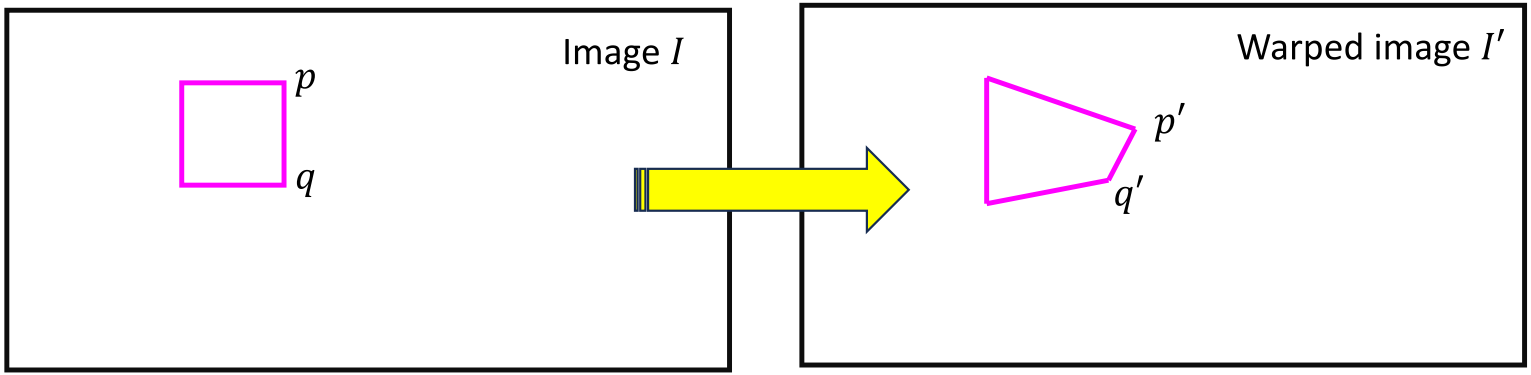
Seams are not visible

Using all 57 images. Laplacian blending.

Brown & Lowe; ICCV 2003

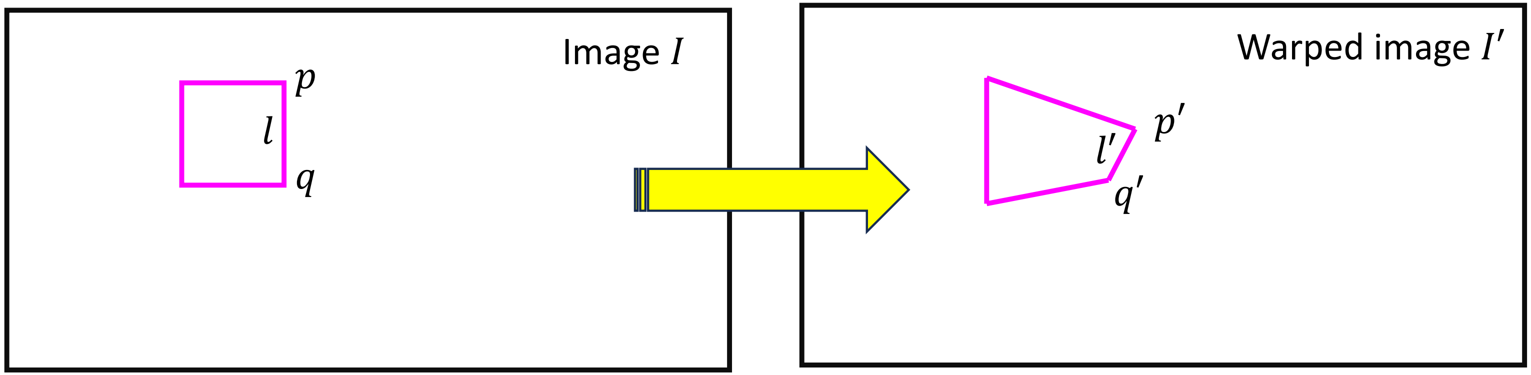
Linear image wraps

- To align multiple photos for image stitching, we must warp these images in such a way that all lines are preserved.
  - Lines before warping remain lines after warping
- Linear image wraps and *homographies*



Linear image wraps

- Definition: an image warp is linear if every 2D line l in the original image is transformed into a line l' in the warped image
- Property: Every linear warp can be expressed as a 3×3 matrix H that transforms homogeneous image coordinates (we won't discuss the proof here)



Warping images using homography

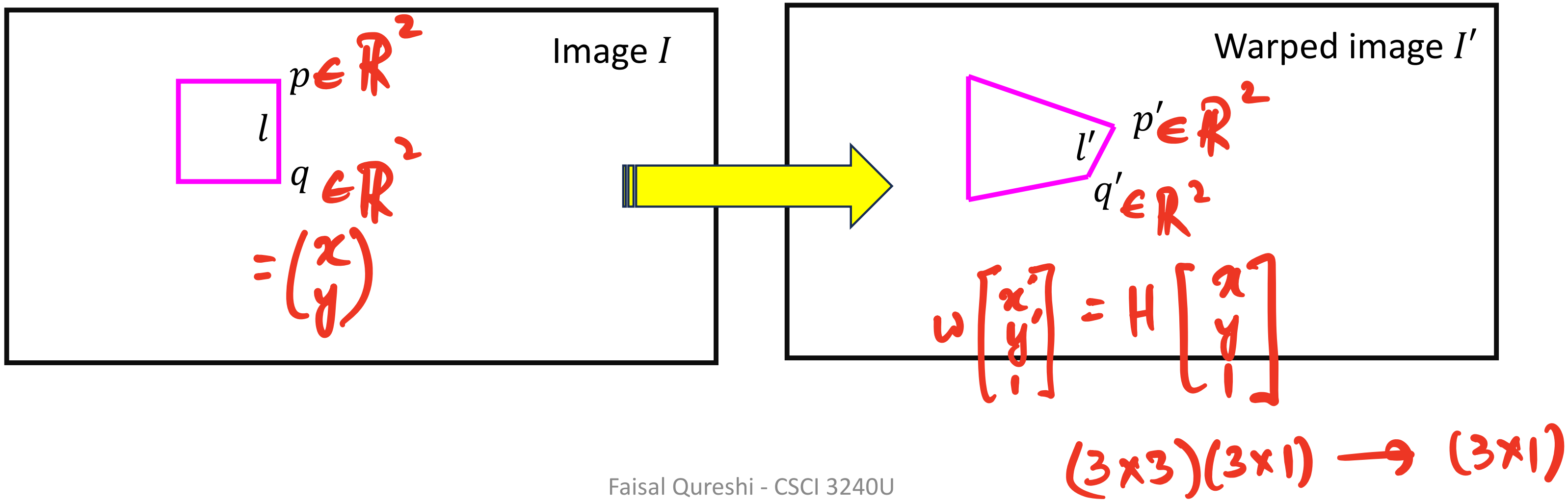
Linear warping equation: $I(\boldsymbol{p}) = I'(H\boldsymbol{p})$

Intensity at pixel in the source image I with homogeneous coordinates $\boldsymbol{p}$

Intensity at pixel in the warped image I' with homogeneous coordinates $H\boldsymbol{p}$

Matrix H is called homography

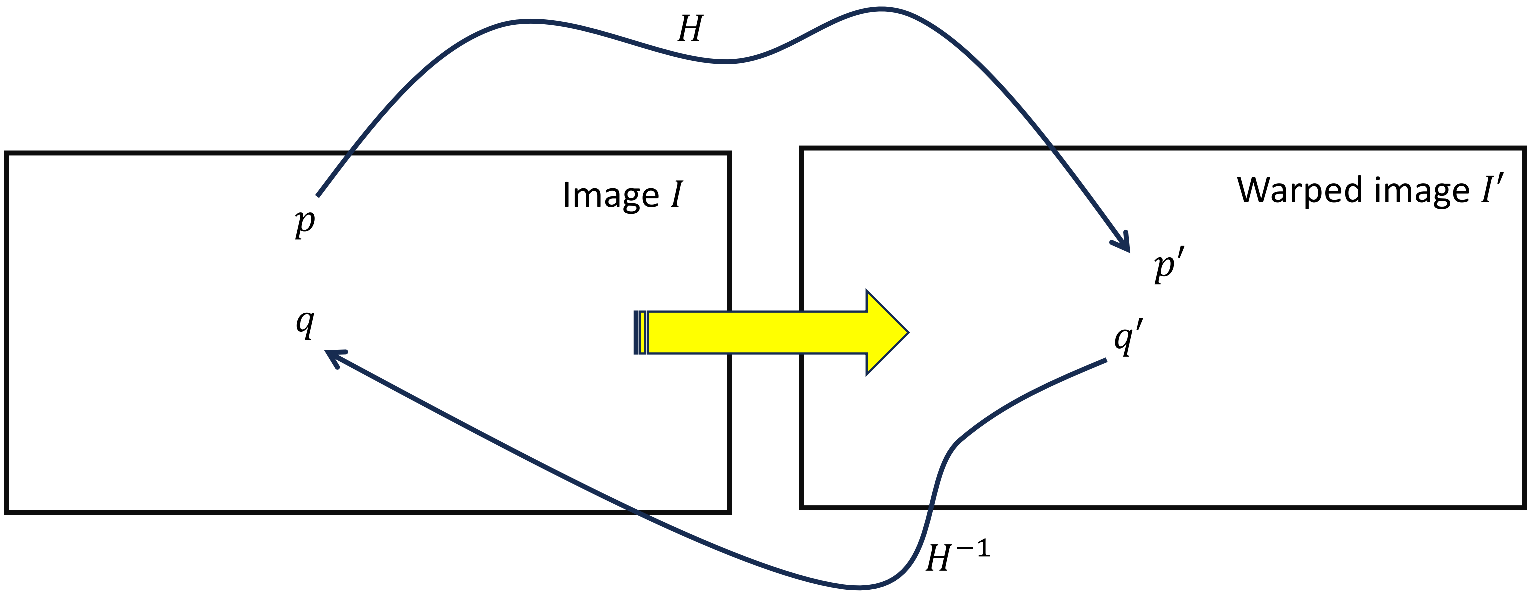
Scaling H by a factor $\lambda \neq 0$ does not change homography



Warping images using homography

Linear warping equation:

$$I(\boldsymbol{p}) = I'(H\boldsymbol{p}) \text{ and also } I'(\boldsymbol{q}') = I(H^{-1}\boldsymbol{q}')$$



Computing warp I' from I and H

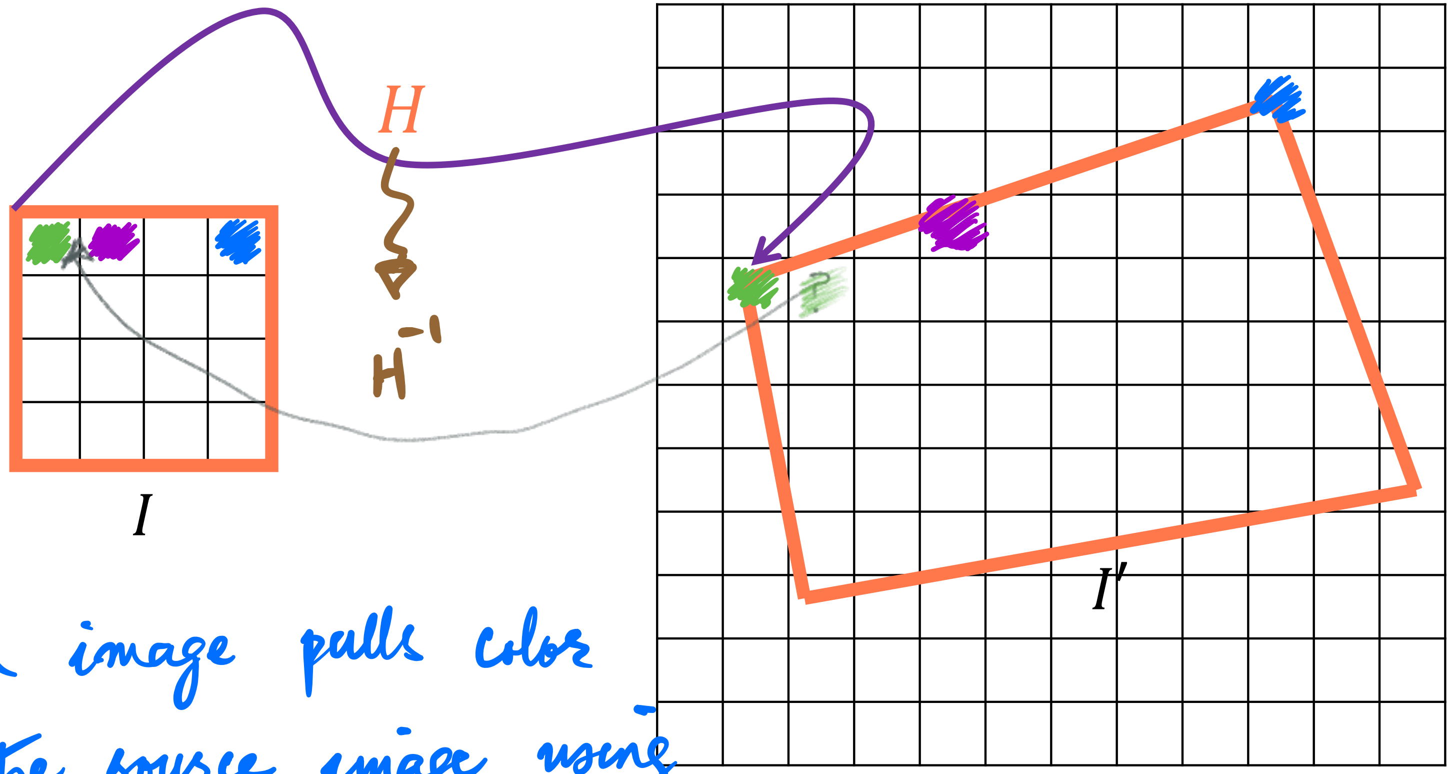
- Compute H^{-1}
- To compute the color of pixel (u, v) in the warped image
  - Compute $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = H^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$
  - Copy color from $I\left(\frac{a}{c}, \frac{b}{c}\right)$

What if location $\left(\frac{a}{c}, \frac{b}{c}\right)$ is not valid pixel locations?

1. Graduate scholarship applications are now open. Please check the graduate studies website.

2. For third-year students, start thinking about summer research opportunities. These will set you up for honors thesis, etc.

Computing warp I' from I and H



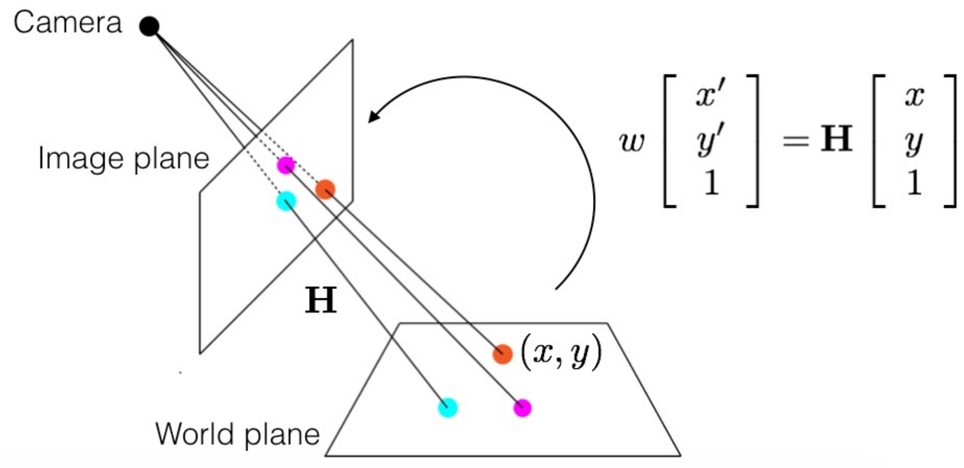
Warped image pulls color from the source image using H^{-1}.

Homography & image mosaicing

- Every photo taken from a tripod-mounted camera is related by a homography
- Assumptions
  - No lens distortion
  - Camera's center of projection does not move while camera is mounted on the tripod
- Problem
  - These homographies that relate photos taken from a tripod-mounted camera are *unknown*
    - We need to estimate them

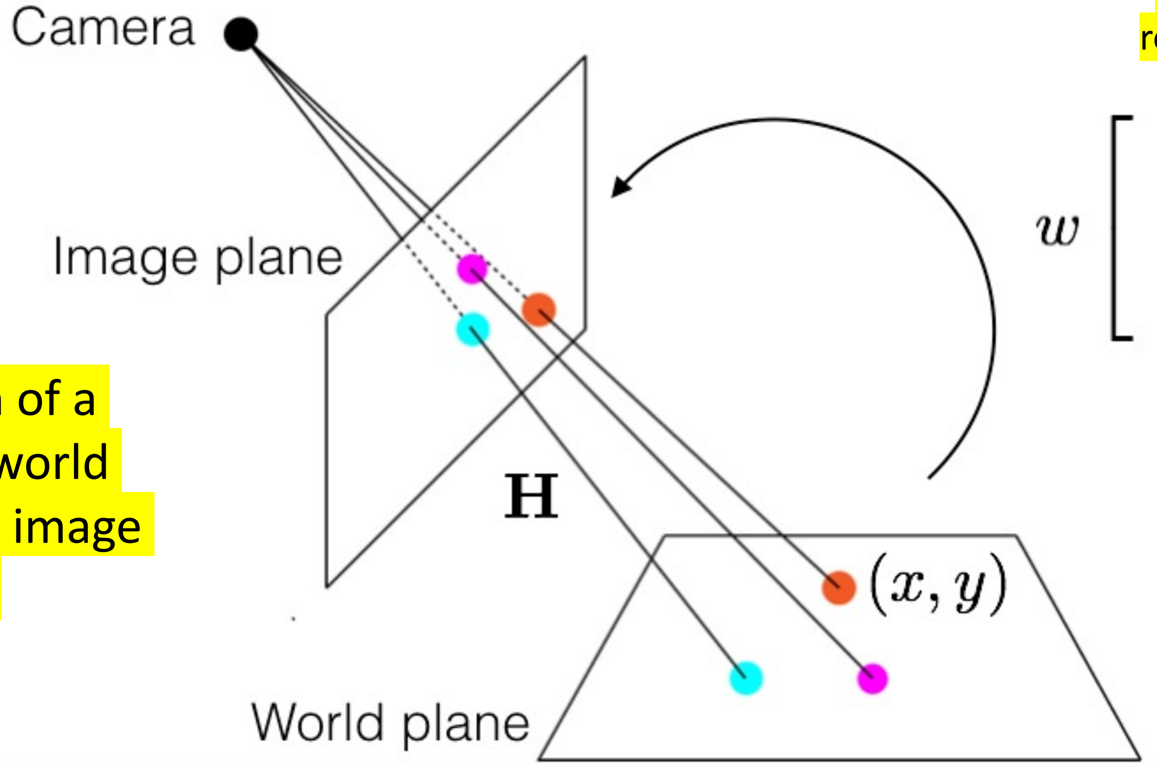
Homography

- Generally speaking, points that lie on two planes are related via homography.



Homography

The location of the point and that of its projection are related via a Homography.

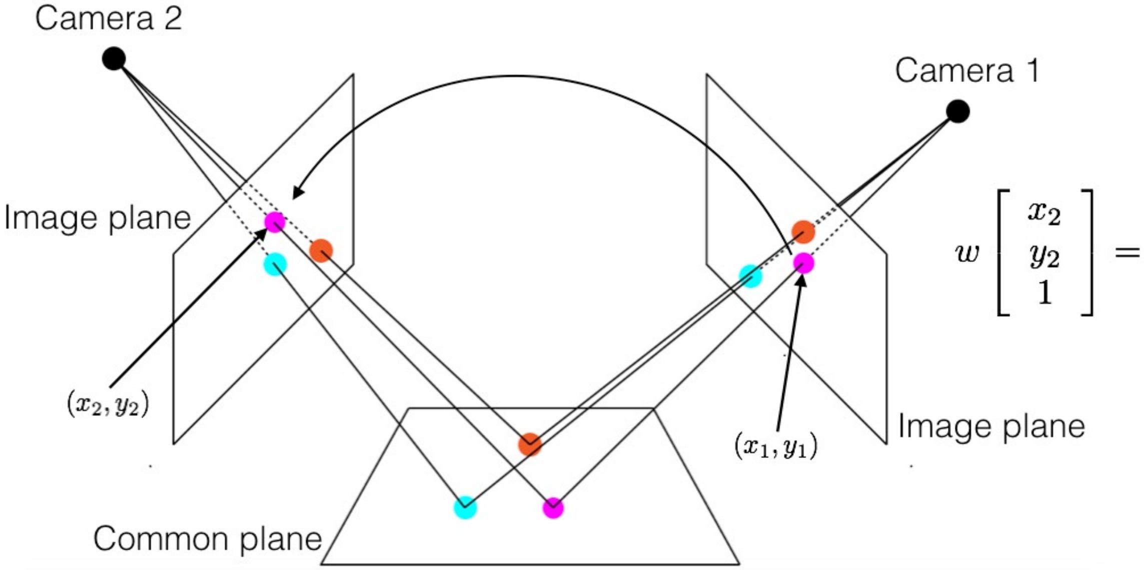


$$w \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projection of a point in a world plane to the image plane

Homography

- Generally speaking, points that lie on two planes are related via homography.
  - This also means that the projections of points (that lie on a common plane) in two cameras are related via homography.



$$w \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \mathbf{H_{12}} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

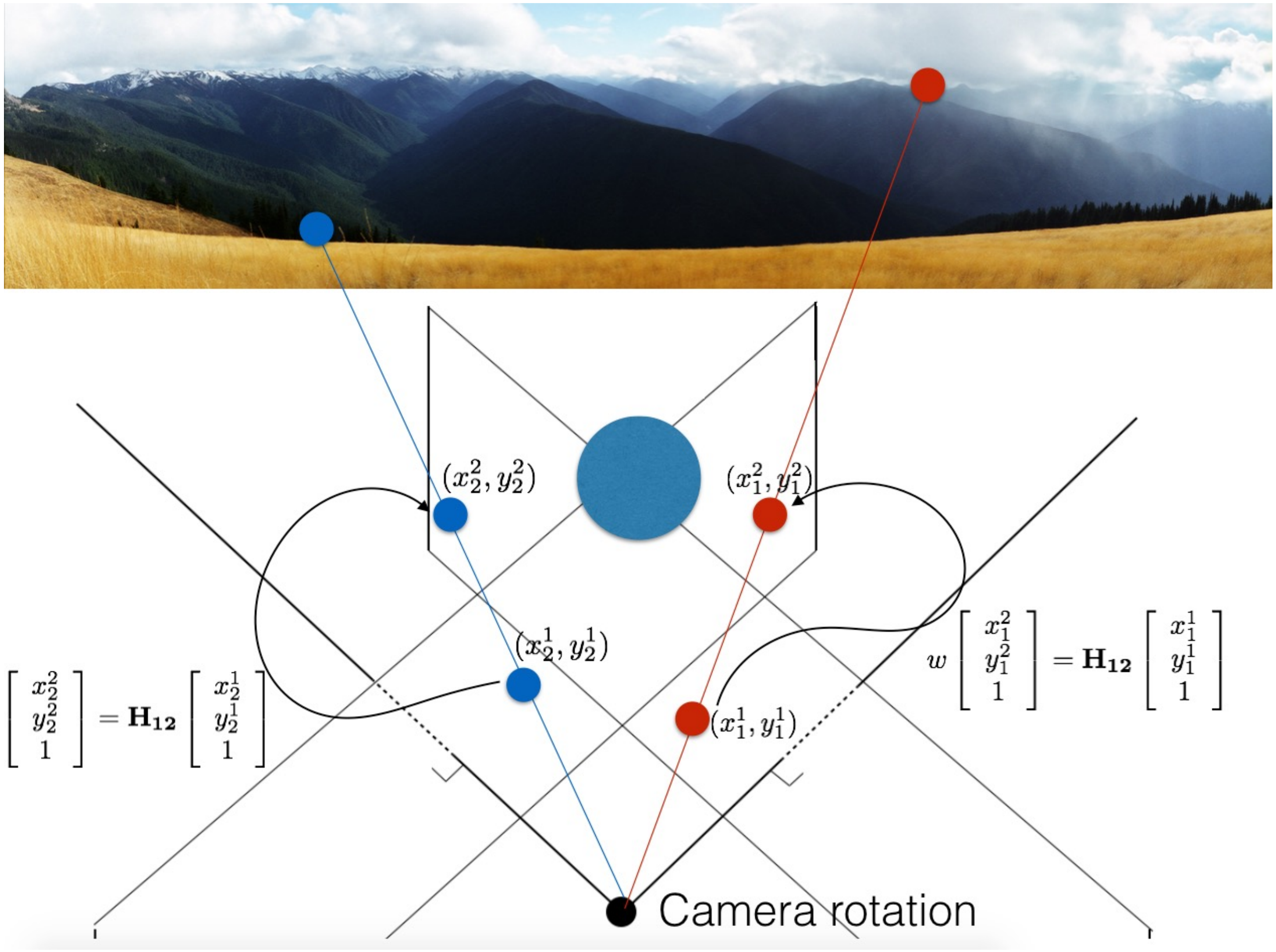
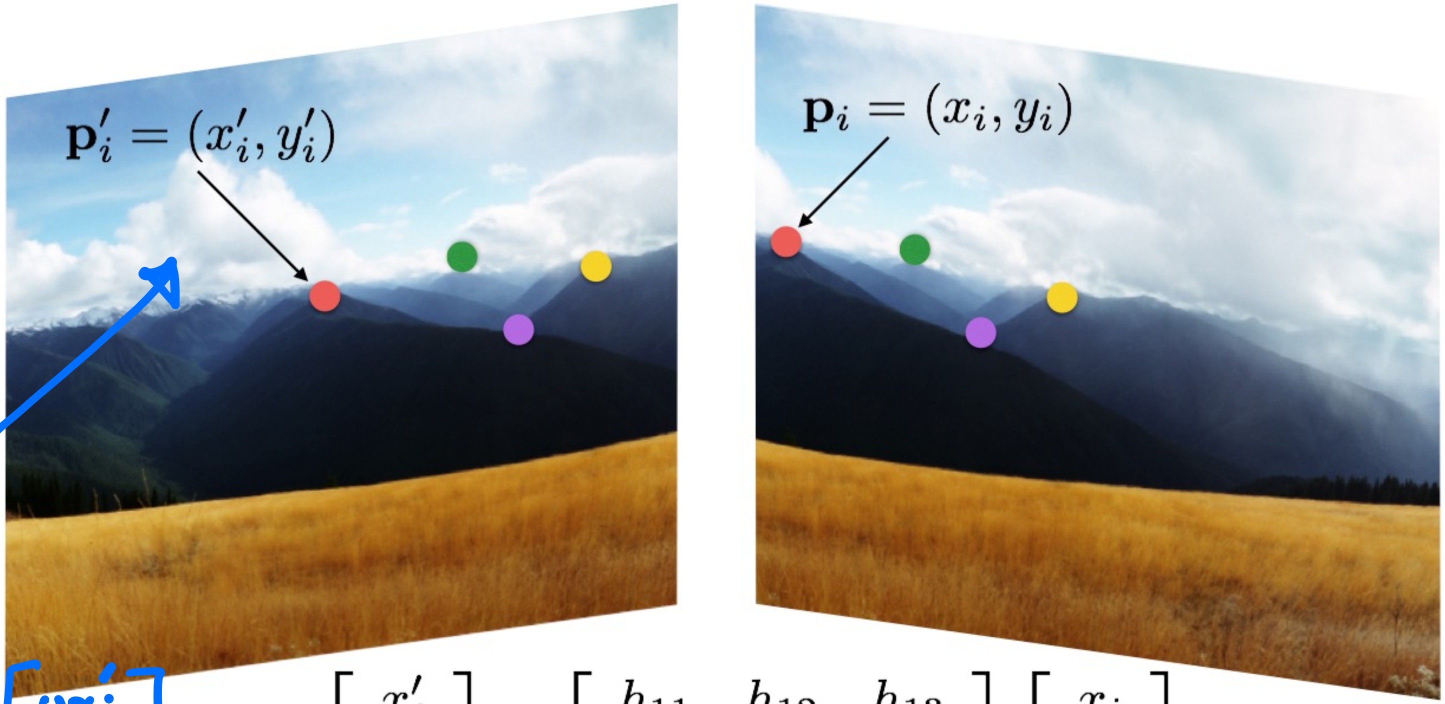
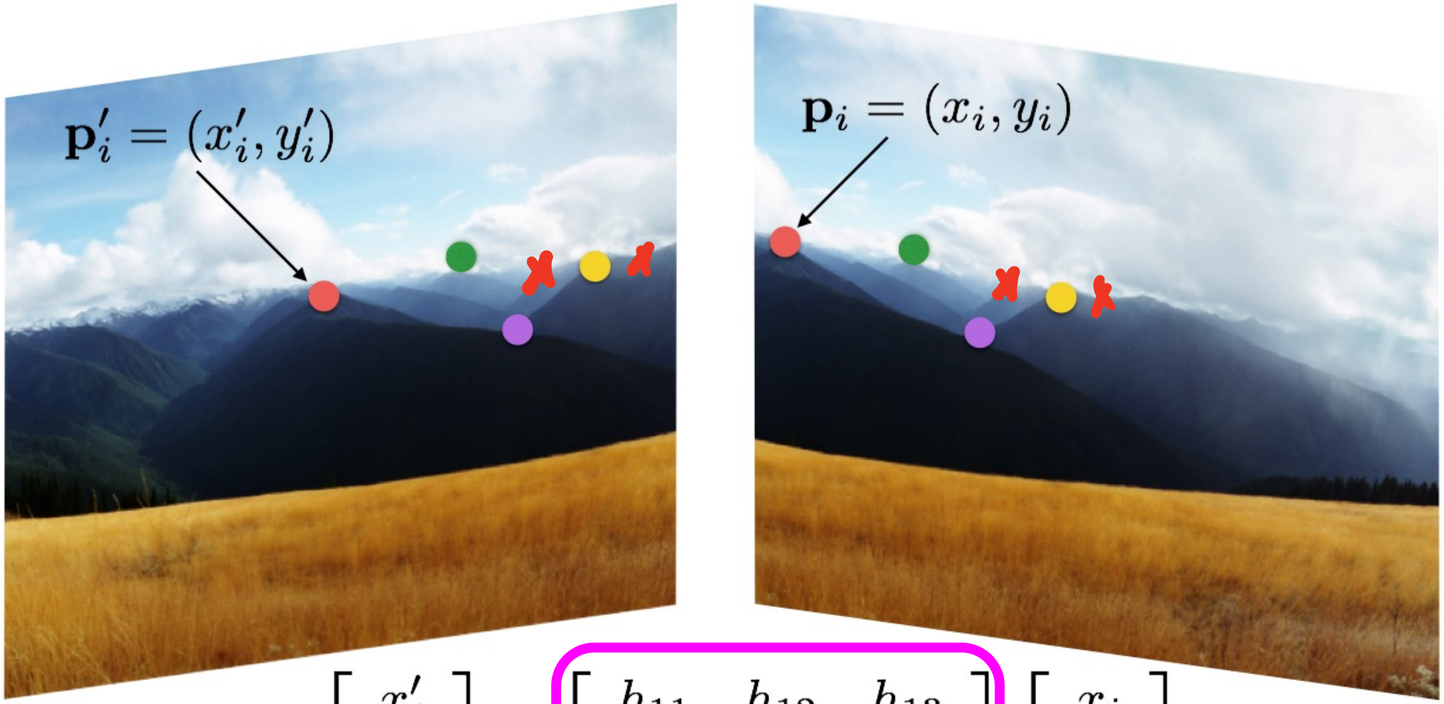

(x_2^2, y_2^2)
(x_1^2, y_1^2)
(x_2^1, y_2^1)
(x_1^1, y_1^1)
$\begin{bmatrix} x_2^2 \\ y_2^2 \\ 1 \end{bmatrix} = \mathbf{H_{12}} \begin{bmatrix} x_2^1 \\ y_2^1 \\ 1 \end{bmatrix}$
$w \begin{bmatrix} x_1^2 \\ y_1^2 \\ 1 \end{bmatrix} = \mathbf{H_{12}} \begin{bmatrix} x_1^1 \\ y_1^1 \\ 1 \end{bmatrix}$
Camera rotation

Image stitching



$$\begin{bmatrix} w x'_i \\ w y'_i \\ w \end{bmatrix} = w \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Image stitching



$$w \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

How do we solve H?

Solving homography

$$w\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

How many degrees-of-freedom?

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \begin{bmatrix} 3x \\ 3y \\ 3 \end{bmatrix}, \quad \begin{bmatrix} kx \\ ky \\ k \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

$$\begin{bmatrix} A \end{bmatrix}\begin{bmatrix} \vec{x} \end{bmatrix} = \begin{bmatrix} \vec{b} \end{bmatrix}$$

Solving for homography (Step 1)

- Re-write homography relationship as homogeneous equations

$$x_i' = \frac{h_{11} x_i + h_{12} y_i + h_{13}}{h_{31} x_i + h_{32} y_i + h_{33}} \qquad y_i' = \frac{h_{21} x_i + h_{22} y_i + h_{23}}{h_{31} x_i + h_{32} y_i + h_{33}}$$

$$\Rightarrow h_{11} x_i + h_{12} y_i + h_{13} = h_{31} x_i x_i' + h_{32} y_i x_i' + h_{33} x_i'$$

$$\Rightarrow h_{11} x_i + h_{12} y_i + h_{13} - h_{31} x_i x_i' - h_{32} y_i x_i' - h_{33} x_i' = 0 \quad (A)$$

$$h_{21} x_i + h_{22} y_i + h_{23} - h_{31} x_i y_i' - h_{32} y_i y_i' - h_{33} y_i' = 0 \quad (B)$$

Solving for homography (Step 2)

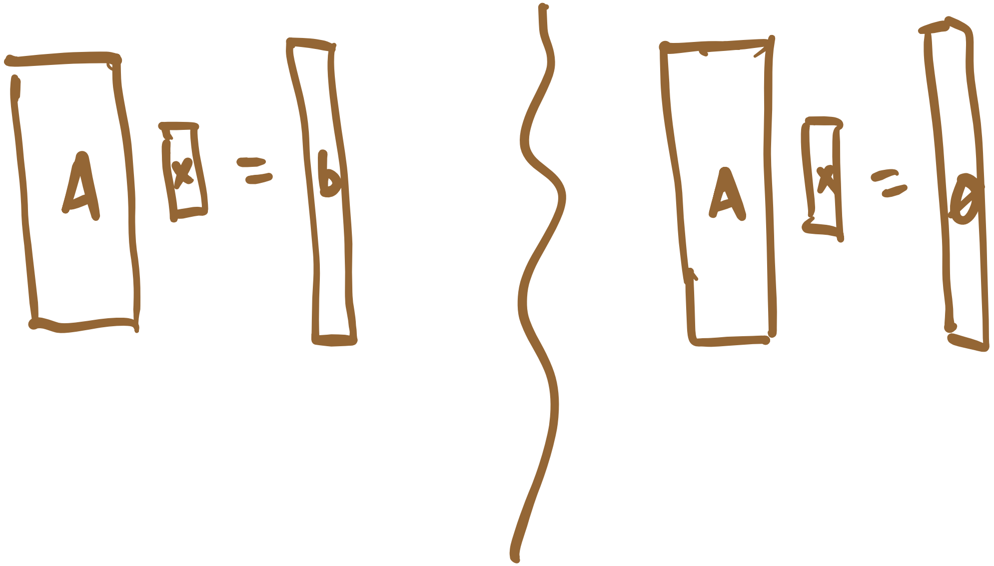
- We can then write these as matrix-vector product

$$
\underbrace{\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i x_i' & -y_i x_i' & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -x_i y_i' & -y_i y_i' & -y_i' \\ & & & & \vdots & & & & \end{bmatrix}}_{\in \mathbb{R}^{20\times 9}}
\underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\in \mathbb{R}^{9\times 1}\ ?}
=
\underbrace{\begin{bmatrix} [0] \\ [0] \\ \vdots \end{bmatrix}}_{\in \mathbb{R}^{20\times 1}}
$$

(A) is the first row and (B) the second row.

Solving for homography (Step 3)

- Given n correspondences between two images, setup $A\boldsymbol{x} = 0$ and solve for $\boldsymbol{x}$.



Solving $Ax = 0$

- Estimate using least-square fitting

$$x^* = \underset{x}{\operatorname{argmax}} \; ||Ax||^2 \;\; \text{s.t.} \;\; ||x|| = 1$$

- The solution is the right *null-space* of A; therefore, the solution is the eigenvector corresponding to the smallest eigenvalue of $A^T A$

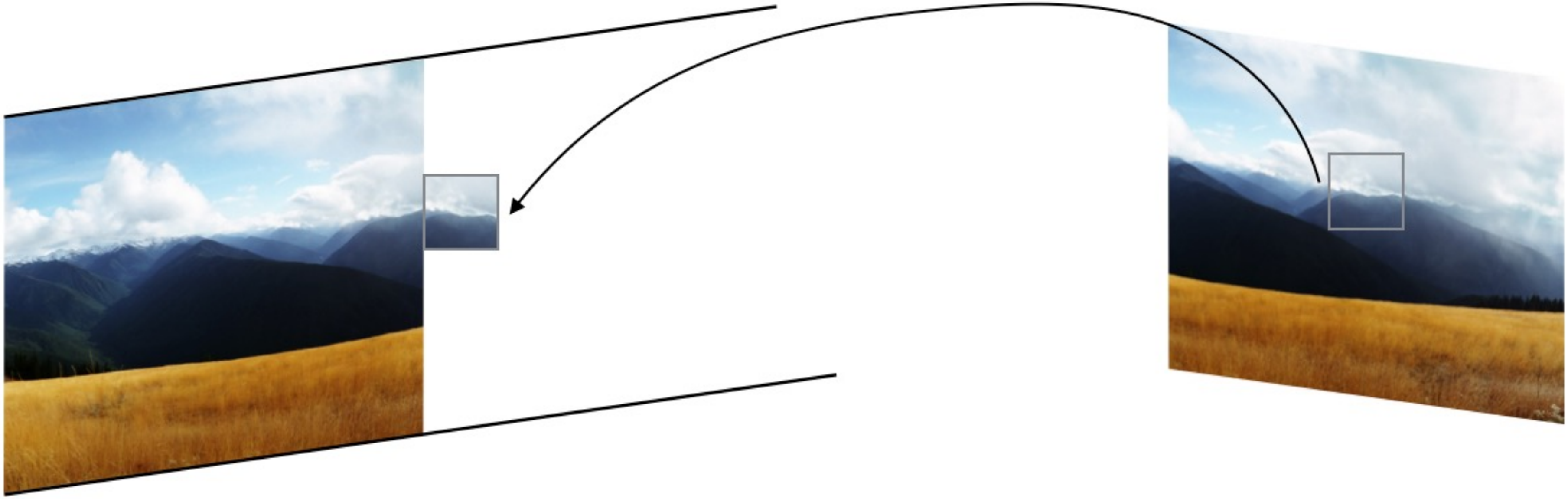
$Ax = 0 \rightarrow$ Compute A^TA

Eigenvectors / eigenvalues of A^TA.

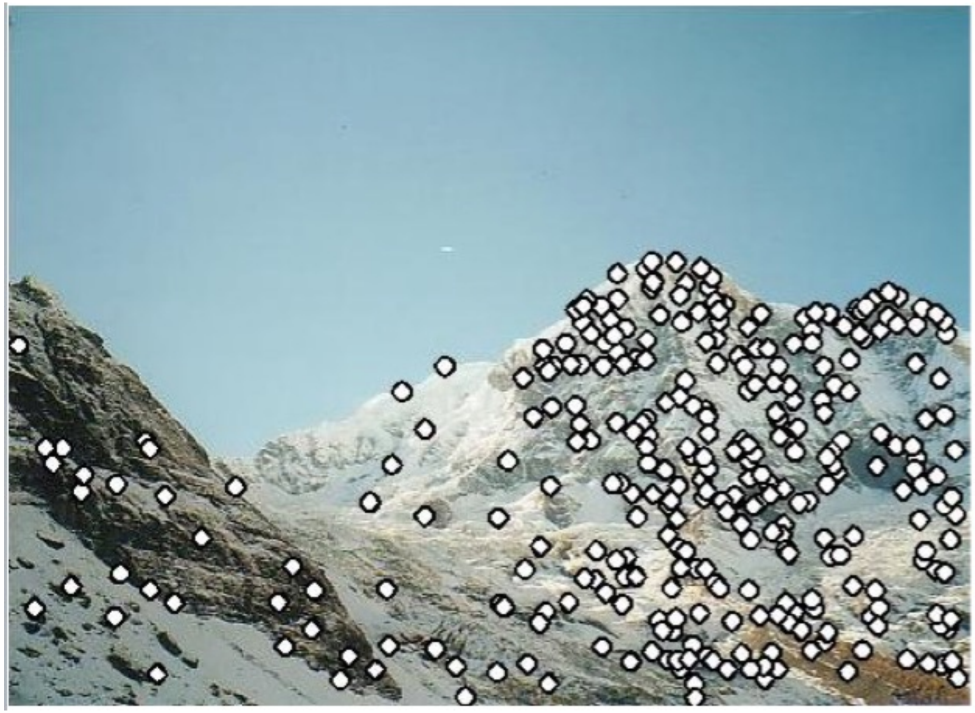
Eigenvector corresponding to the smallest eigenvalue is the solution.

Image stitching

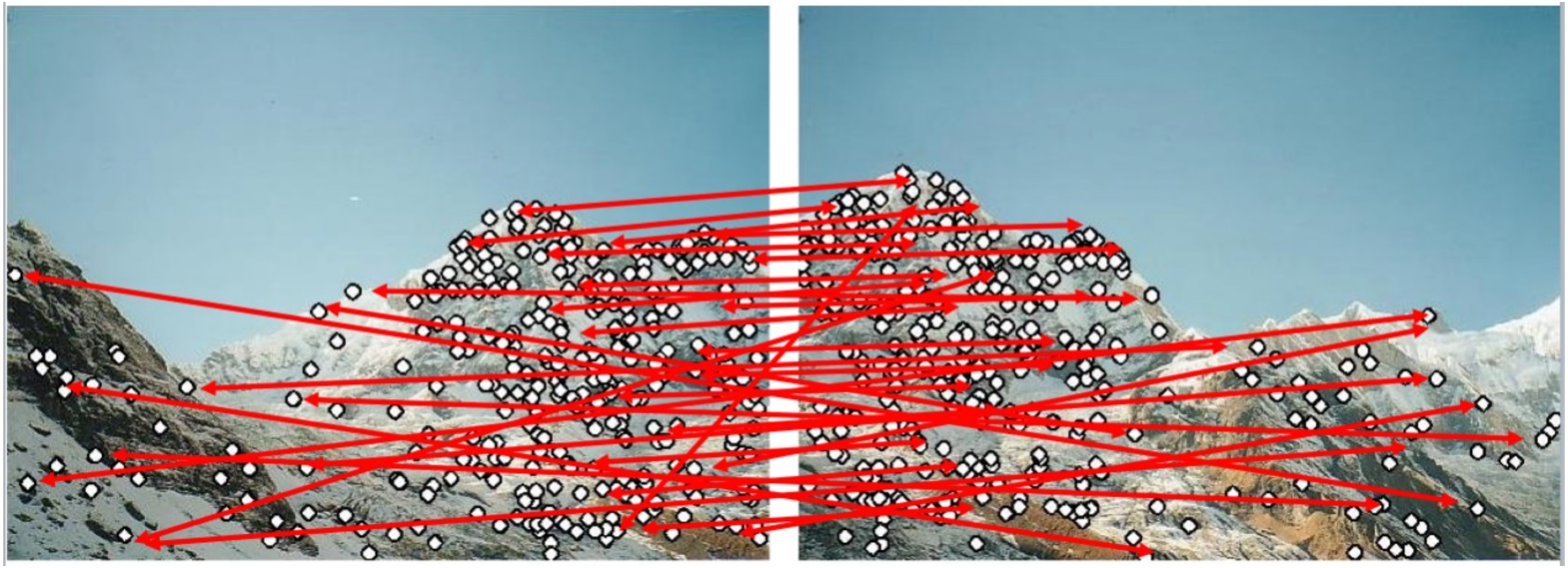
- Estimate homography
- Use it to fill the colors from the “other” image



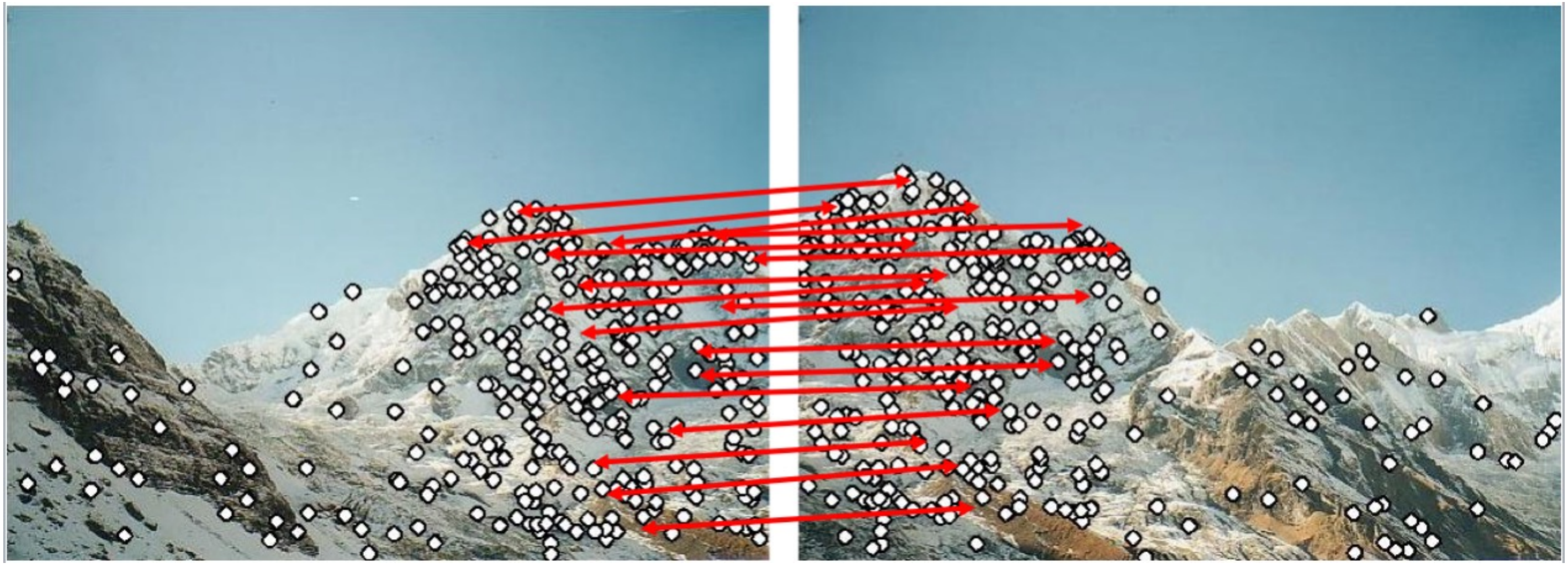
Extract features



Find matches



Use RANSAC to estimate homography



Perform image stitching

$$\left(\frac{fx}{z}, \frac{fy}{z}\right) \leftarrow \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Quiz $f(x) = 3x^2 + x$ at $x = 3$.

$$\frac{\partial f}{\partial x} = 6x$$

$$\frac{\partial^2 f}{\partial x^2} = 6$$

$$I = \{ G_0, G_1, \ldots, G_n \}$$

↑

I

$32 \times 32 = 1024$ 1 byte

$16 \times 16 = 256$

$8 \times 8 = 64$

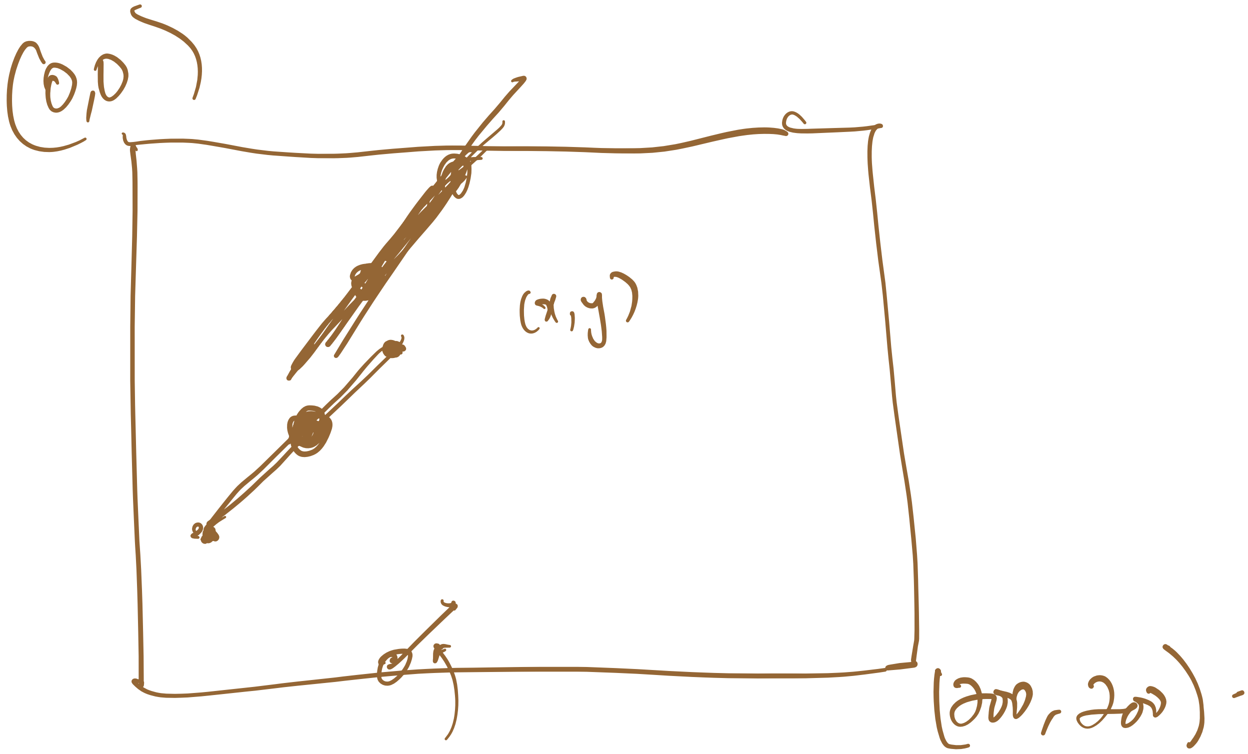
$4 \times 4 = 16$

$2 \times 2 = 4$

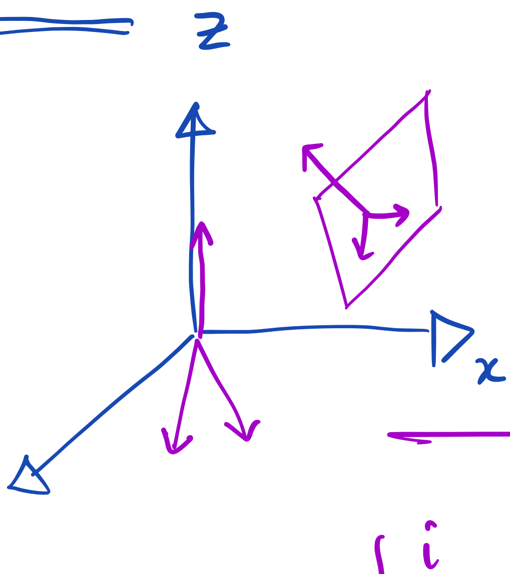
$1 \times 1 = 1$

$$\begin{bmatrix} 64 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 64/4 \\ 8/4 \end{bmatrix} = \begin{bmatrix} 16 \\ 2 \end{bmatrix} \quad (16)$$

$\vec{a} \cdot \vec{b}$ $\vec{a} \cdot \vec{b} = \emptyset$



Cross-product



$\vec{a} = (a_x, a_y, a_z)$

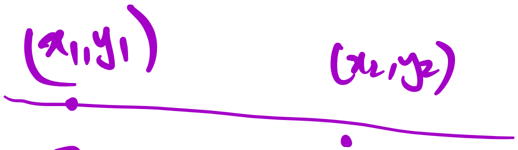
$\vec{b} = (b_x, b_y, b_z)$

$v1 = (a, b, c)$

$v2 = (d, e, f)$

$$\begin{vmatrix} i & j & k \\ a & b & c \\ d & e & f \end{vmatrix}$$

$$= i(bf - ec) - j(af - de) + k(ae - db)$$



$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

$$\begin{vmatrix} i & j & k \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$