Image Gradients

Computational Photography (CSCI 3240U)

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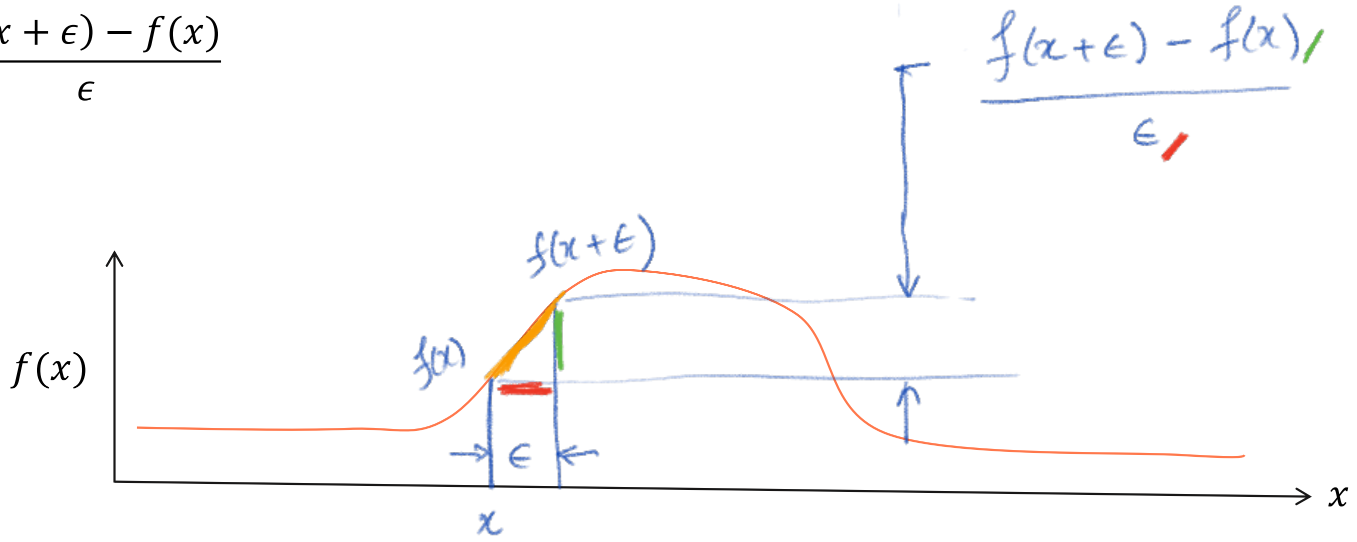
Today's lecture

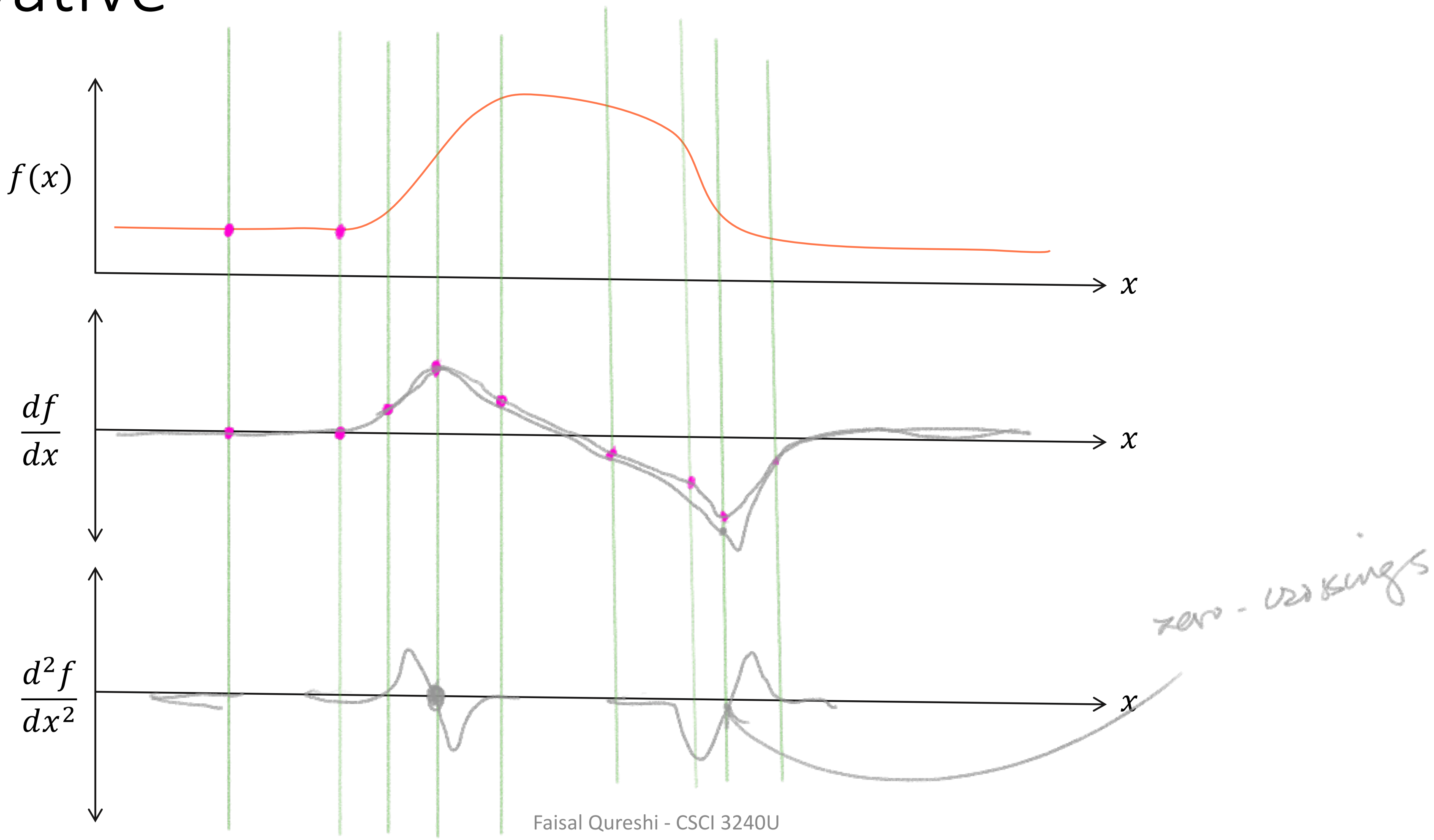
- Why do we care about image gradients?
- Computing image gradients
- Sobel filters
- Gradient magnitude and directions
- Visualizing image gradients

Derivative

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

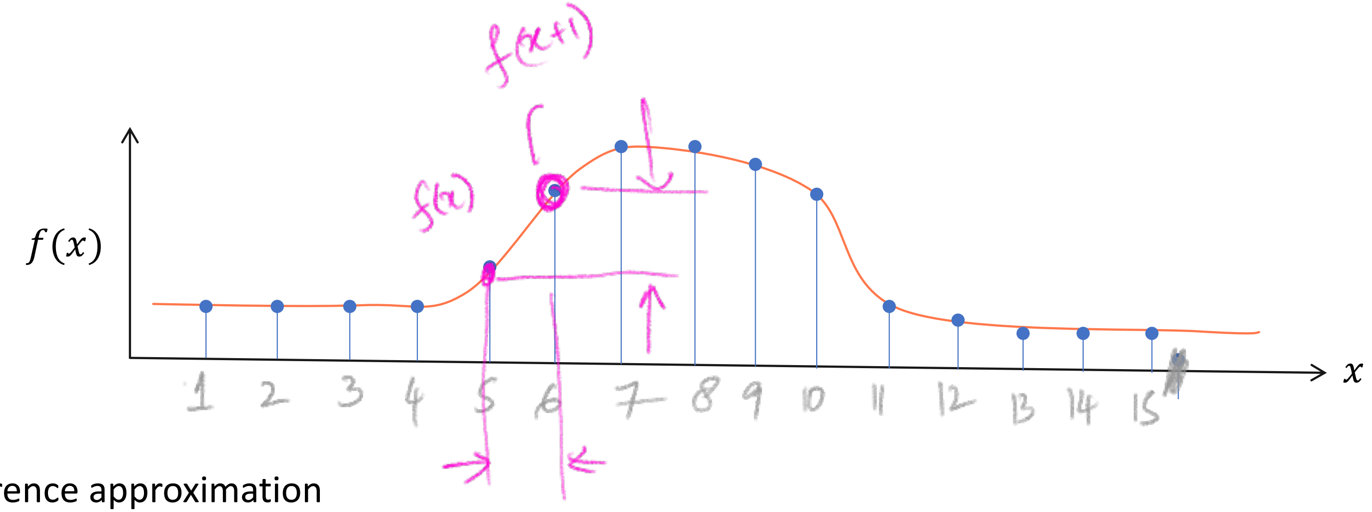


Derivative



Derivative

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$



Finite-difference approximation

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

Use finite difference approximation to compute image derivatives

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

$I =$

1	1	9	8	6	0	0
0	1	2	3	4	5	6

$I' =$

0	8	-1	-2	-6	0	?

$I'' =$

8	-9	-1	-4	6	?	?

Zero-crossing between x=0 and x=1 and again between x=3 and x=4

Use finite difference approximation to compute image derivatives

$$\frac{df}{dx} \approx \frac{\Delta f}{\Delta x} = \frac{f(x+1) - f(x)}{(x+1) - x} = f(x+1) - f(x)$$

$I =$

0	1	2	3	4	5	6
1	**1**	**9**	**8**	**6**	**0**	**0**

$I' =$

0	**8**	**-1**	**-2**	**-6**	**0**	**?**

kernel

DO NOT FORGET TO FLIP THE KERNEL !!

$I * [1, -1] =$

0	8	-1	-2	-6	0	?

convolution operator

[1, 0, -1]

Partial derivatives

$I \neq f(x)$

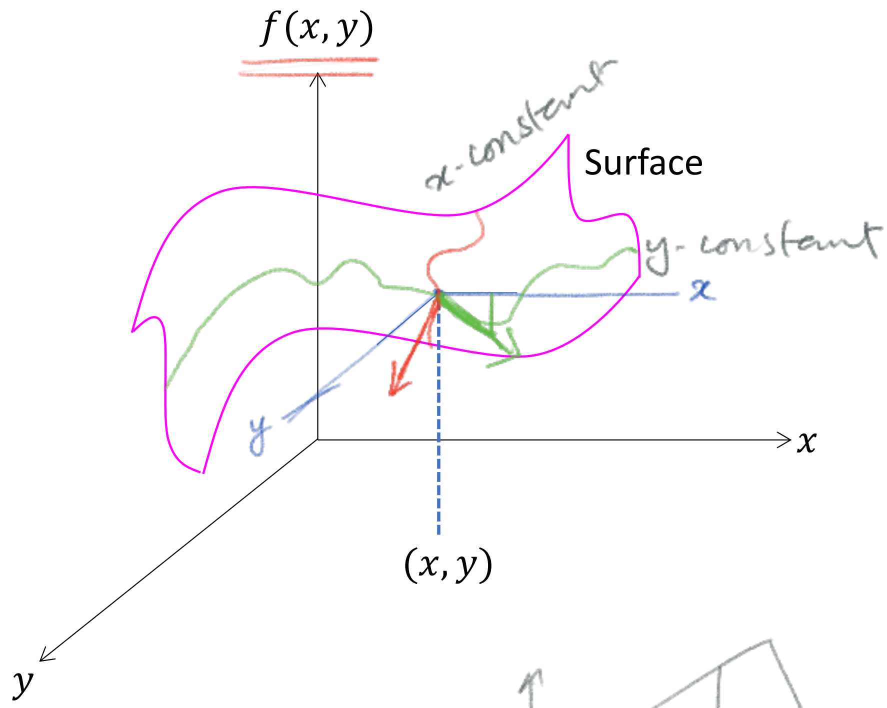
$I_{grayscale} = f(x, y)$

$I_{rgb} = f(x, y, c)$

$I_{HSI} = f(x, y, \lambda_1, \lambda_2, \ldots, \lambda_{100})$

$\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y}$

Partial derivatives.



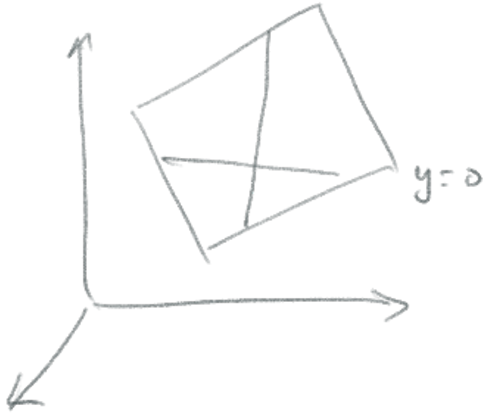


Image derivatives in x and y directions

$I =$

1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

→ x

↓ y

$I \rightarrow \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix}$ ← gradient ∇I

$I_x = I * [1, -1] =$

0	8	-1	-7	
0	0	0	0	
2	2	3	-7	
-2	-1	6	-2	

$I_y = I * [1, -1]^T =$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

7	7	-1	0	7
-7	-5	-3	0	-7
4	0	-3	0	5

Image gradient ∇I

$$\nabla I = \left[\frac{\partial I(x,y)}{\partial x}, \frac{\partial I(x,y)}{\partial y}\right]$$

$I_x =$ ($\frac{\partial I}{\partial x}$)

0	8	-1	-7	
0	0	0	0	
2	2	3	-7	
-2	-1	6	-2	

$I_y =$ ($\frac{\partial I}{\partial y}$)

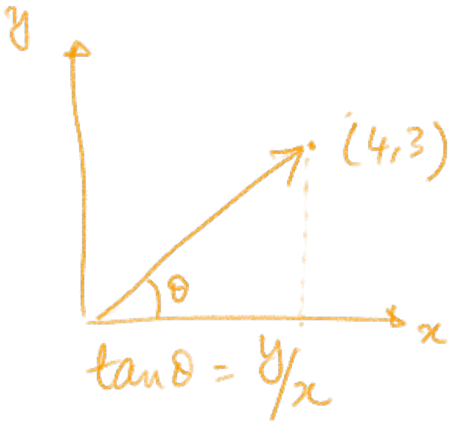
7	7	-1	0	7
-7	-5	-3	0	7
4	0	-3	0	5

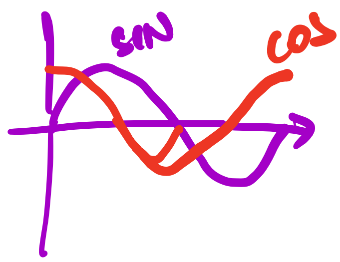
$I =$

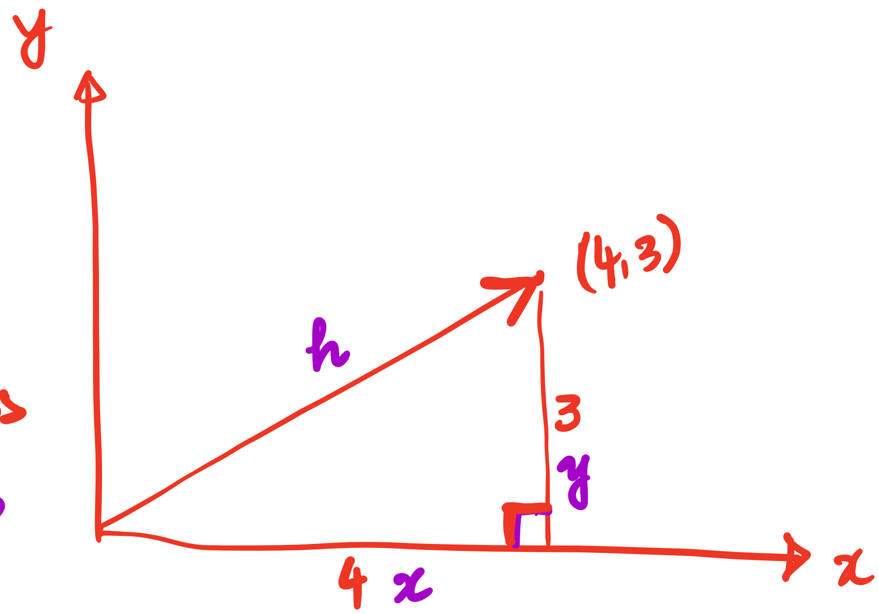
1	1	9	8	1
8	8	8	8	8
1	3	5	8	1
5	3	2	8	6

x

y







$$x = h\cos\theta$$
$$y = h\sin\theta$$

$$h^2 = x^2 + y^2$$
$$h = \sqrt{x^2 + y^2}$$
↑ magnitude.

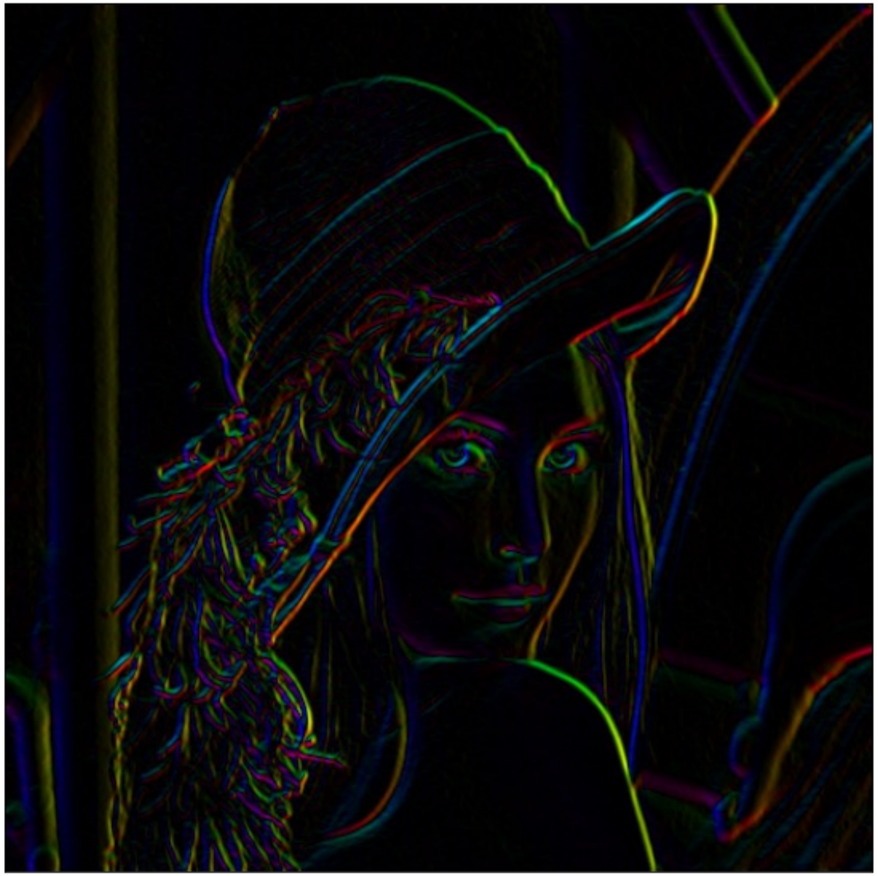
$$h = \frac{x}{\cos\theta}$$
$$h = \frac{y}{\sin\theta}$$

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta}$$
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$
$$\Rightarrow \tan\theta = \frac{y}{x}$$
$$\Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Gradient direction and magnitude

$$\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

$$\theta = \tan^{-1}\left(\frac{\partial I}{\partial y} \Big/ \frac{\partial I}{\partial x}\right)$$



Filters for computing Image derivatives

- Sobel filters

$$H_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } H_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- Prewire

$$H_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } H_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

- Roberts

$$H_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } H_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Image noise and gradients

Scheme 1.

$I_G = G * I$

$I_{Result} = D * I_G$

$I_{result} = D * (G * I)$

Scheme 2.

$I_{result} = (D * G) * I$

Deriv. of Gaussian.

$G_D = D * G$

$I_{result} = G_D * I$

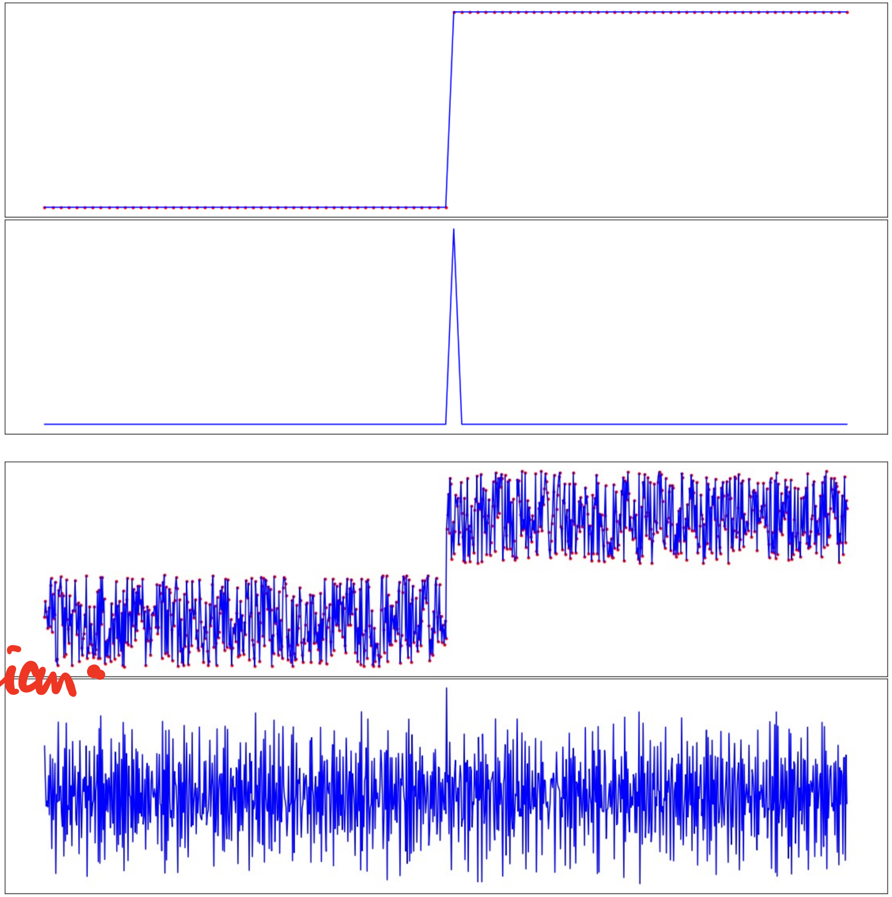
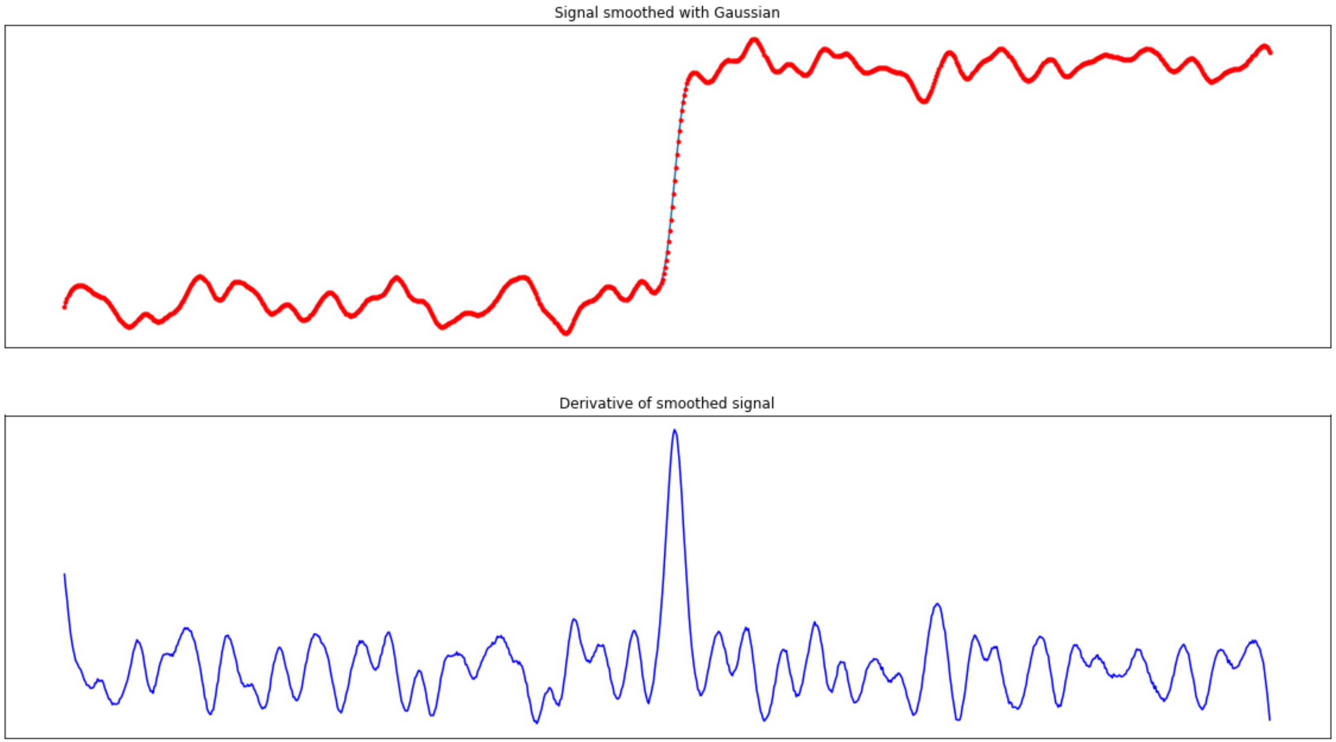


Image noise and gradients



Summary

- Image gradients
- Finite-difference approximation filters
- Gradient magnitude and direction
- Image noise and gradients