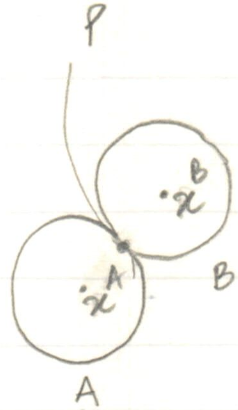
Rigid Body Collisions



Consider two balls A and B.

Radius of ball A: r^A
" B: r^B

Position of ball A: x^A
" B: x^B

Velocity of ball A: v^A
" B: v^B

Mass of ball A: m^A
" B: m^B

* There are no rotational effects.

Velocity of collision point P for ball A: $v^{AP} = v^A$
" B: $v^{BP} = v^B$

Collision Detection

- compute normal $n = (p^A - p^B)/|p^A - p^B|$
- compute relative velocity $v^{AB} = v^{AP} - v^{BP}$

$v^{AB} \cdot n = 0 \rightarrow$ resting contact
$< 0 \rightarrow$ collision imminent
$> 0 \rightarrow$ A and B moving away.

Collision Response

1. Use Newton's Law of Restitution for Instantaneous Collision with no friction.

   ↓

   impulse: an infinite force applied for a very short duration. Impulse is equal to the change in momentum.

$$J = \Delta P$$
$$= mv_1 - mv_2$$
$$\Rightarrow v_2 = v_1 - \frac{J}{m}$$

v_1 is the velocity of a body before collision and v_2 is the velocity of the body after collision.

no gravity

no friction

conservation of momentum.

J for first body is equal to $-J$ of the second body.

② Empirical model of frictionless collisions.

$$v_2^{AB} \cdot n = -e\, v_1^{AB} \cdot n$$

$$\downarrow$$

Relative velocity between the two bodies after collision (along the collision direction n) is a function of the relative velocity between the two bodies before collision.

e is called the Coefficient of Restitution.

- $e = 1$ elastic collision, no loss of kinetic energy.
- $e = 0$ perfectly inelastic, total loss of kinetic energy.
- $0 < e < 1$ some loss of kinetic energy.

Given ① and ② above, lets solve for the velocities of balls A and B after collisions.

(Refer pg. 1) v_1^{AP} = velocity of P in A, before collision

v_1^{BP} = velocity of P in B before collision.

v_2^{AP} = velocity of A after collision

v_2^{BP} = velocity of B after collision

From 1

$$v_2^{AP} = v_1^{AP} + \frac{jn}{m^A} \quad \text{— A}$$

$$v_2^{BP} = v_2^{BP} - \frac{jn}{m^B} \quad \text{— B}$$

Subtract B from A.

$$\left(v_2^{AP} - v_2^{BP}\right) = \left(v_1^{AP} - v_2^{BP}\right) + \left(\frac{1}{m^A} + \frac{1}{m^B}\right) jn$$

$$\Rightarrow \quad v_2^{AB} = v_1^{AB} + \left(\frac{1}{m^A} + \frac{1}{m^B}\right) jn \quad \text{— C}$$

From 2

$$v_2^{AB} \cdot n = -e v_1^{AB} \cdot n \quad \text{— D}$$

Using C and D

$$-e v_1^{AB} \cdot n = v_1^{AB} \cdot n + \left(\frac{1}{m^A} + \frac{1}{m^B}\right) j \underbrace{n \cdot n}_{1}$$

unit vectors

$$\Rightarrow j = \frac{-(1+e)\, v_1^{AB} \cdot n}{\left(\frac{1}{m^A} + \frac{1}{m^B}\right)}$$

Strategy

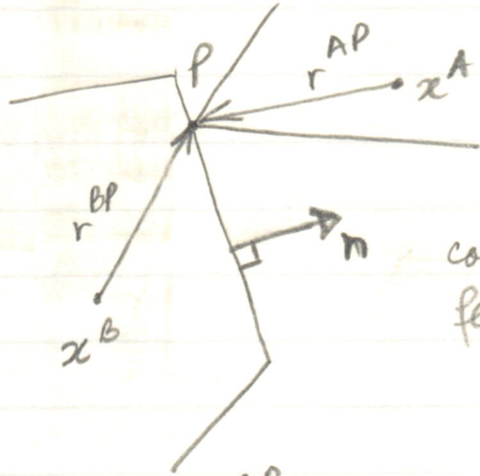
You are given $v_1^A, v_1^B, m^A, m^B, e, x^A, x^B$
Compute n and then compute j

j for $A = -j$ for B.

Compute $v_2^A = v_1^A - \frac{jn}{mA}$

and $v_2^B = v_1^B - \frac{jn}{mB}$

Rigid Body Collision With Rotational Effects.



collision will be felt along n.

Collision point P in A : r^{AP}
" B : r^{BP}

Before collision

Velocity of P in A: $v_1^A + \omega_1^A \times r^{AP} = v_1^{AP}$
" B: $v_1^B + \omega_1^B \times r^{BP} = v_1^{BP}$

linear

angular

we are interested in v_2^A, v_2^B, ω_2^A and ω_2^B i.e., velocities after collision.

Mass of body A: m^A and inertia tensor: I^A (world)
" B: m^B " I^B "

From 1 pg. 2

$$v_2^A = v_1^A + \frac{jn}{m^A} \quad \text{and for rotational component}$$

$$\omega_2^A = \omega_1^A + (I^A)^{-1} r^{AP} \times jn$$

Similarly for body B

$$v_2^B = v_1^B - \frac{jn}{m^B}$$

$$\omega_2^B = \omega_1^B + (I^B)^{-1} r^{BP} \times jn$$

We can use 1 and 2 to get

$$j = \frac{-(1+e)\, v^{AB} \cdot n}{\left(\frac{1}{m^A} + \frac{1}{m^B}\right) + n \cdot (I^A)^{-1}(r^{AP} \times n) \times r^{AP} + n \cdot (I^B)^{-1}(r^{BP} \times n) \times r^{BP}}$$

Strategy.

1. compute n.
2. compute j.
3. compute v_2^A, ω_2^A, v_2^B and ω_2^B.
4. update the state of the two bodies.
5. Continue with simulation.