Image Interpolation

Computational Photography (CSCI 3240U)

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How do we resize images?

Original

Upscaling

Downscaling

Let’s consider a 1D image

7	4	3

We want to increase its width by a factor of 2

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Upscaling

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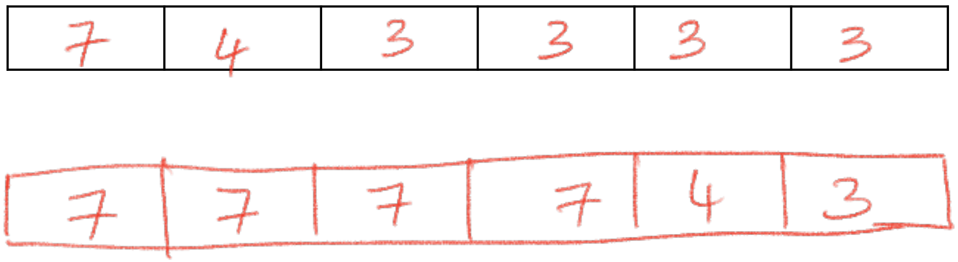
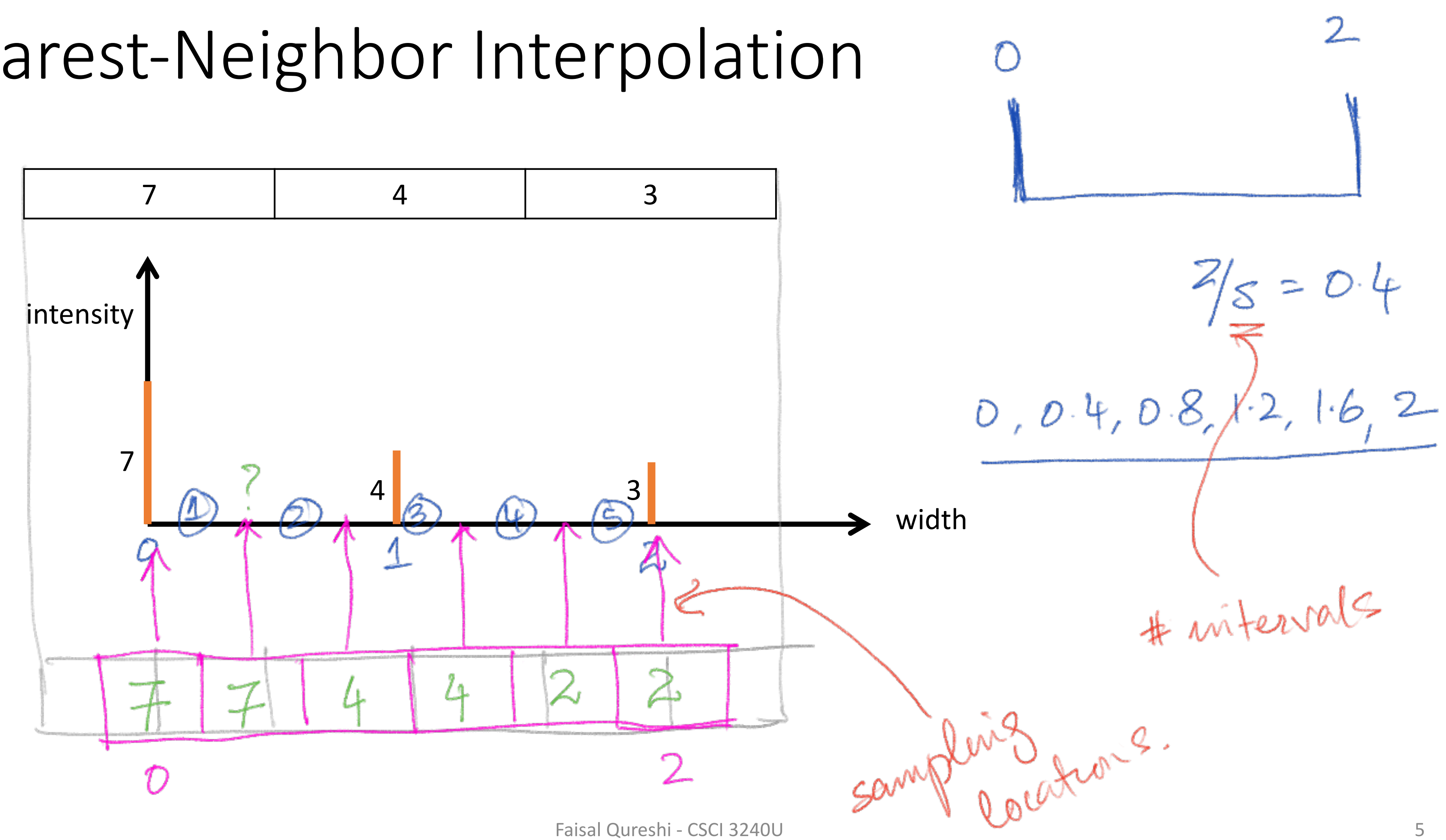


Nearest-Neighbor Interpolation



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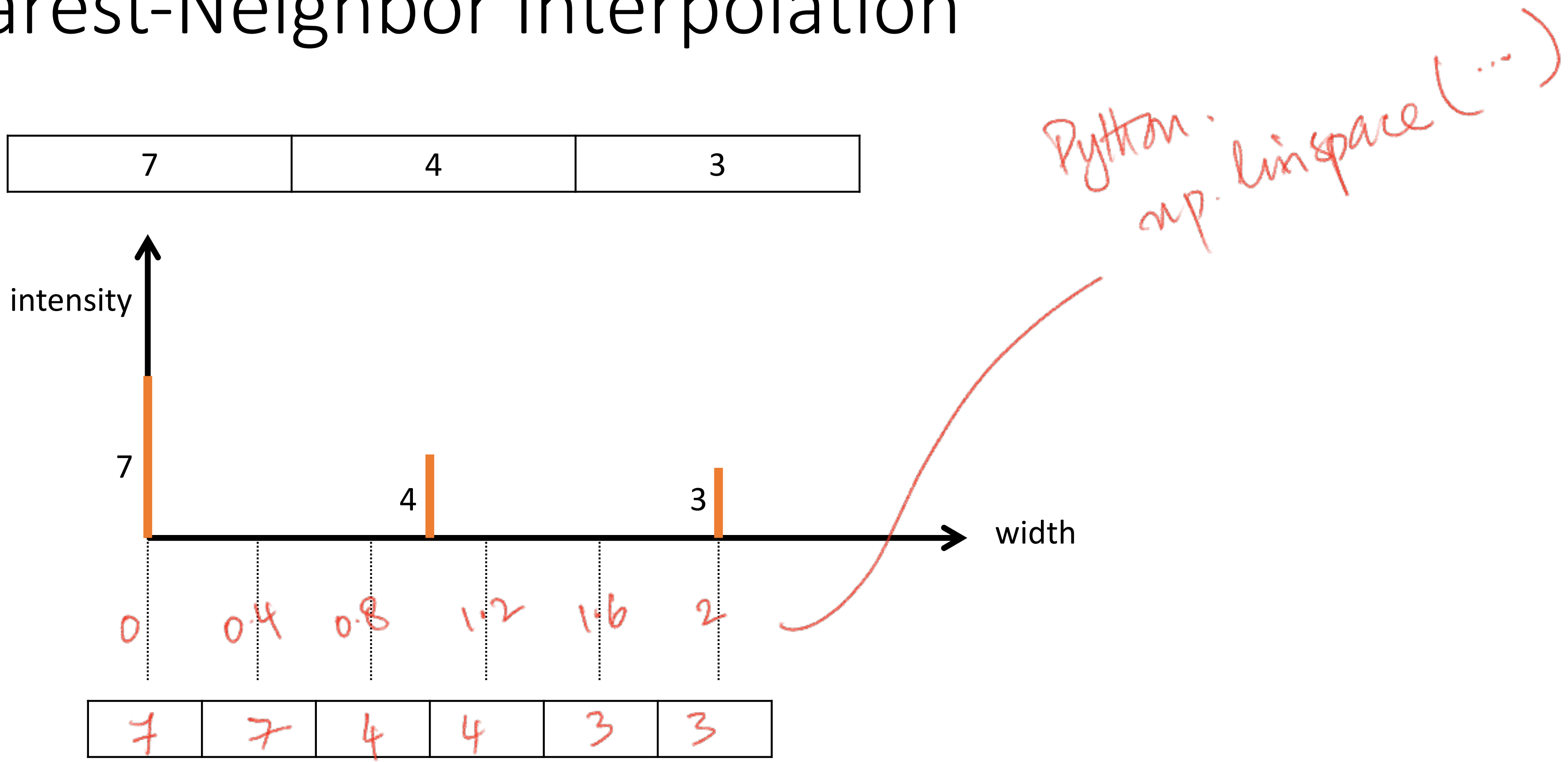

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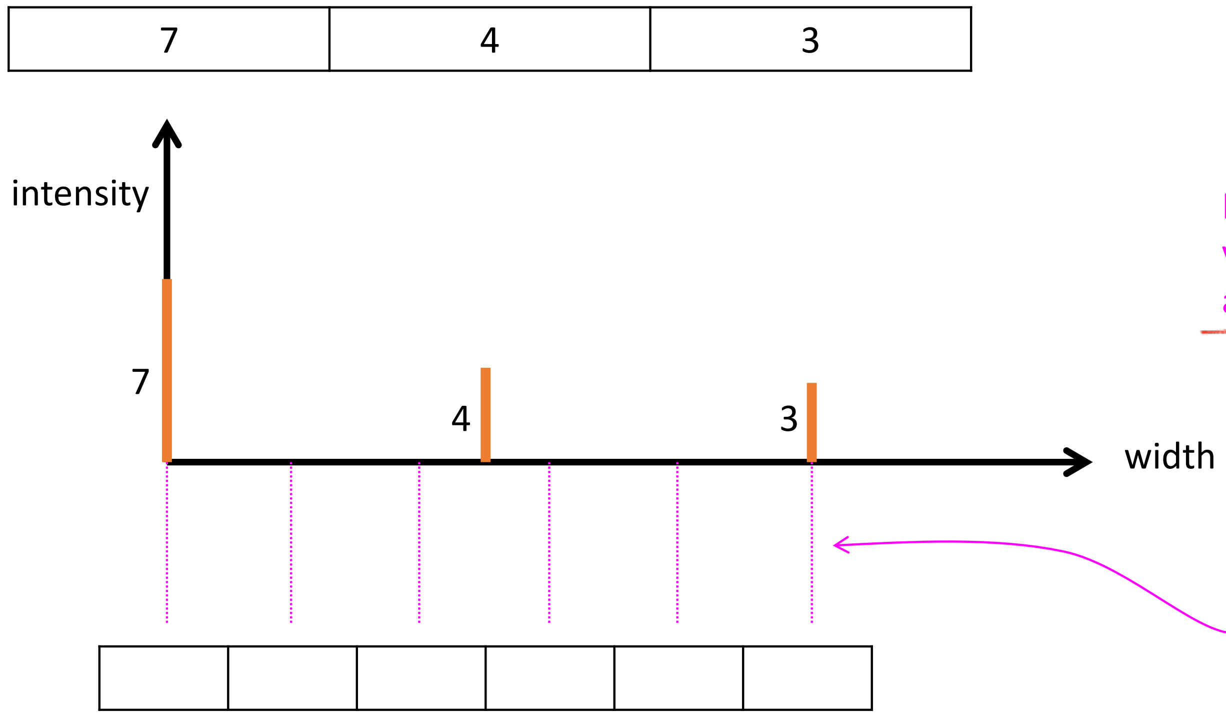
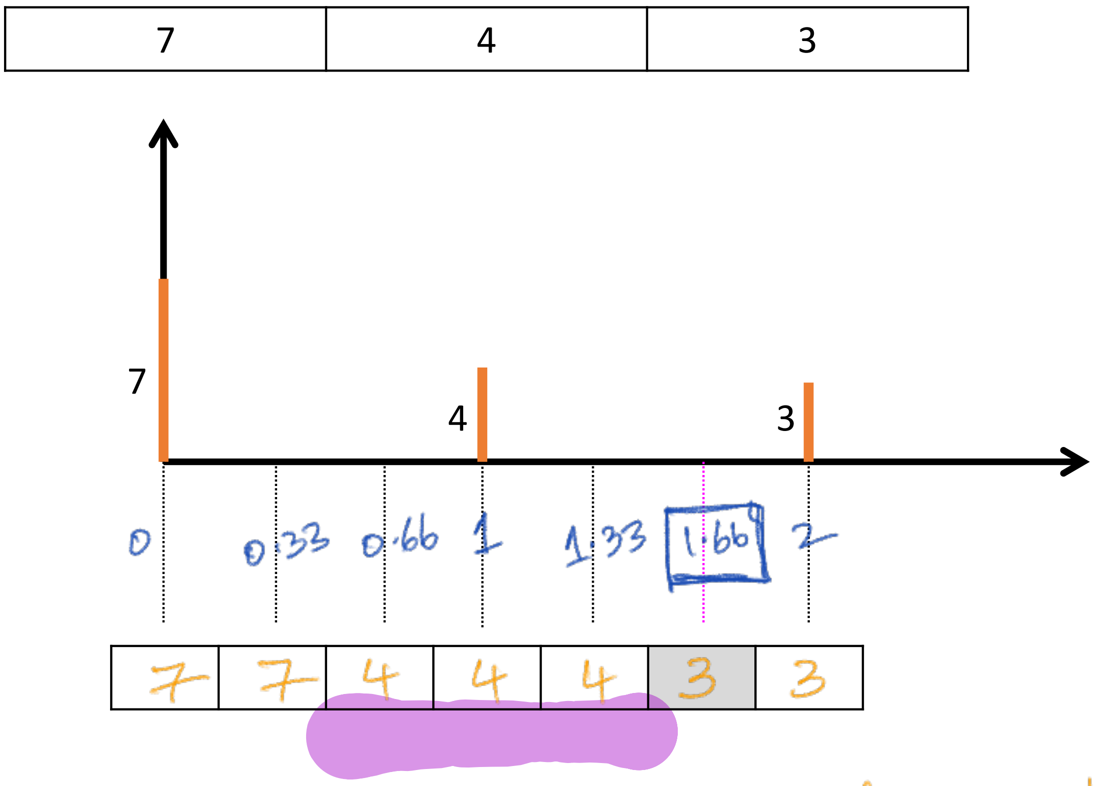


How do we compute location values for the 6 pixels between 0 and 2?

$$\frac{\#pixels(original) - 1}{\#pixels(result) - 1}$$

Nearest-Neighbor Interpolation



Practice Questions

1. What is the location (between 0 and 2) of the shaded pixel?
2. Compute the value for the shaded pixel?

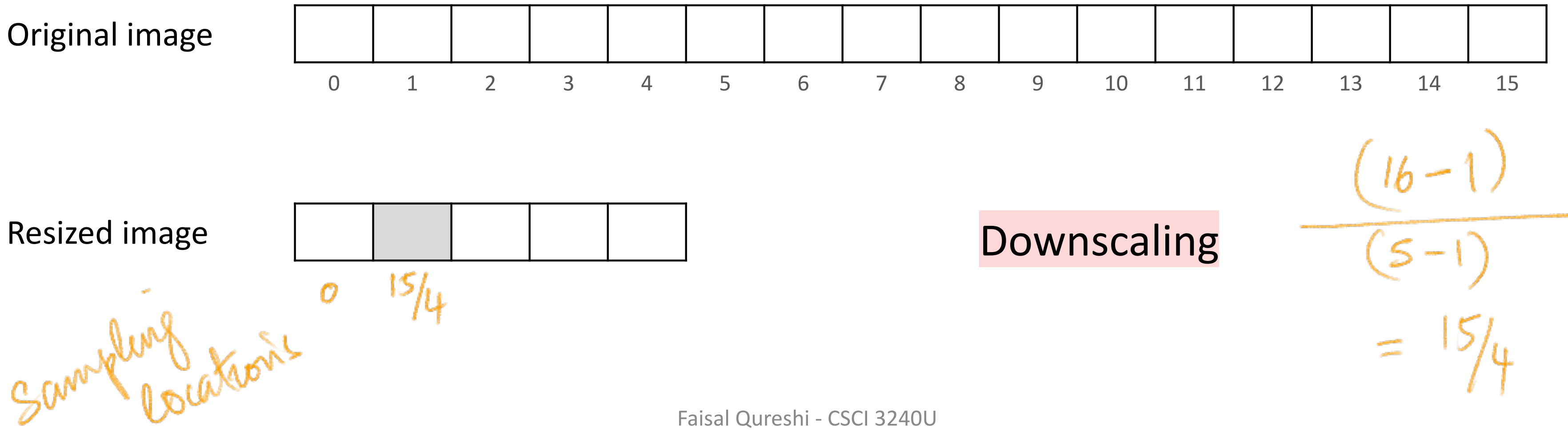
1. $\frac{3-1}{7-1} = \frac{2}{6} = \frac{1}{3}$
   $= 0.33$

Problem: some pixels are being used more often than others.

Nearest-Neighbour Interpolation

Practice Question

Consider a 16-pixel 1D image. You are asked to resize it to a 5-pixel 1D image. What is the location of pixel 2 (between 0 and 15) of the new image?



Downscaling

$$\frac{(16-1)}{(5-1)} = 15/4$$

Nearest-Neighbour Interpolation

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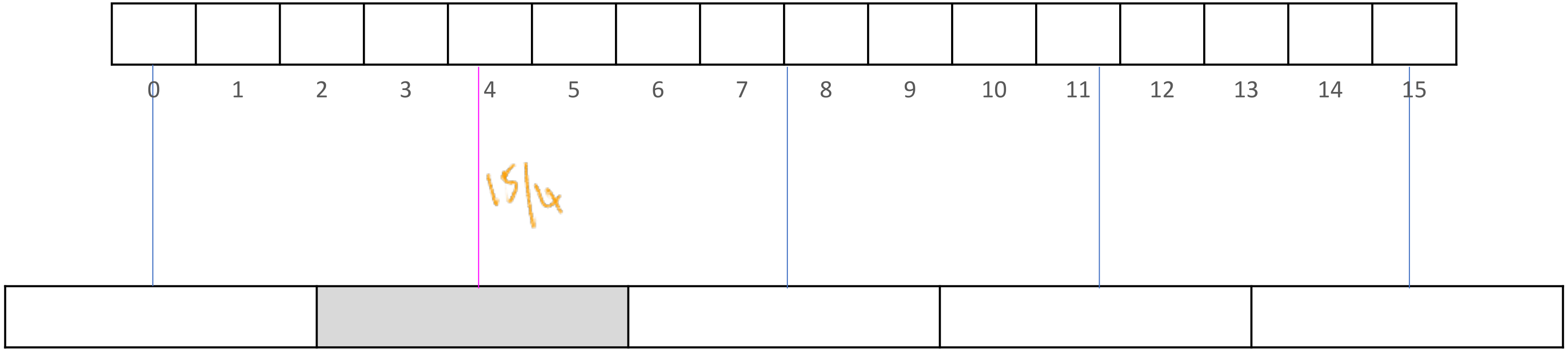


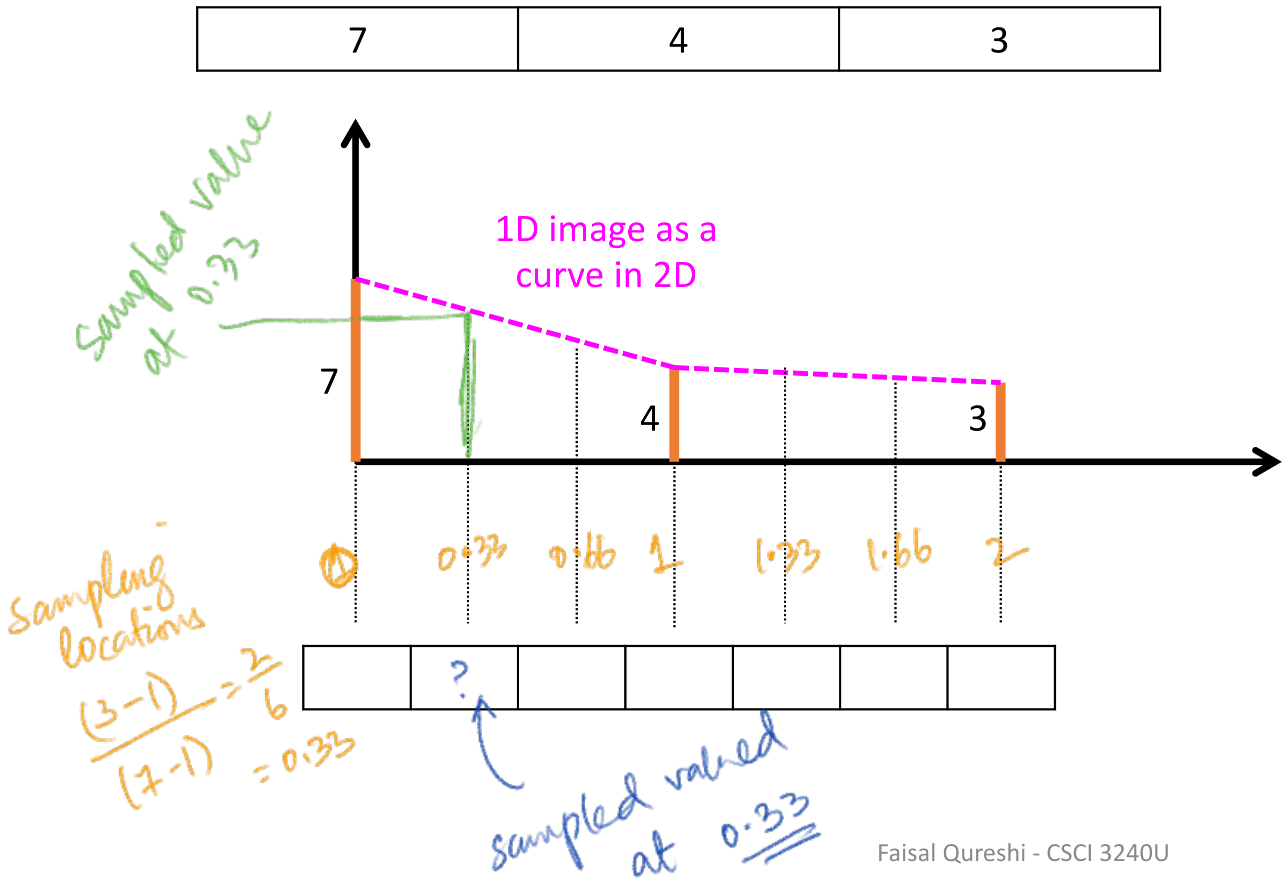
Nearest-Neighbor Interpolation

- Easy to implement.
- Results in blocky or pixelated results
- Does not consider neighboring pixels
- Losses details and smoothness
- Use other methods, e.g., bilinear, bicubic, etc., for higher-quality image resizing

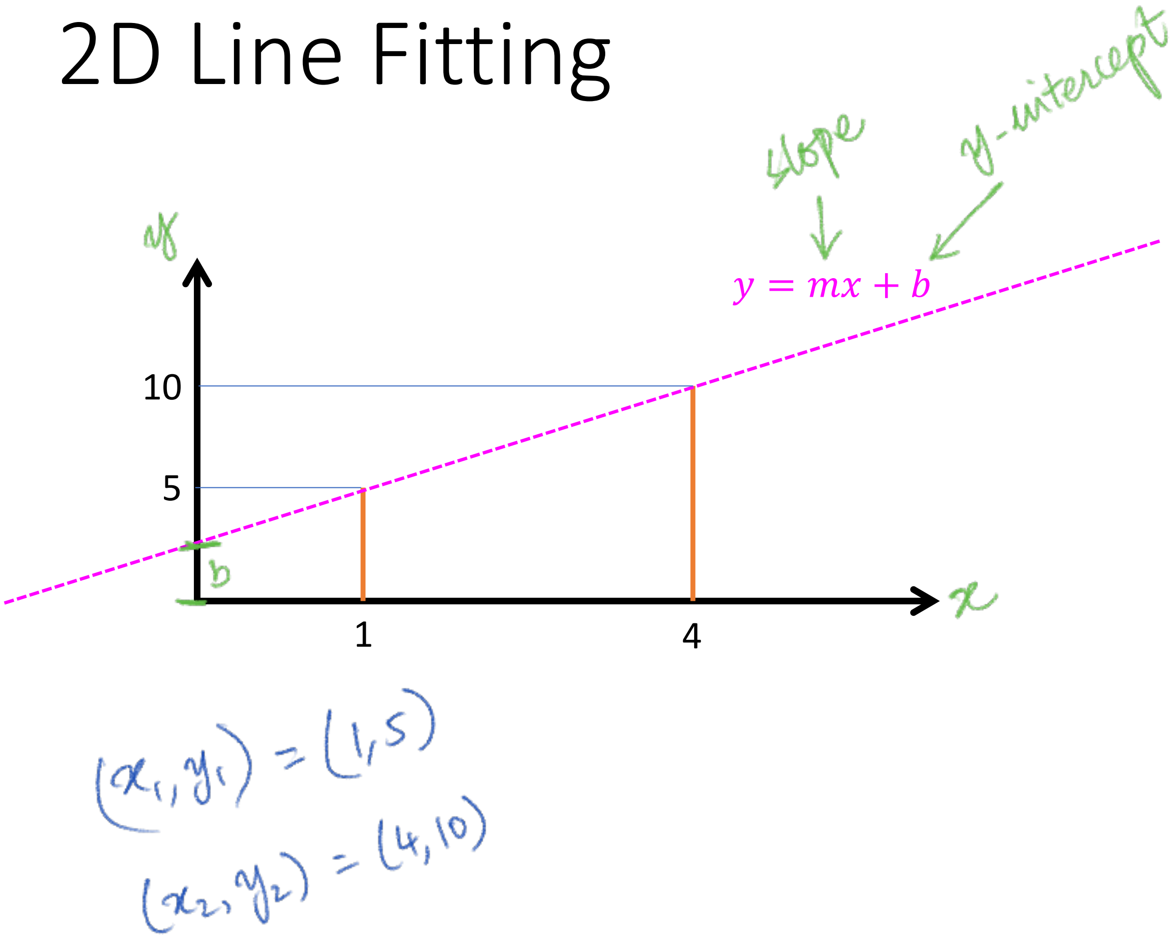
Nearest-Neighbor Interpolation

Nearest-Neighbor Interpolation

Linear Interpolation



2D Line Fitting

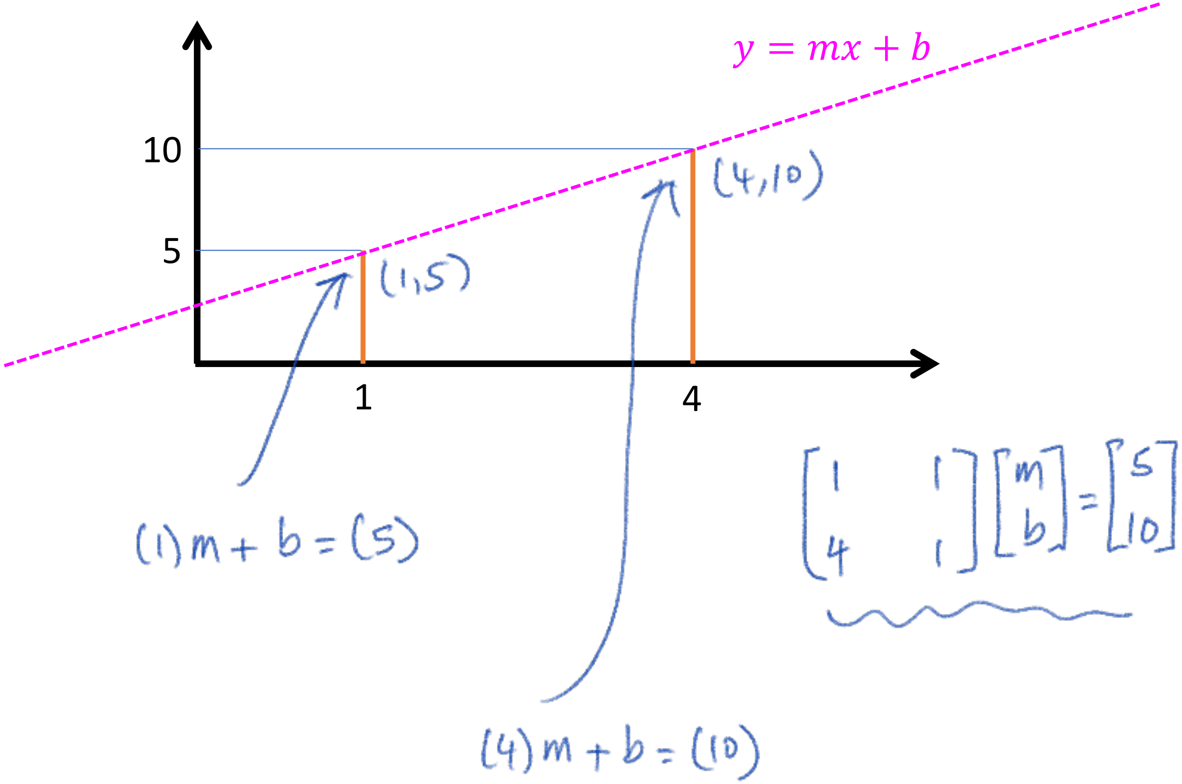


2D Line Fitting

A line between (x_1, y_1) and (x_2, y_2) is given by:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

2D Line Fitting



2D Line Fitting

A line between (x_1, y_1) and (x_2, y_2) is given by:

Matrix form

Re-write

$$x_1 m + b = y_1$$

$$x_2 m + b = y_2$$

as

$$A \vec{x} = \vec{b}$$

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

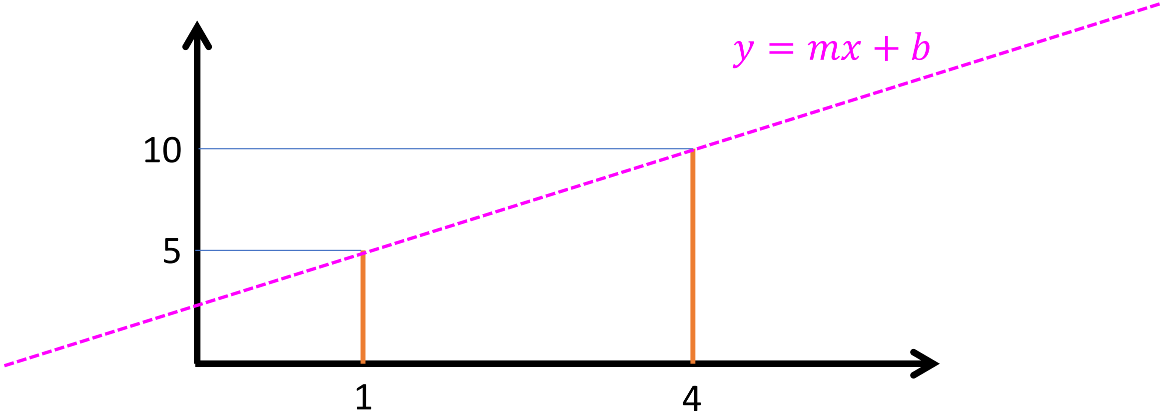
$$\vec{x} = A^{-1} \vec{b}$$

$$\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2D Line Fitting



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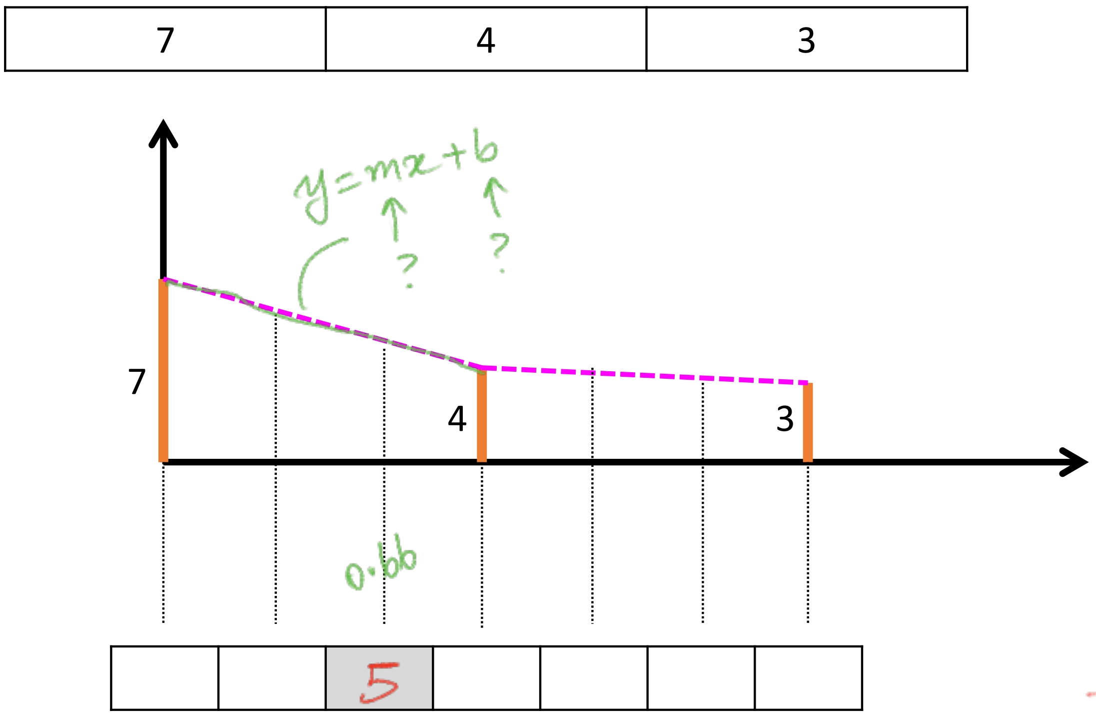
Practice Question

Estimate (fit) the dotted-line shown on the left.

Linear Interpolation



Example

Compute the value for shaded pixel?

1. Sampling location = 0.66
2. Setup line:

$(x_1, y_1) = (0, 7)$

$(x_2, y_2) = (1, 4)$

$$\frac{y-7}{4-7} = \frac{x-0}{1-0}$$

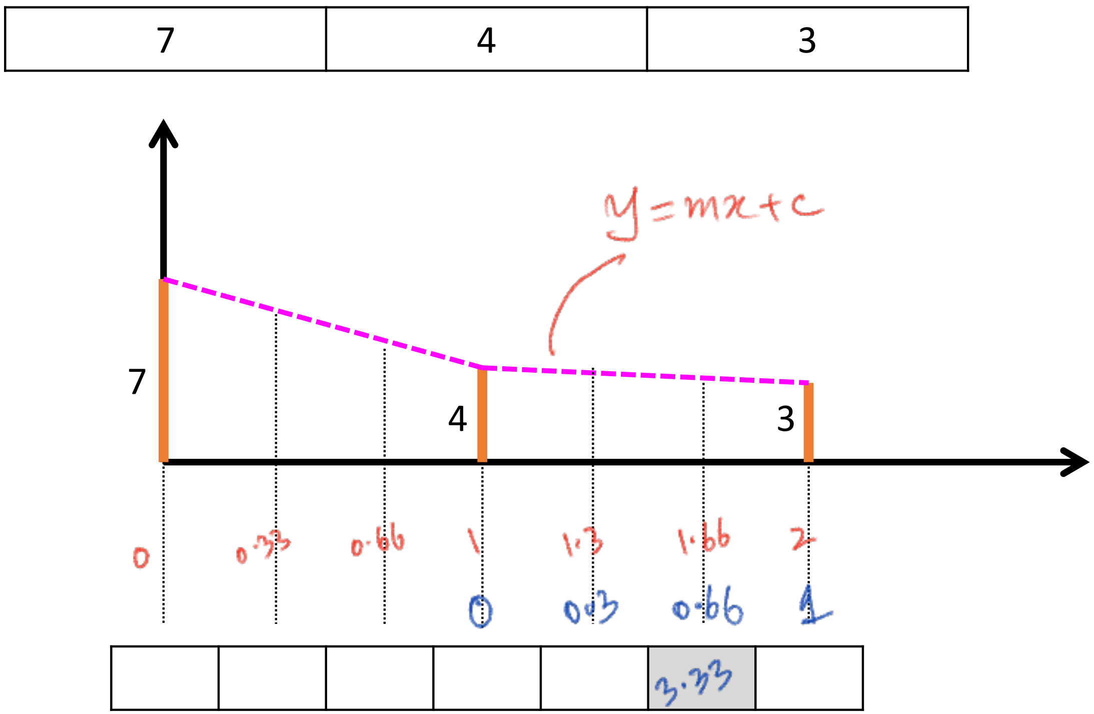
$$\Rightarrow y - 7 = -3x$$

$$\Rightarrow y = -3x + 7$$

$$y = -3\left(\frac{2}{3}\right) + 7$$

$$\Rightarrow y = 5$$

Linear Interpolation



Practice Question

Compute the value for shaded pixels?

① Sampling location = 1.66 → 0.66

② Setup the line.

$(x_1, y_1) = (1, 4) \rightarrow (0, 4)$

$(x_2, y_2) = (2, 3) \rightarrow (1, 3)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - 4}{3 - 4} = \frac{x - 0}{1 - 0}$$

$$\Rightarrow y - 4 = -x$$

$$\Rightarrow y = -x + 4$$

Images as surfaces

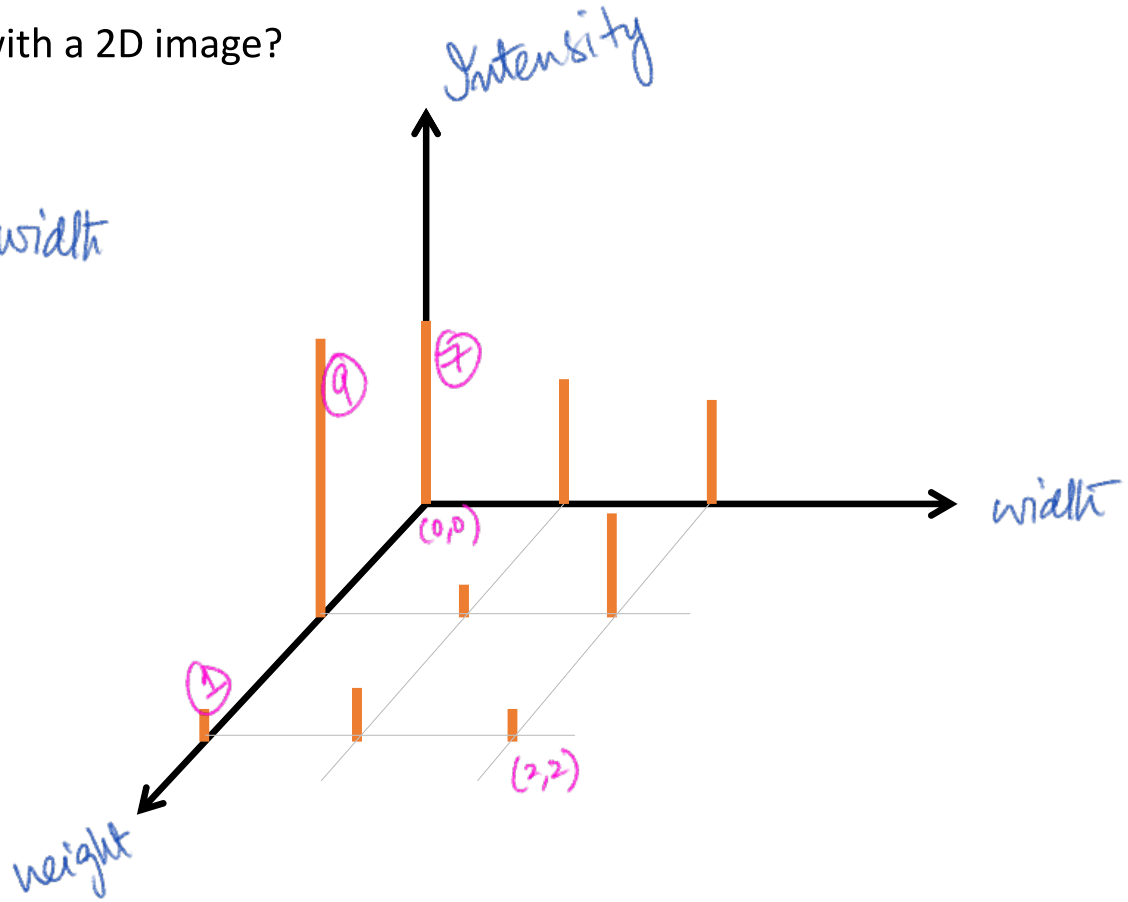
Images are not just 1D. How do we deal with a 2D image?

Consider the following 3x3 2D image.

7	4	3
9	1	3
1	2	1

width

height



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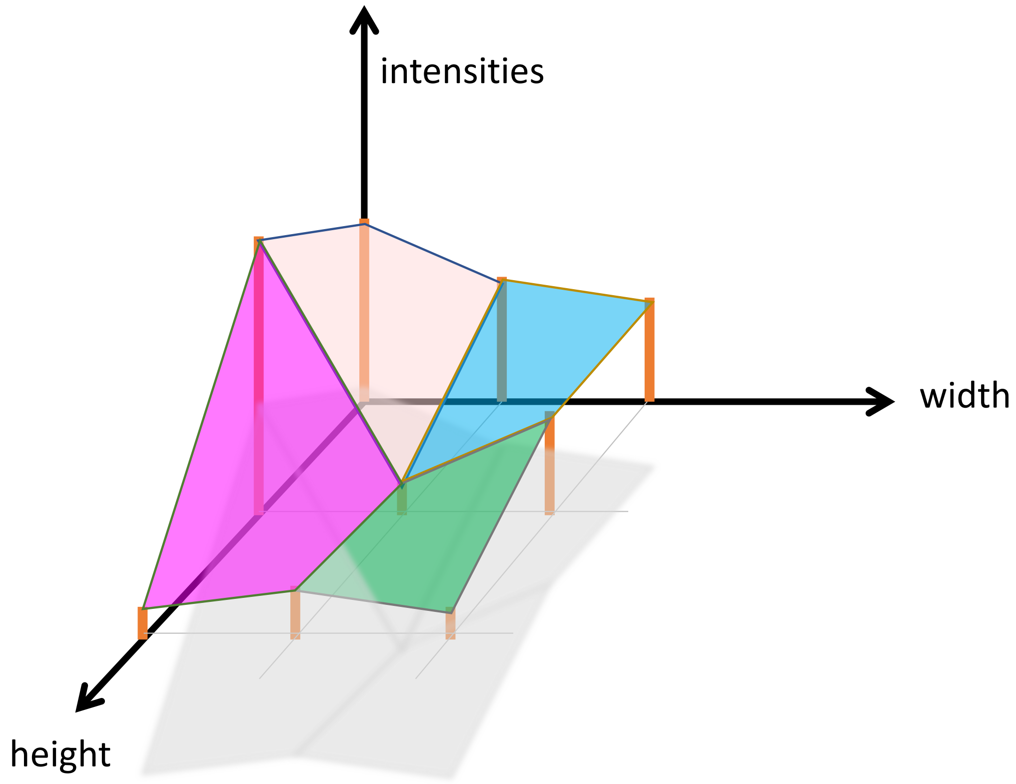

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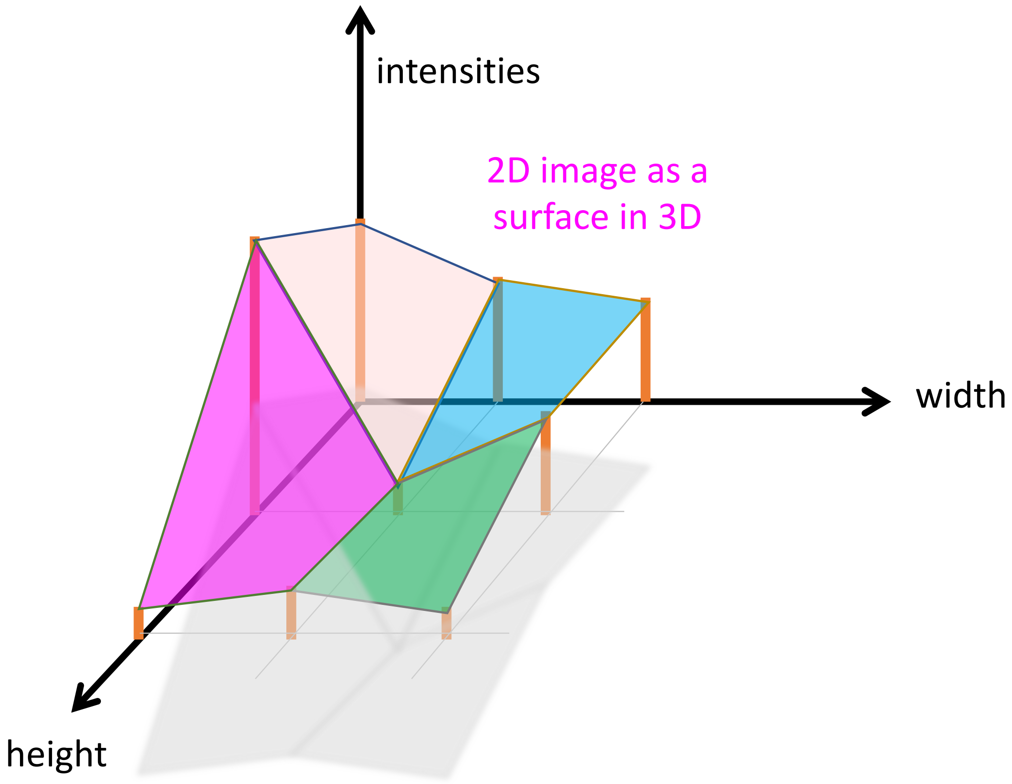

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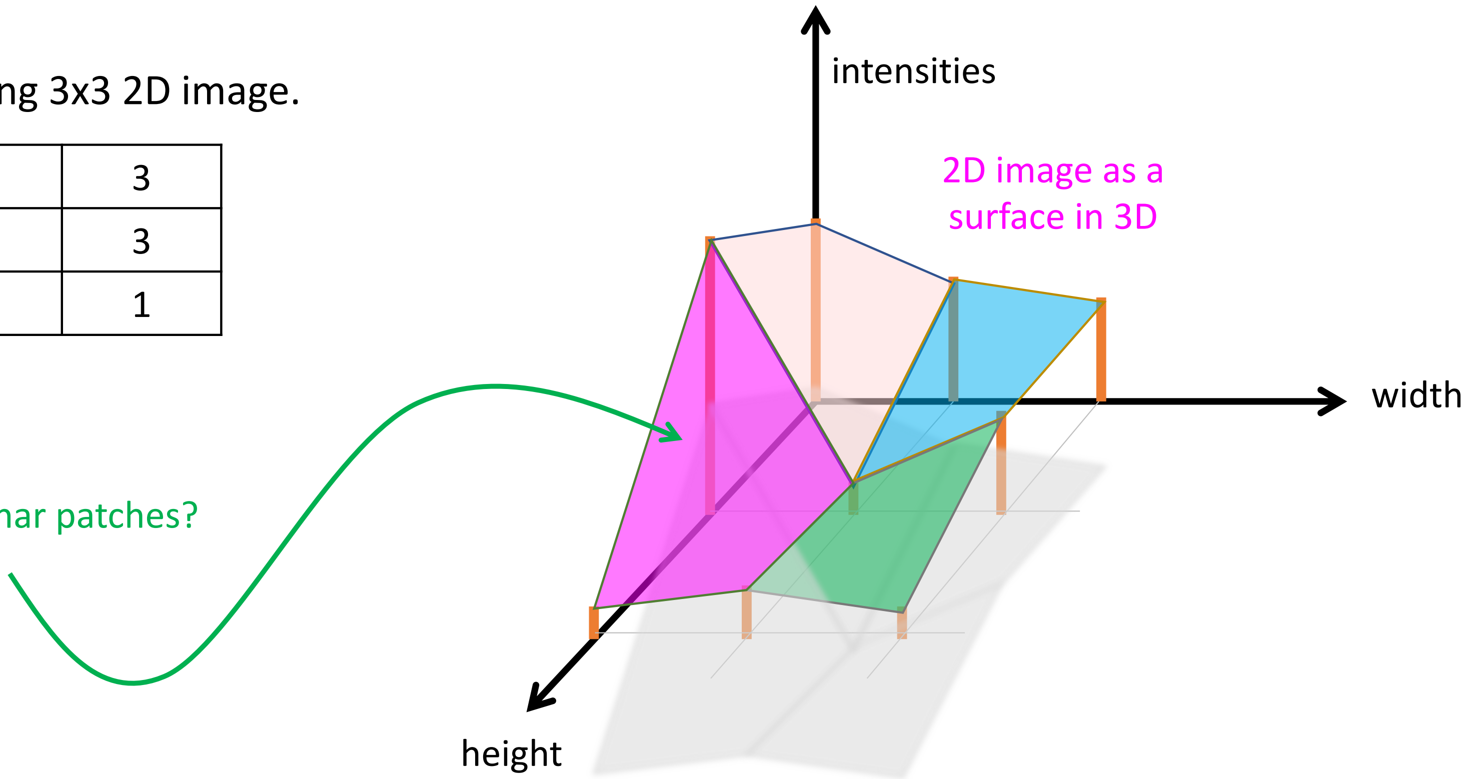
Are these planar patches?



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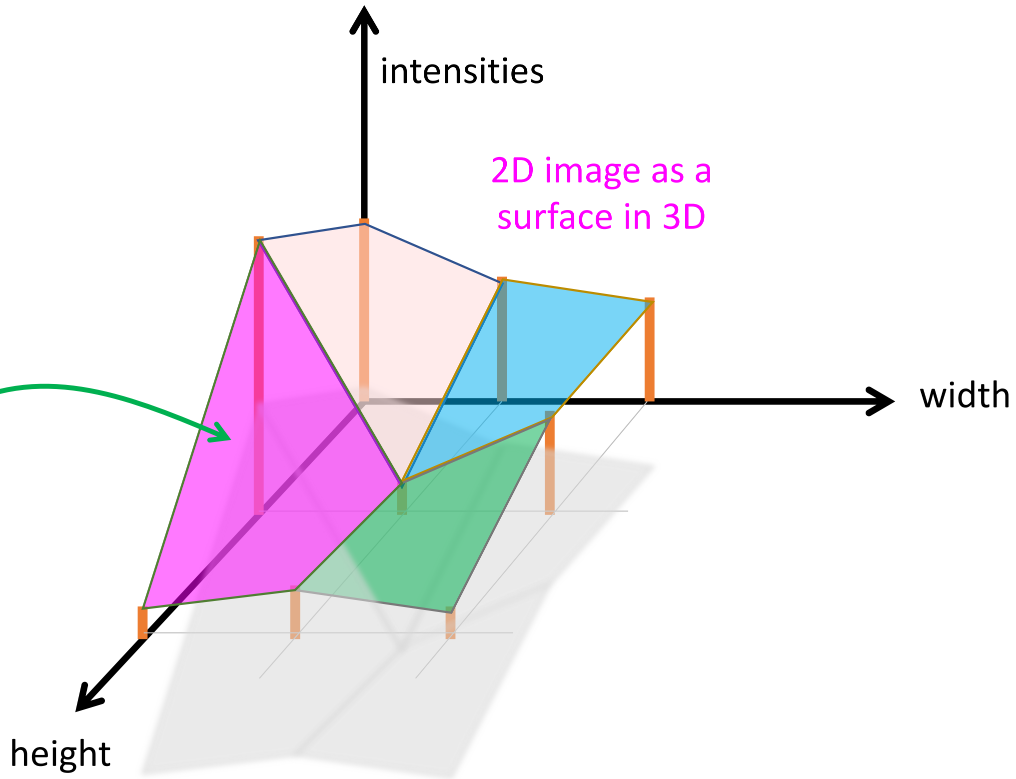
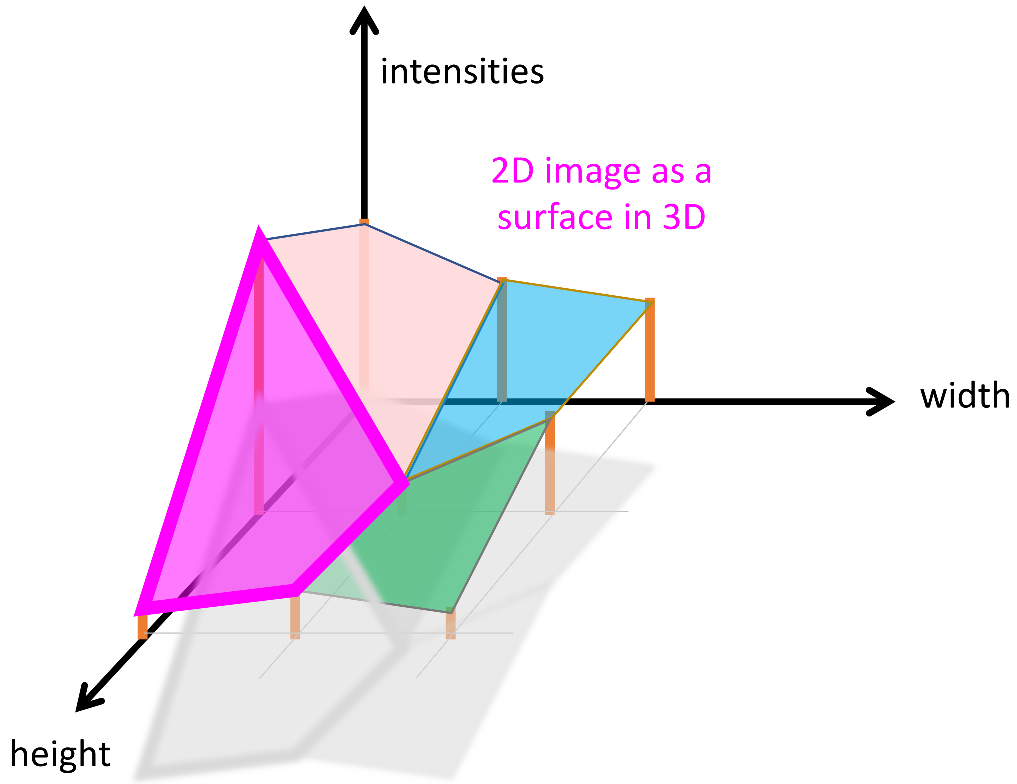
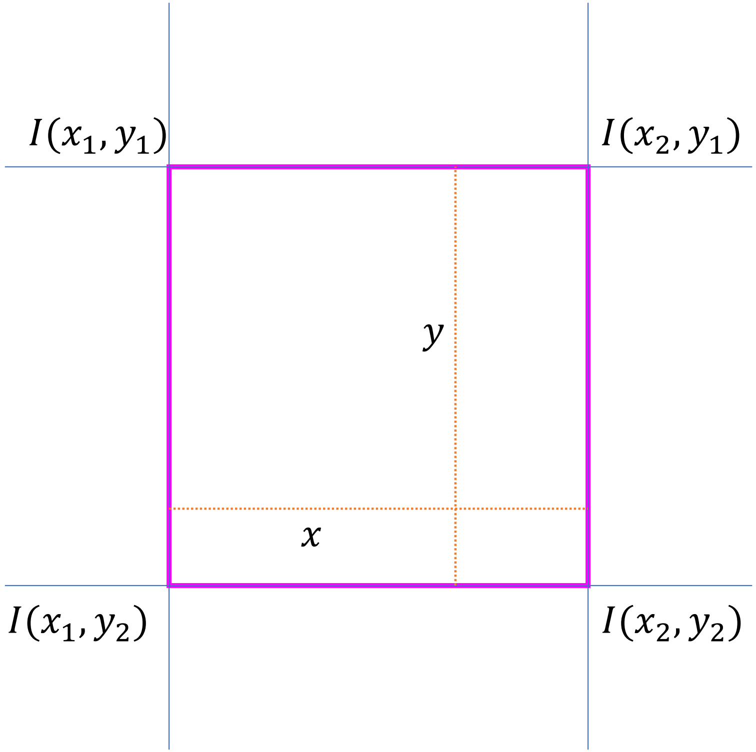


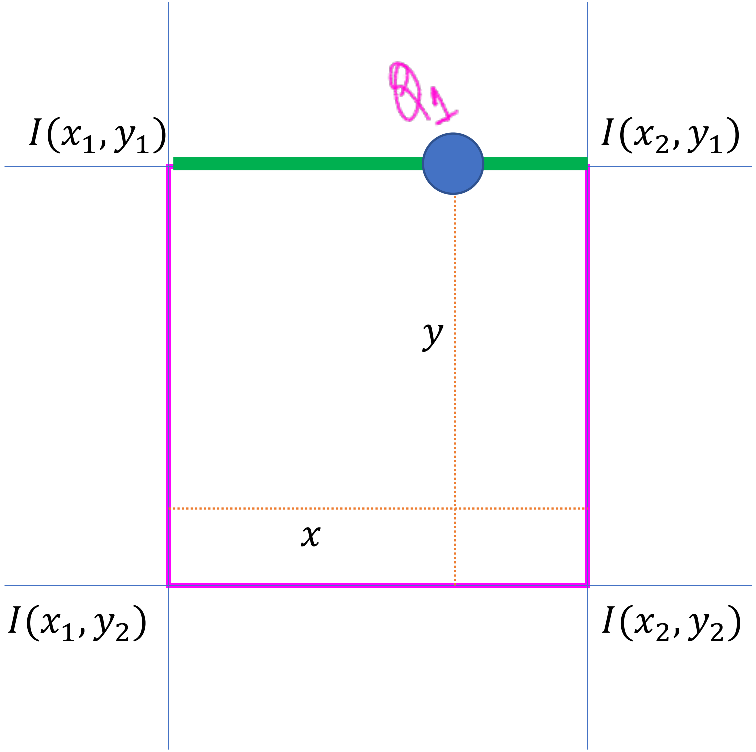
Are these planar patches? **No**

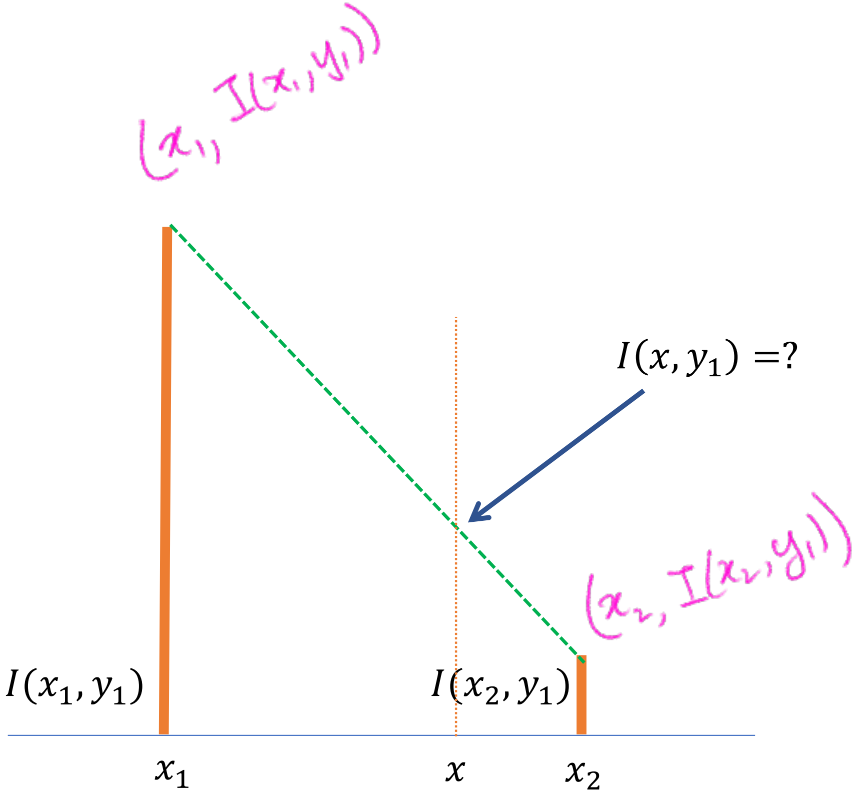
Images as surfaces





Bi-linear interpolation





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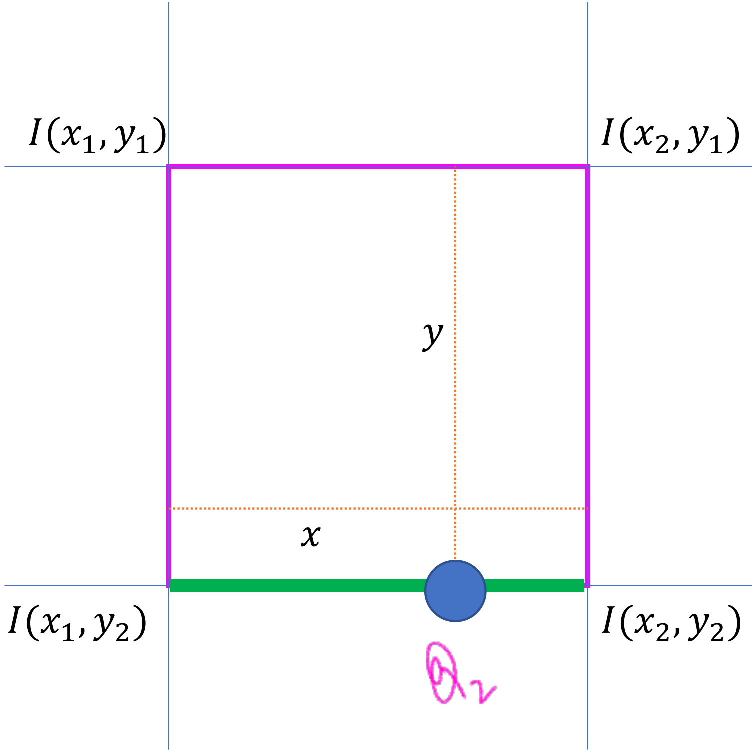

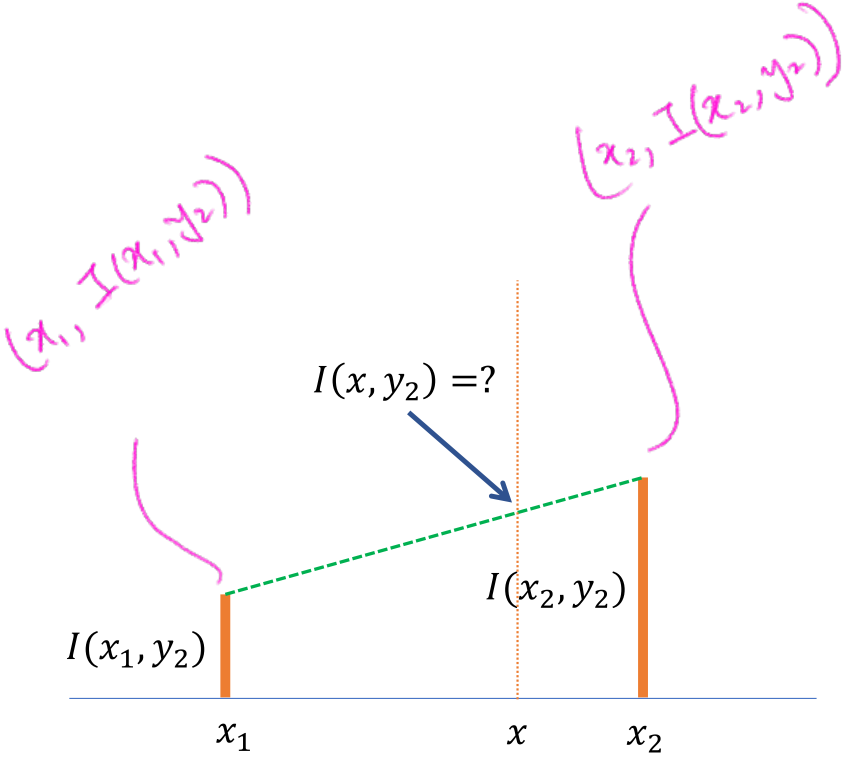

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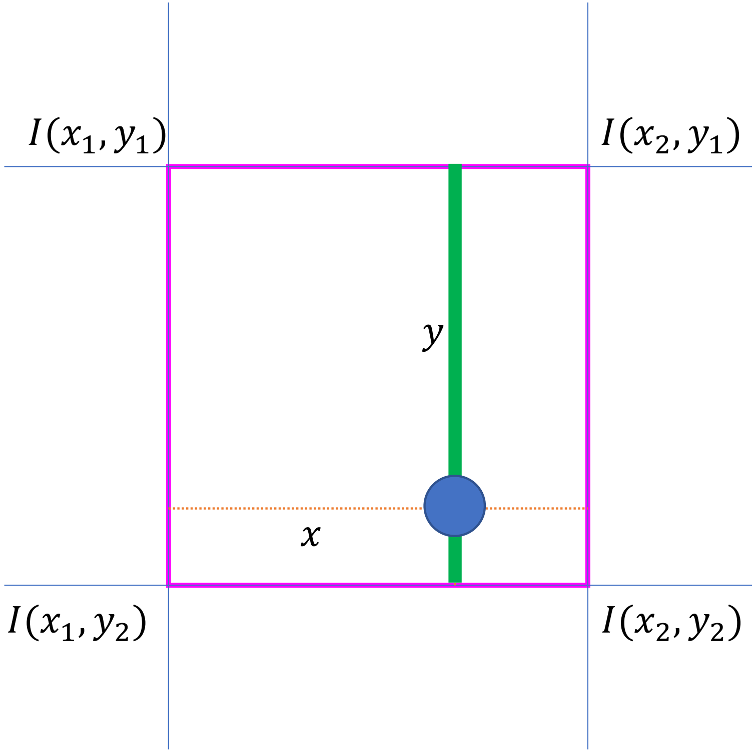

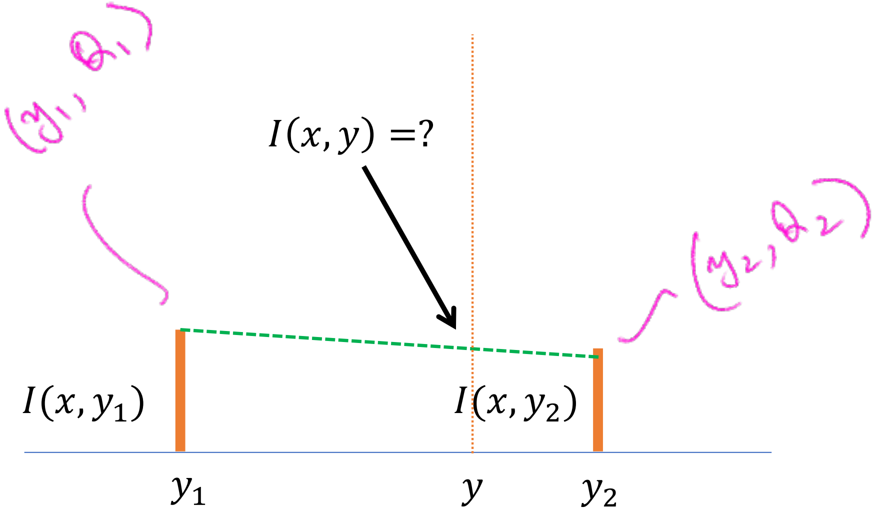

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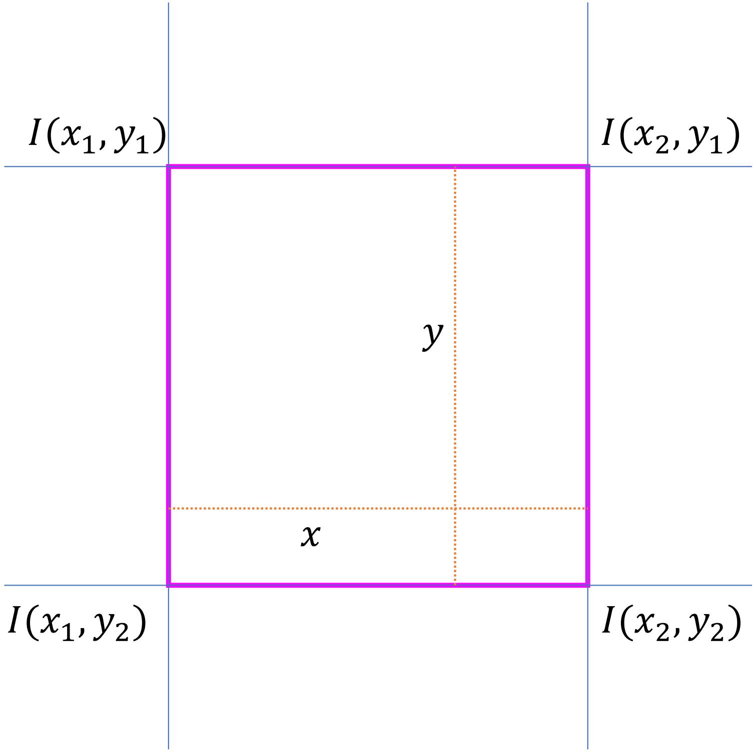


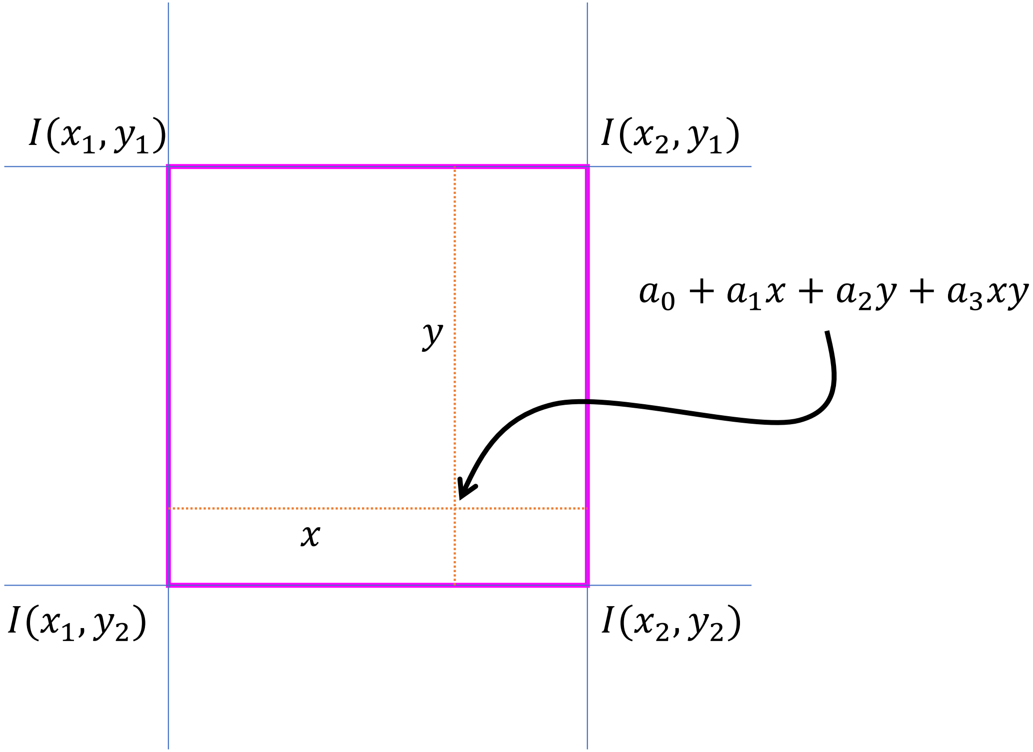
Multi-linear polynomial

$$f(x, y) = a_0 + a_1 x + a_2 y + a_3 xy$$

Then for $i, j \in [1,2]$

$$I(x_i, y_i) = a_0 + a_1 x_i + a_2 y_j + a_3 x_i\, y_j$$

Bi-Linear interpolation



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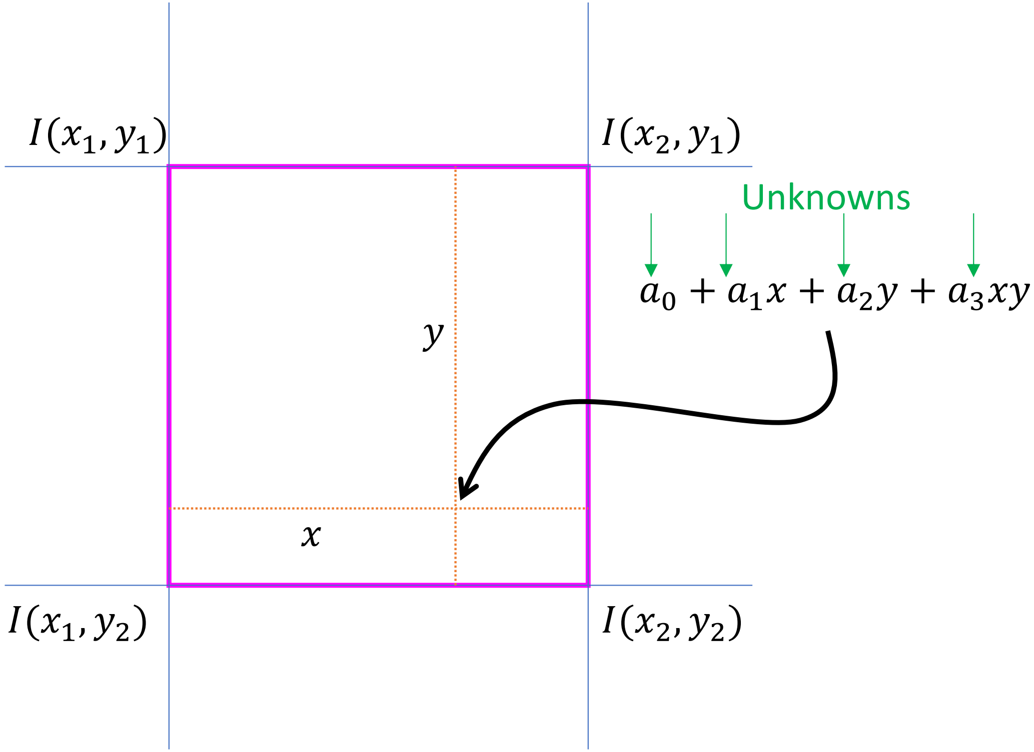

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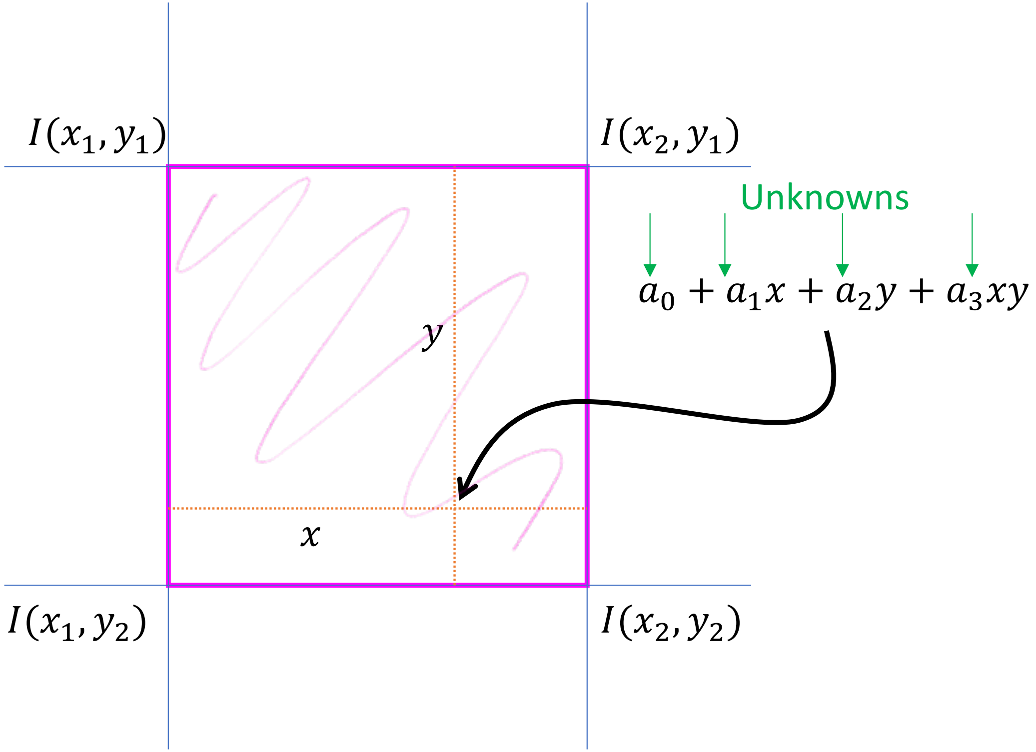

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Solve for the unknowns using the following system of equations

$$\begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_1 & x_2 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ 1 & x_2 & y_2 & x_2 y_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_2 \end{bmatrix} = \begin{bmatrix} I(x_1, y_1) \\ I(x_2, y_1) \\ I(x_1, y_2) \\ I(x_2, y_2) \end{bmatrix}$$

Bilinear interpolation: Pros

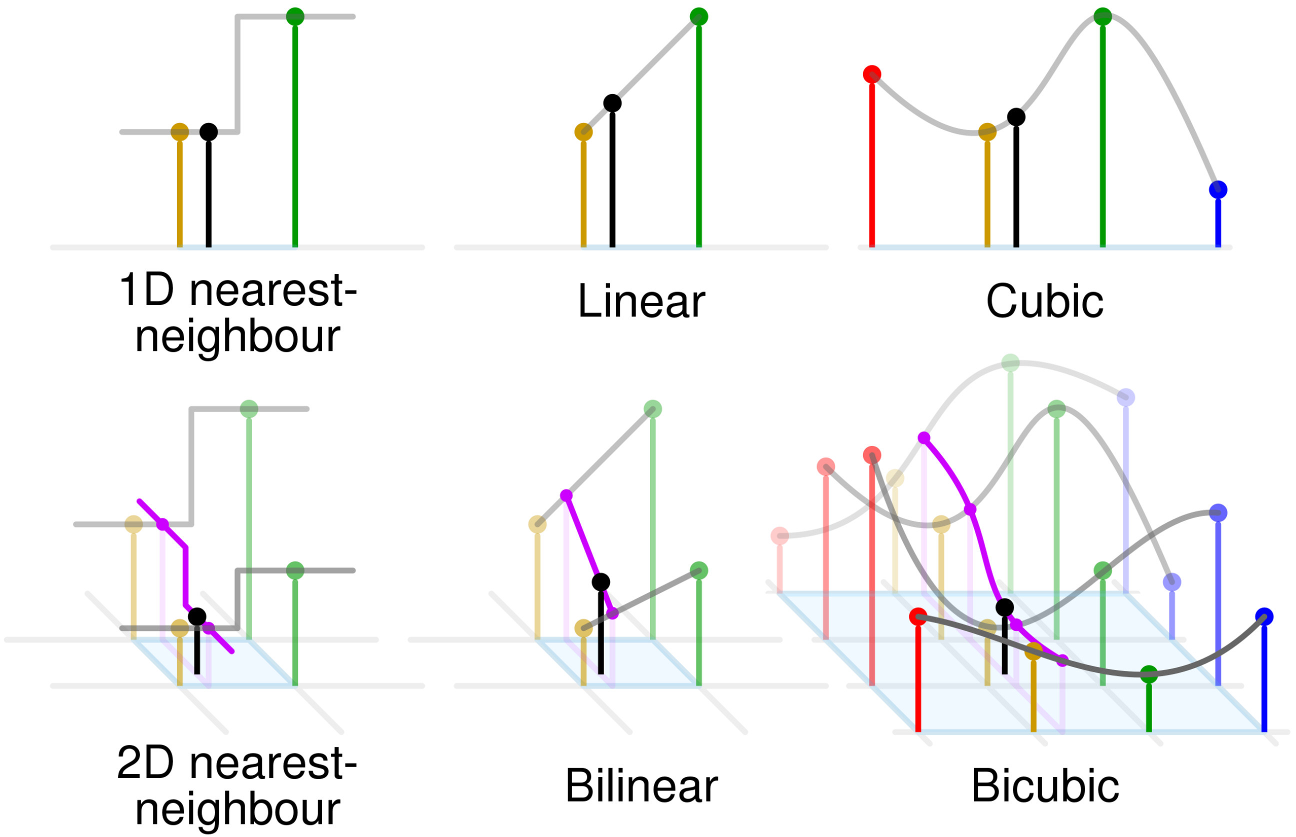
- Smoothing Effect, which helps reduce jagged edges and pixelation.
- Simple to Implement, requires fewer calculation and computational inexpensive as compared to other mathods
- Maintains linearity between the known data points, which can be desirable in certain applications, such as computer graphics.

Bilinear interpolation: Cons

- Loss of sharpness and fine details
- Color artifacts
- No consideration for high-frequency components
  - Not suitable for images with intricate patterns or textures
- Not ideal for large scaling
- Limited accuracy and it may not be suitable for photometric applications

Summary

- Image interpolation methods
- Nearest neighbor interpolation
- Bilinear interpolation



(CMG Le. Wikipedia)